One More Reference on Self-Reference

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There is a plethora of papers about self-reference in the logical and paramathematical literature of our century, not to mention whole books. Some of them purport to establish important mathematical facts, though of a kind that leaves most of the so-called working mathematicians rather impassive. If a typical working mathematician shows an interest in this phenomenon, it is more on the recreational side, in his Gardener mood. This is why in *The Mathematical Intelligencer* the place occupied by self-reference is disproportionate with the importance an ordinary reader of this journal would attach to it in his working mood. This is also why in a paper on self-reference one is likely to find in *The Intelligencer* one can hardly expect to learn much. One just gets the thrill of something amusing, but also whimsical, confused and a little bit unnerving, intriguing but not worth pursuing seriously - something basically hollow. One abandons such papers without enlightenment, without any sense of achievement. Like every amusement, in excessive quantities they may lead to boredom.

In this respect, [1] is no exception. However, its author seems to pretend that it outdoes papers published up to now in journals like *The Intelligencer,* because in it self-reference is not limited to isolated paradoxical sentences, but the paper is thoroughly infected by it. By moving from local to global self-reference, [1] may have achieved the dubious merit of being globally hollow. Hopefully, its small size saves it from being globally boring.

An unquestionable advantage of [1] is that reading it does not require the physical effort most mathematicians inflict upon their readers with such insouciance. The paper has only one reference, given to the reader together with the paper. So there is no need to rise from your chair and search through your badly-ordered shelves, or dig into your worse-ordered xerox copies, or write memos to be taken on your next walk to the library. And, even if this searching, digging, walking, proves successful, one is likely to find that the reference provided by the author is hardly useful if one does not consult references provided by the author referred to (typically, some of these will lead into a mist of preprints, dissertations, unpublished papers, unwritten papers, papers in exotic journals, papers in Slavic idioms). Can anyone show, without dirty tricks like the Axiom of Choice, that the set of references can be well-ordered? Anyway, with [1] we are at least spared the physical effort, though the well-ordering business is in a shape as bad as ever.

This slight advantage of [1] can hardly compensate for the rest we find, or, rather, fail to find, in it. We shall now concentrate on the result the author attempts to prove in [1] and show that it leads to absurdity. In Theorem 1 of [1] it is asserted that a certain theorem is false. We shall now refute this assertion:

THEOREM 1. *Theorem 1 of* [1] *is false.*

Proof. Let p be an abbreviation for Theorem 1 of [1]. Then the equivalence numbered (1) in the proof of p amounts to:

(1) *p if and only if not p.*

To verify this equivalence the reader must consult reference [1] of [1]. If we suppose p, from (1) we obtain *not p.* Since *p* also implies itself, we get the absurdity *p and not p*. Hence, we may conclude *not p. Q.E.D.*

This reasoning is self-evidently correct. Only a *substructural logician* that rejects Gentzen's structural rule of *contraction* (a creature by whom the pages of *The Intelligencer* have not yet been visited, but likely to charge soon in his *linear* or similarly garish costume) would deny us the right to consider a premise used twice as a single premise, which is what we do when, with the help of (1), we infer that *p* implies an absurdity. This expedient for evading paradox is known since Curry's work on the combinator *W*. But should we go so far as to accuse the author of [1] of substructurality?

Note that we use in our proof only the left-to-right direction of (1). One would suppose that the author of [1] mentions the other direction too in order to infer *p* from the conclusion *not p* he has reached, but for some reason he refrains from doing that.

Since Theorem 1 of [1] is false, its proof must have gone wrong somewhere, and the belief in the self-evidential correctness of his reasoning the author of [1] expresses just after the proof of the theorem is unwarranted. The desperate expedient of switching to an extravagant logic, of which we suspected him (we hope unjustly), is just a symptom of his malaise.

The fact that *The Intelligencer* has accepted to print a paper containing a demonstrable falsehood is bad enough, but in this respect *The Intelligencer* is not worse than most mathematical journals. What a serious mathematical journal should refuse to print is the rhetorical part of [1], where the author makes euphuistic remarks about the literature on self-reference and the misgivings of its recreational aspects. The referee of a serious journal would not fail to note that the set of mathematical references, which the author finds so unwieldy when he complains about the labours of reference-hunting, is after all finite and can certainly be well-ordered (all the more so if one applies judiciously not *Choice* but *Personal Choice*, viz. the principle that one may ignore as many references as one wishes). This referee should also be able to suggest a good number of references to supplement, or perhaps supplant, the author's cherished single reference. Although what he tries to exhibit is very well known from other sources, in a remarkably self-conceited way, the author's only reference is a self-reference. His paper should probably have been rejected because it is nothing but a hollow extravaganza - the injury of falsehood coupled with the insult of pomposity and egotism.

The fact that in the penultimate paragraph of [1] the author himself is led to judge severely these defects can hardly excuse him. For if his judgement is true, the defects are real, and if his judgement is false, this makes a defect. He should better follow a famous authority on self-reference whose words from the *First Epistle to the Corinthians* (4: 3) he quotes at the end of his last sentence: "I judge not my own self."

Reference

[1] K. Døsen, One more reference on self-reference, *The Mathematical Intelligencer* vol. 14 (1992), no 4, pp. 4-5

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Short biographical note accompanying a snapshot of K. Døsen with a camera on his eye

Although the author is a logician, this is his first piece on self-reference (which, remarkably enough, is already cited). He has published papers on nonclassical logics. *The Mathematical Intelligencer* has published a short note about him accompanied by a photographic self-portrait. However, his hobby is not photography but orthography. His attempt to link the latter with logic may have something to do with the outlandish spelling of his name.