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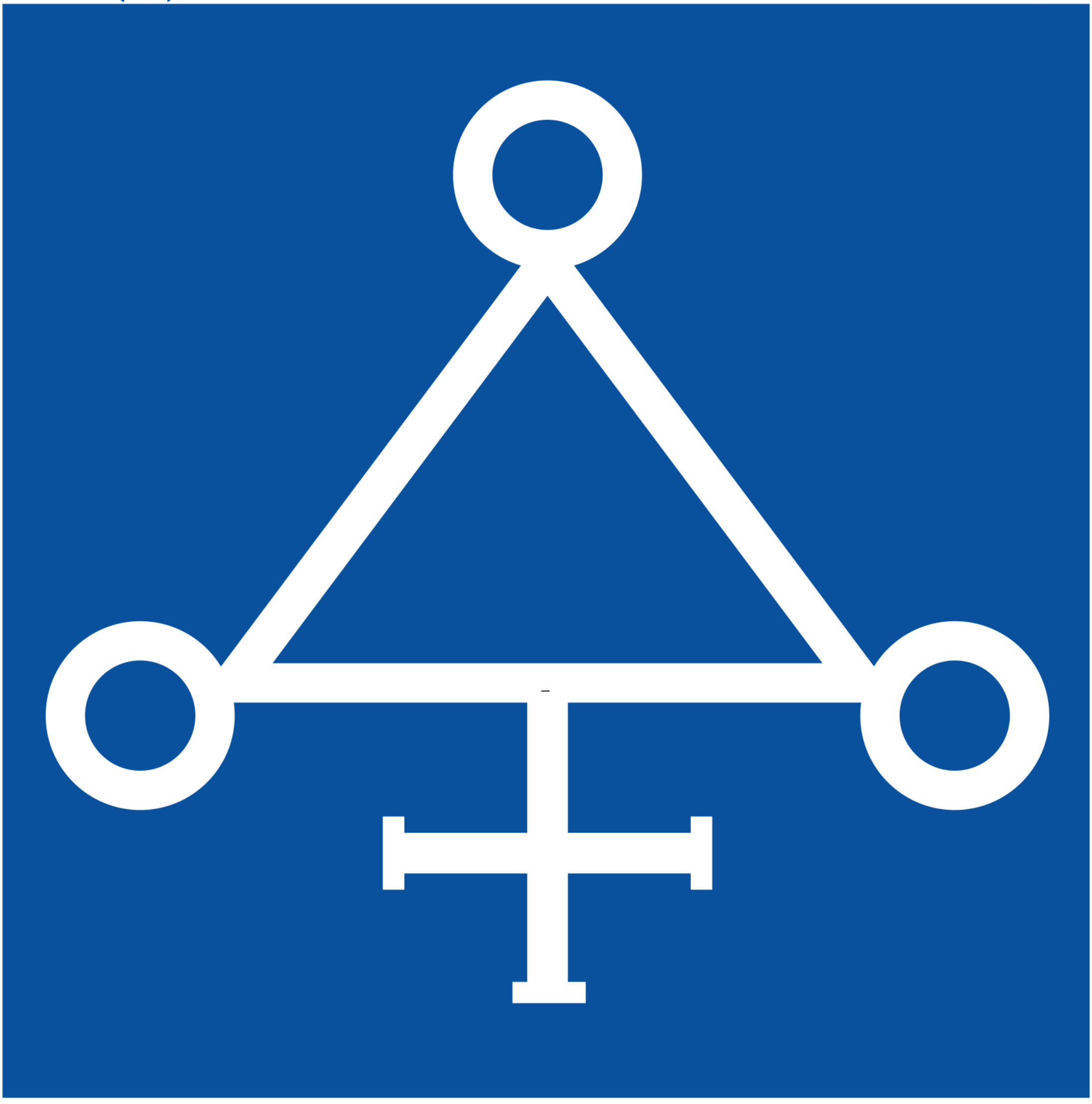

МУЗЕЈ НАУКЕ И ТЕХНИКЕ
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RECOGNITION OF MUSIC MELODIES IN SPECTRAL GRAPH THEORY[4]

Abstract

A spectral graph theory approach is described for representing melodies as graphs, based on intervals between the notes they are composed of. These graphs (or digraphs) are then indexed using eigenvalues of some graph matrices. The eigenvalues are used to define a spectral distance between graphs. Two graphs are considered as similar if their spectral distance is small. This makes it possible to find melodies similar to a given melody. Our contribution includes some improvements of the basic graph model as well as the selection of graph matrices which are used in indexing melodies. We recommend the matrix AA^T, where A is the adjacency matrix of a digraph. The spectrum of AA^T is called the non-negative spectrum or *N*-spectrum of the digraph. We survey some properties of the *N*-spectrum. We also present some examples and musicians' intuitive approach to similarity of melodies. Our contributions are presented within a short review of the huge areas of music recognition and spectral recognition of graphs.

Keywords: spectral graph theory, spectral recognition, music melodies, data mining

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1. Introduction

Spectral graph theory is a mathematical theory in which linear algebra and graph theory meet. Spectral graph theory is a very well developed mathematical field[5] but also an engineering discipline[6]

For any graph matrix *M* we can build a spectral graph theory in which graphs are studied by means of eigenvalues of the matrix *M*. This theory is called *M-theory.* In order to avoid confusion,to any notion in this theory a prefix *M*-could be added (e.g., M-eigenvalues). Frequently used graph matrices are the adjacency matrix A, the Laplacian $L = D — A$ and the signless Laplacian $Q = D + A$, where D is a diagonal matrix of vertex degrees. The spectral graph theory includes all particular theories together with interaction tools.

In the last twenty years or so, it has been recognized that graph spectra have several important applications in computer sciences.[7] Graph spectra appear in the literature in Internet technologies, computer vision, pattern recognition, data mining, multiprocessor systems, statistical databases and in many other areas. There are thousands of papers in which these topics are treated.

In surveys[8] of the applications of graph spectra in Computer Science, applications in the following branches of Computer Science have been identified:

1. Expanders and combinatorial optimization, 2. Complex networks and the Internet topology, 3. Data mining, 4. Computer vision and pattern recognition, 5. Internet search, 6. Load balancing and multiprocessor interconnection networks, 7. Anti-virus protection versus spread of

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[5] Dragoš Cvetković, Michael Doob and Horst Sachs, *Spectra of Graphs, Theory and Application*, 3rd edition (Heidelberg-Leipzig: Johann Ambrosius Barth Verlag, 1995); Dragoš Cvetković, Peter Rowlinson and Slobodan K. Simić, *An Introduction to the Theory of Graph Spectra* (Cambridge: Cambridge University Press, 2009).

[6] Daniel A. Spielman, "Spectral Graph Theory and its Applications", in *48th Annual IEEE Symposium on Foundations of Computer Science* (Los Alamitos: IEEE, 2007), 29-38.

[7] Dragoš Cvetković and Ivan Gutman, eds., *Application of Graph Spectra, Zbornik radova* 13, 21 (Belgrade: Mathematical Institute SANU, 2009); Dragoš Cvetković and Ivan Gutman, eds., *Selected Topics on Application of Graph Spectra, Zbornik radova* 14, 22 (Belgrade: Mathematical Institute SANU, 2011); Dragoš Cvetković and Slobodan K. Simić, "Graph spectra in Computer scince", *Linear Algebra and its Applications,* 434, 6 (2011): 1545-1562.

[8] Cvetković and Simić, "Graph spectra in Computer science", 1545-1562; Branko Arsić et al, "Graph spectral techniques in computer sciences", *Applicable Analysis and Discrete Mathematics,* 6, 1 (2012):1-30.

knowledge, 8. Statistical databases and social networks, 9. Quantum computing, 10. Bio-informatics, 11. Coding theory, 12. Control theory.

It is not unusual that graph specta appear in computer science since graphs themselves are quite relevant in computer sciences. Graphs that are treated in computer sciences using graph spectra typically represent either some physical networks (computer network, Internet, biological network, etc.) or data structures (documents in a database, indexing structure, etc.) In the first case the graphs usually have a great number of vertices (thousands or millions) and they are called *complex networks* while in the second case graphs are of small dimensions.

Moreover, in some applications in data mining graph spectra are used to encode graphs themselves.[9] The following examples are illustrative in this respect.

The indexing structure of objects appearing in computer vision (and in a wide range of other domains such as linguistics and computational biology) may take the form of a tree. An indexing mechanism that maps the structure of a tree into a low-dimensional vector space using graph eigenvalues is developed.[10]

In several databases the data are often represented as graphs. Very frequently graphs are indexed by their spectra.[11]

In order to introduce a suitable graph matrix, note that for any real matrix A, not necessarily a square matrix, the matrices AA^T and A^TA are symmetric. Therefore they have real eigenvalues which are non-negative. Non-zero eigenvalues of AA^T and A^TA are the same. Square roots of these eigenvalues are called *singular* values of A.

Let A be the adjacency matrix of a digraph D. The eigenvalues of AA^T and A^TA are the same and they constitute the N-spectrum of D.

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9 Fatih M. Demirci, Reiner H. van Leuken and Remco C. Veltkamp, "Indexing through laplacian spectra", *Computer Vision and Image Understanding,* 3 (2008): 312, DOI 10.1016/j.cviu.2007.09.012; Lei Zou et al, "A novel spectral coding in a large graph database", in *Proceedings of the 11th international conference on extending database technology (EDBT"08)* (2008), 181-192.

10 Ali Shokoufandeh et al, "Indexing using a spectral encoding of topological structure", *IEEE Trans. Comput. Vision Pattern Recognition,* 2 (1999): 491-497.

11 Alberto Pinto et al, "Indexing music collections through graph spectra", in *Proceedings of the 8th International Conference on Music Information Retrieval (ISMIR'07) (2007),* 153-156; Shokoufandeh et al, "Indexing using a spectral encoding of topological structure", 491-497; Zou et al, "A novel spectral coding in a large graph database", 181-192.

In the paper[12] a spectral graph theory approach is presented for representing melodies as graphs, based on intervals between the notes they are composed of. These graphs are then indexed using their Laplacian spectrum. This makes it possible to find melodies similar to a given melody.

The query for such a database is given by a graph. To find similar data in the database it is necessary to compare subgraphs of the query graph with subgraphs of the graphs stored in the database. One should efficiently select a small set of database graphs, which share a subgraph with the query. Instead of comparing subgraphs one can compare their spectra. While the subgraph isomorphism problem is NP-complete, comparing spectra can be done in polynomial time.

The model from the paper[13] has been discussed and improved in the paper.[14]

The rest of the paper is organized as follows. Section 2 presents a short review of the area of spectral recognition of graphs, Section 3 gives a description of the procedure for recognizing music melodies including our improvements. Section 4 studies some basic properties of the *N*-spectrum of a digraph. Section 5 presents some examples and computational experiments. Section 6 gives concluding remarks.

2. Spectral recognition of graphs

Spectral recognition of graphs is in the core of applications of spectral graph theory to Computer Science. We mention here main points[15] according to our paper. [16]

At some time, in the childhood of spectral graph theory, it was conjectured that non-isomorphic graphs have different spectra, i.e. that graphs are characterized by their spectra. Very quickly this conjecture was refuted and numerous examples and families of non-isomorphic graphs with the same spectrum (cospectral graphs) were found. Still some graphs are

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[12] Alberto Pinto, *MIREX2007 - Graph spectral method*, unpublished.

[13] Ibid.

[14] Dragoš Cvetković and Vesna Manojlovic, "Spectral recognition of music melodies", in *SYM-OP-IS* (2013), 269-271.

[15] Presented at the Conference on Applications of Graph Spectra in Computer Science, Barcelona, July 16 to 20, 2012.

[16] Dragoš Cvetković, "Spectral recognition of graphs", *Yugoslav Journal of Operations Research,* 20, 2 (2012): 145-161.

characterized by their spectra and several mathematical papers are devoted to this topic. In applications to computer sciences, spectral graph theory is considered as very strong. The benefit of using graph spectra in treating graphs is that eigenvalues and eigenvectors of several graph matrices can be quickly computed. Spectral graph parameters contain a lot of information on the graph structure (both global and local) including some information on graph parameters that, in general, are computed by exponential algorithms. Moreover, in some applications in data mining, graph spectra are used to encode graphs themselves. The Euclidean distance between the eigenvalue sequences of two graphs on the same number of vertices is called the spectral distance of graphs. Some other spectral distances (also based on various graph matrices) have been considered as well. Two graphs are considered as similar if their spectral distance is small. If two graphs are at zero distance, they are cospectral. In this sense, cospectral graphs are similar. Other spectrally based measures of similarity between networks (not necessarily having the same number of vertices) have been used in Internet topology analysis, and in other areas. The notion of spectral distance enables the design of various meta-heuristic (e.g., tabu search, variable neighbourhood search) algorithms for constructing graphs with a given spectrum (spectral graph reconstruction).

Several spectrally based pattern recognition problems appear in many areas (e.g., image segmentation in computer vision, alignment of protein-protein interaction networks in bio-informatics, recognizing hard instances for combinatorial optimization problems such as the travelling salesman problem).

3. Spectral recognition of music melodies

A *melody* M is a finite sequence of pitches (or corresponding notes) p_1, p_2,... , p_m. The usual 12-tone system is used. Our considerations are related to a digraph G whose vertex set $V = \{1, 2,..., 12\}$ represents pitch classes. For example, the same vertex represents *c* in all octaves. The digraph G has all possible arcs (oriented edges) and loops.

A melody M considered as a sequence of vertices of G determines in G a closed walk consisting of arcs (p_1,p_2), (p_2,p_3),..., (p_m,p_1). Note that some of the arcs in this walk may be repeated and also that the arc (p_m,p_1) does not actually represent an interval between the pithes in the melody. The vertex set V together with arcs from this closed walk determines a (multi-)digraph G_M associated with the melody M.

The digraph G_M, as a labelled digraph, determines uniquely pitch classes of the melody M. However, different melodies can have isomorphic associated digraphs. Nevertheless, this representation is good enough in the task of finding similar melodies.

This model appeared in papers[17] and it does not take into account the duration of pitches. An obvious way to overcome this is to introduce weights on arcs in G_M. The weight of the arc (p_i, p_{i+1}) would denote the duration of the pitch p_i.

Since all arcs of a digraph G_M lie on a closed walk, G_M consists of a strongly connected component and a number of isolated vertices.

The papers[18] consider spectra of the adjacency matrix and the Laplacian of G_M for indexing G_M.

Two melodies are considered as similar if the corresponding graph spectra are close one to the other. In particular, one should consider the Euclidean distance in $\mathbf{R}^{12}$ between the eigenvalue sequences and require that this distance is small. A number of melodies similar to a given melody M is than obtained from melody database by general retrieval procedures.[19]

A number of objections to this procedure can be made. The main objections are related to the choice of graph matrices. Both the adjacency matrix and the Laplacian matrix are generally non-symmetric for digraphs. Therefore the corresponding spectra are complex which causes some difficulties. It is well-known that main results of spectral graph theory are related to undirected graph where graph matrices are symmetric and eigenvalues are reals. In addition, the Laplacian matrix is unappropriate for digraphs with loops since no loop can be recorded in the Laplacian matrix.

In general, adjacency matrix is not good since arcs not laying in closed walks are not reflected in the spectrum (this does not applies to the considered situation). Also, the reference to the paper[20] for benefits of using Laplacian matrix is not relevant here since this paper considers undirected graphs.

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[17] Pinto, *MIREX2007 - Graph spectral method;* Pinto et al, "Indexing music collections through graph spectra", 153-156.

[18] Ibid.

[19] Demirci, Leuken and Veltkamp, "Indexing through laplacian spectra".

[20] Willem Haemers and Edward Spence, "Enumeration of cospectral graphs", *European Journal of Combinatorics,* 25, 2 (2004): 199-211.

We have pointed out in the paper[21] that in treating music melodies by graph spectra the multidigraph G_M should be indexed by eigenvalues of AA^T where A is the adjacency matrix of G_M, including the case when G_M is a weighted (multi-)digraph. In this way the mentioned objections in the approach from papers[22] would be overcome.

4. The N-spectrum of a digraph

We present here basic properties of the N -spectrum of a digraph.

Let $D = (V(D), A(D))$ be a digraph of order n, with the set of vertices $V(D) = \{vi, v_2,..., v_n\}$ and whose adjacency matrix is $A = [a_{ij}]$. Structural versus spectral properties of digraphs related to the matrices AA^T and A^TA will be considered in this section. Some of these results also appear elsewhere.[23] The N -spectrum of a digraph was not onsidered earlier in mathematical literature[24] but appears in applications.[25]

The matrices AA^T and A^TA are non-negative square and symmetric. One can easily check that these matrices are also positive semi-definite, so that their eigenvalues are non-negative.

The statement given by the following proposition is also well known.[26]

Proposition 1. *The (i, j)-entry of the matrix AA^T (A^TA) of D is equal to the number of common out-neighbours (in-neighbours) of v_j and v_j. Diagonal entries of the matrix AA^T (A^TA) represent out-degrees (in-degrees) of the vertices of D.*

Proof. The (i, j)-entry of the matrix AA^T is equal to the sum of all products $a_{il}a^T_{lj}$, for each l= 1, 2,..., n. Further, $a_{il}a^T_{lj}$ = 1 if a_{il} = 1 and a^T_{lj} = 1 hold, i.e. if $v_{i;}$ is the common out-neighbour of v_i and v_j. The case of A^TA is treated in a similar way. ■

[21] See footnote 14.

[22] See footnote 17.

[23] Irena Jovanović, "Spectral recognition of graphs and networks" (PhD Thesis, School of Mathematics, University of Belgrade, 2015); Irena Jovanović, "Non-negative spectrum of a digraph", *Ars Mathematica Contemporanea,* 12, 1 (2017): 167-182.

[24] Richard A. Brualdi, "Spectra of digraphs", *Linear Algebra and its Applications,* 432 (2010): 2181-2213.

[25] Ami N. Langville and Carl D. Meyer, "A survey of eigenvector methods for web information retrieval", *SIAM Review,* 47, 1 (2005): 135-161.

[26] Ibid.

According to the previous remarks and proposition, we can introduce the following notation: $N_{out} = AA^T$ and $N_{in} = A^TA$. The characteristic polynomial det ($\lambda I — N_{in}$) of N_{in} is the N_{in}- *characteristic polynomial* of *D*, while the characteristic polynomial det($\lambda I — N_{out}$) of N_{out} is the N_{out}-*characteristic polynomial* of *D*. The spectrum of N_{out} and of N_{jn} are the same and they are denoted by the single name - the *N-spectrum.* So, the characteristic polynomials *N(x)* of these matrices can be named the *N -polynomials.* However, we underline that through the investigation we mainly considered the $N_{out}(D)$ matrix of *D*, whose spectrum we denote by $\eta_1 \geq \eta_2 \geq ... \geq \eta_n$. The *N-spectral radius* $\rho_N(D)$ of *D* is defined to be the spectral radius of $N_{out}(D)$, i.e. $N_{in}(D)$.

Remark 1. *For the N-spectrum $\eta_1, \eta_2, ..., \eta_n$ of a digraph D the following holds:*

- *The numbers $\eta_1, \eta_2, ..., r_n$ are real and non-negative,*
- *$\eta i + \Pi 2 + ... + \eta_n = trN_{out}$ (= trN_{in}) = $\Sigma_{i=1}^{\eta}$ outdeg(v_i) (=$\Sigma_{i=1}^{\eta}$ indeg(v_i)),*
- *D consists only of isolated vertices if and only if $\eta_1 = \eta_2 = ... = \eta_n = 0$.*

We say that a digraph *D* is *r-regular* if the in-degree and the out-degree of each its vertex are equal to *r*.

Lemma 1. *The* N-*spectral radius* $\rho_N(D)$ *of an r-regular digraph D = (V(D), A(D)) of order* n *is* r^2.

Remark 2. *The eigenvector that corresponds to the N-eigenvalue* r^2 *of an r-regular digraph* D *is all-1 vector.*

Example 1. *The complete digraph of order n is the digraph* $\overleftrightarrow{K}_n$ *in which for each pair of vertices there is an arc, including a loop at each vertex. Thus,* $\overleftrightarrow{K}_n$ *has* n^2 *arcs and it is n-regular digraph.*

Since the in-degree and the out-degree of each vertex of this digraph is n, and every pair of its vertices has n common out-neighbours, apropos n common in-neighbours, the N-characteristic polynomial of this digraph is:

$$N_{\overleftrightarrow{K}_n}(x) = (x — n^2)x^{n-1},$$

and thus its N-spectrum consists of: n^2, $[0]^{n-1}$.

Here an eigenvalue λ of the multiplicity *k* is denoted by $[\lambda]^k$.

The *complement* $D^C = (V(D^C), A(D^C))$ of a digraph *D = (V(D), A(D))* has vertex set $V(D^C) = V(D)$ and $a \in A(D^C)$ if and only if $a \notin A(D)$. Also, there is a loop at vertex V_1 in D^C if and only if there is no loop at v_i in *D*. Similarly to the proof of the corresponding theorem for regular graphs we can prove the following:

Proposition 2. *If the N-eigenvalues of an r-regular digraph D of order n are* $n_i(D)$, *i* = 1, 2,... ,*n, then the N-eigenvalues of* D^C *are* $n_1(D^C) = n^2 - 2nr + r^2$ *and* $n_i(D^C) = n_i(D)$, *i* = 2, *3*,... ,*n.*

5. Some examples and experiments

In conference presentation[27] the third author illustrated the main ideas of our approach by the following two examples of pairs of similar melodies, provided by the second author:

1.1. Pyotr Ilyich Tchaikovsky: Piano Concerto No. 1, B-flat minor, movement I;

1.2. Pyotr Ilyich Tchaikovsky: song *Den li tsarit.*

2.1. Antonio Vivaldi: *Si fulgida per te,* aria di Abra from oratorio Juditha triumphans,

2.2. Antonio Vivaldi: *Io son quel gelsomino,* aria from the opera Arsilda, regina di Ponto.

Example 1.1. is Tchaikovsky piano concerto in B-flat minor, beginning of the first movement, not the introduction, but the theme written in orchestral part. While piano is playing chords, orchestra plays the first theme in form of D-flat major arpeggiated cord.

Example 1.1. Tchaikovsky piano concerto in B-flat minor

Comparative example 1.2. is song by Tchaikovsky *Den li tsarit,* theme in soprano line.

Example 1.2. Tchaikovsky *Den li tsarit*

This example is in E-major key, which doesn't change anything regarding the recognition of this motif, since transposition doesn't affect the recognition of melody, because all parameters are preserved (melody, harmony and rhythm). More importantly for our purposes, a melody can still be uniquely identified after it has undergone transposition (we still

[27] Vesna Todorčević, "Some remarks on spectral recognition of music melodies", *Book of Abstracts* (2016), 41-42.

recognize a familiar tune in a different key as being the same tune).[28] In this case, only meter is changed, in the example 01 there is 3/4 and in the example 02 there is 9/8 meter.

Example 2.1. is aria of Abra from the oratorium Juditha triumphans by Antonio Vivaldi, *Si fulgida per te.* It is written in A-minor, melody is easily recognized by ascending triplets in soprano. Even visually, without listening to music, this theme is graphically very significant.

Example 2.1. Antonio Vivaldi: Si *fulgida per te*

Comparative example 2.2. is also by Antonio Vivaldi, *Io son quel gelsomino,* aria from the opera Arsilda regina di Ponto. Key is G-minor, meter is the same, only this melody begins at another part of the bar in 12/8 meter.

Example 2.2. Antonio Vivaldi: *Io son quel gelsomino*

Musicians consider identifying the melody by default, like, for instance, identifying words and sentences. When musician hears a melody, this auditory stimulus is immediately transposed into sequences of sound grouped by expectations, based on former experience. This is pointing to complexity of perception of melodic aspects of music, respectively general level of organization, based on gestalt principles.[29]

Visual recognizing of sheet music is regularly used, but it is always connected to sound. A musician always hears the music he is looking at, and this is always happening in his intrinsic imagination, because he learned to do so since childhood, during a period of learning music. Similar phenomenon occurs when we are reading some text, we recognize meaning of the words, not only seeing signs on the paper (unless we are looking at some lines written in unknown language).

[28] Youngmoo E. Kim et al, "Analysis of a contour-based representation for melody", in *Proceedings of 1st International Symposium on Music Information Retrieval* (ISMIR 2000), accessed August 16, 2018, https://pdfs.semanticscholar.org/bc27/f7d4a549a0d3dd36b74afab6ae7ba5514e6b.pdf.

[29] Ksenija M. Radoš, *Psihologija muzike* (Beograd: Zavod za udžbenike i nastavna sredstva, 1996), 106.

Musicians perceive music as an integral aural stimulus, which includes recognizing of the rhythm, harmony and tempo, not only the melody. Perceptual organization cannot be defined by only one component for instance by height, or by rhythm but by interaction, or even collision of different components.[30]

Musicians and nonmusicians perceive music as holistic entirety, not only listening to some outstanding thematic material, like first bar of some sonata, symphony, or aria. Motif in the middle of a musical piece can be recognized as prominent melody, as well. However, it is possible that listeners do not attend to individual notes, but rather analyze the overall shape or structure of the melody.[31]

For further illustration we quote the third pair of similar melodies.

3.1. Mozart's aria of Dona Elvira from the opera Don Giovanni, *Mi tradi,* the very beginning of aria.

Example 3.1. Wolfgang Amadeus Mozart, aria of Donna Elvira, *Mi tradi*

3.2. Comparative example is Mozart's concert aria for bass/bariton K.612, one motif from the middle of aria, not the thematic material as in the former example.

Example 3.2. Wolfgang Amadeus Mozart, concert aria
for bass/bariton K.612

These examples match completely in melody, but keys and meters are different. In the example 3.1.there is 4/4 meter, but in the example 3.2. there is 6/8 meter.

Musicians and nonmusicians both experience music as whole stimuli, but trained musicians tend to analyse what they hear or see written on paper. They rely to their perennial experience of learning music by

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[30] Ibid., 107.

[31] Matthew D. Schulkind, Rachel J. Posner and D. C. Rubin, "Musical Features That Facilitate Melody Identification: How Do You Know It's "Your Song When They Finally Play It?", *Music Perception: An Interdisciplinary Journal,* 21, 2 (2003): 217-249.

gathering informations, so to say, on higher level. Accordingly, it might be that listeners without musical training tend to rely on momentary, distinctive features to recognize melody, whereas musicians draw on the information in a melodic sequence in a cumulative fashion.[32]

Melodies recognized as similar by musicians are recognized as similar by musicians are recognized as similar by our formal procedure as well.

In his final examination work[33] at Faculty of Organizational Sciences M. Putnikovic has implemented in *R* programming language a procedure for finding similar music melodies.

The aim of this project was to examine the eectiveness of various mathematical methods for computing similarity of melodies. In particular, eigenvalues of dierent types of graphs were used, as well as several string distance measures. Melodies have been transformed into letter notation, so that graphs could be easily constructed. A sequence of notes is understood as a string of letters, each letter representing one note, which enabled us to use the existing methods for computing string sequence similarities.

The dataset contains 17 instances, with compositions by Johann Sebastian Bach and Wolfgang Amadeus Mozart. These composers were chosen as their style and epoch differ significantly both in music theory and practice.

All melodies were transcribed to letter-duration notation. Also, the sheets are for violin, not for pianos, since accords make computability inefficient.

The included melodies are as follows, with scale specified:

Melodies by Johann Sebastian Bach:
Prelude from Suite no. 1 for unaccompanied cello - C major,
Air on G-string - A major,
Menuet from French Suite no. 3 - A minor,
Aria from Goldberg Variations - G major,
Bist Du Bei Mir - D major,
Arioso from Cantata - G major,
Prelude no. 1 from 48 Preludes and Fuges - D major,
Menuet - G major,
Sonata no. 5 BWV 1034 - D major,
Violin Concerto BWV 1056 - F-sharp minor;

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[32] Freya Bailes, "Dynamic melody recognition: Distinctiveness and the role of musical expertise", *Memory & Cognition,* 38 , 5 (2010): 641-650.

[33] Marko Putniković, "Recognition of melodies by means of graphs" (Diploma Thesis, Faculty of Organizational Scences, Belgrade, 2017).

Melodies by Wolfgang Amadeus Mozart:
Menuet from Don Giovanni - D major,
Menuet - D major,
German Dance no. 1 K 605 - A major,
Laudate Dominum - G major,
March of the Priest from The Magic Flute - A major,
Oh Isis Und Osiris - G major,
Lacrimosa Dies Illa - E minor.

A melody can be given in the form of a digraph, where the adjacency matrix can be determined, and then the eigenvalues in the N-spectrum can be calculated. The spectral distance between each pair of melodies has been calculated and clustering methods applied to partition melodies into groups of similar melodies. The hierarchical clustering method proved to be the most accurate one. The author claims that composers and types of melodies (concert, menuet, aria etc.) could have been distinguished.

6. Concluding remarks

A spectral graph theory approach for representing melodies as (multi-) digraphs[34] is improved[35] and illustrated in this paper. These digraphs are then indexed using eigenvalues of some graph matrices. Our contribution includes some improvements of the basic graph model (taking into account the duration of pitches) as well as the selection of graph matrices which are used in indexing melodies. After presenting some shortcomings of the procedure from[36] we have suggested using singular values of the adjacency matrix i.e. the N -spectrum of digraphs considered. The theory has been supported by some examples and preliminary computational experiments.

[34] See footnote 17.

[35] See footnote 14.

[36] See footnote 17.

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ПРЕПОЗНАВАЊЕ МУЗИЧКИХ МЕЛОДИЈА У СПЕКТРАЛНОЈ ТЕОРИЈИ ГРАФОВА

Музичка мелодија, која је састављена од тонова, карактерише се и интервалима између тонова. Овако схваћена мелодија се може представити графовима а они се могу третирати средствима спектралне теорије графова. Ови графови (или диграфови) се могу индексирати помоћу сопствених вредности неких графовских матрица. Помоћу сопствених вредности дефинише се спектрално растојање графова. Две мелодије се сматрају сличним ако је спектрално растојање њихових графова мало. То омогућава да се нађу мелодије сличне задатој мелодији. Наш допринос укључује побољшање основног модела као и избор графовских матрица које се користе за индексирање мелодија. Ми препоручујемо употребу матрице AA^T, где је A матрица суседства једног диграфа. Спектар матрице AA^T назива се N-спектар диграфа. У раду се даје преглед неких особина N-спектра. Наводе се примери сличних мелодија и описује интуитивни приступ музичара проблему препознавања сличних мелодија. Описују се прелиминарни рачунарски експерименти са скицираном теоријом. Наши доприноси су приказани у контексту сажетог прегледа широке области препознавања музике и спектралног препознавања графова.

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