

LOGIC is traditionally described as the theory of deduction. In a correct deduction we pass from sentences called *premises* to a sentence called *conclusion*, usually introduced with *so*, *hence* or *therefore*, so that truth or provability is being preserved. A correct deduction is done in conformity to laws based on the meaning of the words that occur in the premises and the conclusion. The primary task of logic is to systematize and investigate these laws. It is usually put into the definition of logic that it is concerned with *correct* deductions, but separating the correct deductions from the incorrect ones involves considering both.

With more reason, it is often put into the definition of logic that it is concerned with *formal* deduction. This means that logic is concerned with laws of correct deduction based uniquely on the meaning of some particular words called *logical constants*. In describing logical laws one keeps these constants and the rest is replaced by variables, which makes logical *form* appear. In algebraic laws, one has analogously constants for operations and variables for numbers. The usual logical constants are the connectives *and*, *or*, *if*, *if and only if* and *not*, which are studied in *propositional* logic, and the quantifier expressions *for every* and *for some*, and identity (i.e. the relational expression *equals*), which, together with the connectives, are studied in *predicate* logic. If logic is indeed the theory of deduction, then logical constants should presumably be distinguished from other words by the special role they play in deduction.

A close relative of the word *deduction* is *proof*, when it refers to a correct deduction where the premises are true, or acceptable in some sense. A more distant relative is *argument*, because an argument may, but need not, be deductive. The study of argumentation in general belongs more to rhetoric than to logic, and is far less systematic and exact. Even further removed from the science of logic considered here, there are various philosophical endeavours that share with logic not much more than the name, such as, for example, the logic of scientific discovery, or the logic of colour words, or Hegel's logic.

As the theory of deduction, logic was studied in philosophy since the time of Aristotle, whose teachings, exposed in a collection of his writings called the *Organon*, dominated the subject until the 19th century. In that century, the work of George Boole and especially Gottlob Frege inaugurates the tradition of contemporary logic, which is a prosperous, still not very old, field of mathematics. Many deep mathematical structures, of extreme importance and beauty, have been unearthed in it.

Contemporary logic has close ties with philosophy, but it is nevertheless a field of mathematics, and not of philosophy. To distinguish it from the older logic with Aristotelian roots, this science was called *symbolic logic* or *mathematical logic*. The former name, though pretty appropriate, is not very common any more, while in the latter the adjective *mathematical* is redundant, because there is no living kind of logic today except the mathematical one. (The expression *philosophical logic* is used not for a separate subject, but for some investigations within mathematical logic, or at its borders, deemed to be interesting to philosophy.)

In ancient Greece, besides Aristotle and his followers, philosophers of the Megarian and Stoic schools made contributions to logic, which belong to propositional logic. These contributions—in particular, the invention of *material implication* by Philo of Megara—are more significant from the standpoint of contemporary logic than any teaching of Aristotle. (Material implication is the *if* connective such that *if A, B* means *not A or B*, or *not both A and not B*.) It is remarkable that ancient Greek mathematicians were not concerned with logic, though mathematics, as in Euclid's *Elements*, was conceived as a rigorous deductive science.

An ancient logical tradition existed in India and China, but with no visible connection with the Greek tradition and with contemporary logic.

Logic in the Aristotelian tradition flourished in medieval Western universities, where it belonged as much to theology as to philosophy. Read with the spectacles of contemporary logic, medieval logic texts were found to contain interesting contributions. With the rise of modern science and mathematics in the 17th century this scholastic logic fell into disrepute. It was considered by leading minds—not only of scientists and mathematicians, but also of philosophers—as a useless pedantic subject, and after the close of the Middle Ages no significant contributions seem to have been made in it. As an interesting phenomenon for the sociology of science, scholastic logic remained entrenched nevertheless in the philosophical curriculum of universities and grammar schools, and the rise of contemporary logic in the 19th century did not eliminate it completely until these days.

The greatest precursor of contemporary logic was Leibniz, in writings that remained unpublished until the 20th century. His lasting achievement is the analysis of identity.

If before the middle of the 19th century we have the prehistory of logic, the period from that time on until the First World War could be called the ancient history of logic. A most important date of these ancient times is 1879, when Frege in his *Begriffsschrift* introduced what is today the standard conception of logical form, and thereby put logic on a new fruitful basis. It was not however before the second half of the 20th century, after Frege's discoveries were rediscovered and welded to others, that the year 1879 was taken as the end of the era of the *Organon* and the beginning of a new era.

The development of contemporary logic is inseparable from the development of *set theory* and studies in the *foundations of mathematics*. Georg Cantor inaugurated set theory in the 1870s, and Frege's work was motivated by his logicism, which is the doctrine that arithmetic, i.e. the theory of natural numbers, reduces to logic.

The period between the two world wars is the beginning of the classical time of logic (which at some point in the second half of the 20th century turned into "baroque"). This period saw the birth of the major branches of logic: *model theory*, *proof theory* and *recursion theory*. Set theory, whose axiomatization was finalized at the same time, is usually also counted as a branch of logic, but we cannot dwell on it here.

In the 1920s, David Hilbert and his close collaborator Paul Bernays gave to logic the physiognomy it has retained ever since. They opened the way for Kurt Gödel, who published in 1930 a *completeness* result for an axiomatization of predicate logic (Bernays obtained in 1917 an analogous easier result for propositional logic) and in 1931 his famous *incompleteness* results for axiomatic arithmetic. Gödel's completeness theorem is paradigmatic for much of the later work of logicians, and inaugurates model theory, the dominant branch of logic in the second half of the 20th century.

Hilbert's views about the foundations of mathematics are at the root of proof theory. Hilbert's and Bernays' pupil Gerhard Gentzen wrote in 1933 his *Untersuchungen über das logische Schließen*, a fundamental work for logic understood as the theory of formal deduction. Its importance, and relatively slow recognition, can be compared with those of Frege's *Begriffsschrift*. Gentzen's achievement is also an infinitary consistency proof for axiomatic arithmetic.

The 1920s saw the birth of *intuitionistic logic*, the major alternative logic, which formalizes the constructivist foundational views of L.E.J. Brouwer. This logic,

based on an implication different from material implication, is of particular importance for proof theory.

The 1930s saw the birth of recursion theory, whose principal achievement is a precise analysis of the notion of *computable* arithmetical function. The most widely known way to formulate this analysis is through *Turing machines*, an abstract infinite kind of computing machine, but there are several other equivalent approaches (the most natural of which is perhaps through Joachim Lambek's infinite abacus). Recursion theory clarifies the notions involved in *decidability* problems, which together with completeness problems are the standard kinds of problems solved by logicians.

As in mathematics in general, research in logic grew immensely in the second half of the 20th century. This massive growth tends unfortunately to be accompanied by loss of a clear and unified perspective, and deterioration in the standards of rigour and intellectual honesty.

With the close of the century, logic became increasingly drawn into theoretical computer science. Through this connection it became one of the most applied branches of mathematics. (So much for the "useless pedantic" subject of lore.) In *complexity theory*, which is tied to recursion theory and proof theory, logic is concerned with the major open problems of theoretical computing. Logic has influenced theoretical linguistics in the second half of the 20th century, and during the whole century it has also influenced a lot of philosophy oriented towards language. For example, *modal logic*, which is best characterized as the general theory of unary connectives, inspired many philosophers.

With this immense growth, the definition of logic as the science of formal deduction tends to become increasingly inadequate. While in its classical period in the 20th century logic could practically be identified with the foundations of mathematics, that description too ceased to be adequate. Set theory is nowadays better characterized as infinite combinatorics, and model theory is either that or a kind of algebra. It seems that only research within rather narrow and not very influential fields, like the field of proof theory on the border of category theory, could legitimately pretend today to belong to the science of formal deduction.

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