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SPECTRAL RECOGNITION OF MUSIC MELODIES
SPEKTRALNO PREPOZNAVANJE MUZIČKIH MELODIJA

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Rezime: *A spectral graph theory approach is described for representing melodies as graphs, based on intervals between the notes they are composed of. These graphs are then indexed using eigenvalues of some graph matrices. The eigenvalues are used to define a spectral distance between graphs. Two graphs are considered as similar if their spectral distance is small. This makes it possible to find melodies similar to a given melody. Our contribution is related to the selection of graph matrices which are used in indexing melodies.*

Ključne reči: *spectral graph theory, spectral recognition, music melodies, data mining*

1. INTRODUCTION

Spectral graph theory is a mathematical theory in which linear algebra and graph theory meet. Spectral graph theory is a very well developed mathematical field (see, for example,[3], [6]) but also an engineering discipline [13].

For any graph matrix M we can build a spectral graph theory in which graphs are studied by means of eigenvalues of the matrix M. This theory is called *M-theory*. In order to avoid confusion, to any notion in this theory a prefix M- could be added (e.g., M-eigenvalues). Frequently used graph matrices are the adjacency matrix A, the Laplacian $L = D - A$ and the signless Laplacian $Q = D + A$, where D is a diagonal matrix of vertex degrees. *The* spectral graph theory includes all particular theories together with interaction tools.

It was recognized in about the last ten years that graph spectra have several important applications in computer sciences (see, e.g., [4, 5, 7]). Graph spectra appear in the literature in Internet technologies, computer vision, pattern recognition, data mining, multiprocessor systems, statistical databases and in many other areas. There are thousands of such papers.

In surveys [7] and [1] of the applications of graph spectra in Computer Science, applications in the following branches of Computer Science have been identified :

1. Expanders and combinatorial optimization, 2. Complex networks and the Internet topology, 3. Data mining, 4. Computer vision and pattern recognition, 5. Internet search, 6. Load balancing and multiprocessor interconnection networks, 7. Anti-virus protection versus spread of knowledge, 8. Statistical databases and social networks, 9. Quantum computing, 10. Bio-informatics, 11. Coding theory, 12. Control theory.

We have recently published the paper [2] which represents a survey on spectral recognition problems in Computer Science.

Of course, graph spectra appear in computer science since graphs for themselves are relevant. Graphs that are treated in computer sciences using graph spectra typically represent either some physical networks (computer network, Internet, biological network, etc.) or data structures (documents in a database, indexing structure, etc.) In the first case the graphs usually have a great number of vertices (thousands or millions) and they are called *complex networks* while in the second case graphs are of small dimensions.

Moreover, in some applications in data mining graph spectra are used to encode graphs themselves (see, e.g., [9, 14]). The following examples are illustrative in this respect.

The indexing structure of objects appearing in computer vision (and in a wide range of other domains such as linguistics and computational biology) may take the form of a tree. An indexing mechanism that maps the structure of a tree into a low-dimensional vector space using graph eigenvalues is developed in [12].

In several databases the data are often represented as graphs. Very frequently graphs are indexed by their spectra [10], [12], [14].

In [10, 11] a spectral graph theory approach is presented for representing melodies as graphs, based on intervals between the notes they are composed of. These graphs are then indexed using their Laplacian spectrum. This makes it possible to find melodies similar to a given melody.

The query for such a database is given by a graph. To find similar data in the database it is necessary to compare subgraphs of the query graph with subgraphs of the graphs stored in the database. One should efficiently select a small set of database graphs, which share a subgraph with the query. Instead of comparing subgraphs one can compare their spectra. While the subgraph isomorphism problem is NP-complete, comparing spectra can be done in polynomial time.

In the next section we shall present the procedure from [10, 11] for finding melodies similar to a given melody, together with our suggestions for improvements,

2. DESCRIPTION OF THE PROCEDURE

A *melody M* is a finite sequence of pitches (or corresponding notes) $p_1, p_2, \ldots, p_m$. The usual 12-tone system is used. Our considerations are related to a digraph G whose vertex set $V = \{1, 2, \ldots, 12\}$ represent pitch classes. For example, the same vertex represents c in all octaves. The digraph G has all possible arcs (oriented edges) and loops.

A melody M considered as a sequence of vertices of G determines in G a closed walk consisting of arcs $(p_1, p_2), (p_2, p_3), \ldots, (p_m, p_1)$. Note that some of the arcs in this walk may be repeated and also that the arc (p_m, p_1) does not actually represent an interval between the pithes in the melody. The vertex set V together with arcs from this closed walk determines a (multi-)digraph G_M associated with the melody M.

The digraph G_M, as a labelled digraph, determines uniquely pitch classes of the melody M. However, different melodies can have isomorphic associated digraphs. Nevertheless, this representation is good enough in the task of finding similar melodies.

Since all arcs of a digraph G_M lie on a closed walk, G_M consists of a strongly connected component and a number of isolated vertices.

The papers [10] and [11] consider spectra of the adjacency matrix and the Laplacian of G_M for indexing G_M.

Two melodies are considered as similar if the corresponding graph spectra are close one to the other. In particular, one should consider the Euclidean distance in $\mathbf{R}^{12}$ (or in $\mathbf{C}^{12}$) between the eigenvalue sequences and require that this distance is small. A number of melodies similar to a given melody M is than obtained from melody database by general retrieval procedures (see, for example, [9]).

A number of objections to this procedure can be made. The main objections are related to the choice of graph matrices. Both the adjacency matrix and the Laplacian matrix are generally non-symmetric for digraphs. Therefore the corresponding spectra are complex which causes some difficulties. It is well-known that main results of spectral graph theory are related to undirected graph where graph matrices are symmetric and eigenvalues are reals. In addition, the Laplacian matrix is unappropriate for digraphs with loops since no loop can be recorded in the Laplacian matrix.

In general, adjacency matrix is not good since arcs not laying in closed walks are not reflected in the spectrum (this does not applies to the considered situation). Also, the reference to [8] for benefits of using Laplacian matrix is not relevant here since [8] considers undirected graphs.

In order to introduce a suitable graph matrix, note that for any real matrix A, not necessarily a square matrix, the matrices AA^T and A^TA are symmetric. Therefore they have real eigenvalues which are non-negative. Non-zero eigenvalues of AA^T and A^TA are the same. Square roots of the these eigenvalues are called *singular* values of A.

In our oppinion, in treating music melodies by graph spectra the multidigraph G_M should be indexed by eigenvalues of AA^T where A is the adjacency matrix of G_M. In this way the mentioned objections in the approach from [10] and [11] would be overcome.

3. CONCLUDING REMARKS

A spectral graph theory approach for representing melodies as digraphs from [10] and [11] is described. These digraphs are then indexed using eigenvalues of some graph matrices. Our contribution is related to the selection of graph matrices which are used in indexing melodies. After presenting some shortcomings of the procedure from [10] and [11] we have suggested using singular values of the adjacency matrix of digraphs considered.

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