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INVESTIGATION OF THE DIVIDED CIRCLE OF THE BELGRADE LARGE VERTICAL CIRCLE

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SUMMARY: Corrections for 1080 diameters at 10' spacing of the 2' divided circle of the Belgrade Large Vertical Circle are determined according to Nikolić's method. All the diameter corrections are within ± 1".5 limits, the accuracy being ±0.11. The values of the corrections are tabulated and illustrated graphically.

1. INTRODUCTION

In the period 1976–1980 observations, by absolute method, have been carried out with the Large Vertical Circle (LVC) of the Belgrade Observatory. These observations are aimed at elaborating a catalogue of absolute declinations of 308 bright stars in the zone $+65^{\circ}$ to $+90^{\circ}$ declination (Teleki et al., 1981). Certain preparatory works were thereby necessitated, one of them being the circle investigation. The method used was that of Nikolić (1965). The method has been given preference for its efficiency. The investigation was executed in the period February 8 – March 12, 1980.

The first ever investigation of the LVC division errors has been performed in 1964 at 4° according to both Nikolić's and Bruns' methods (Nikolić, 1968a), Fig. 2.

All the measurements comprised by the present investigation are made visually, five microscope pairs having been mounted for the purpose. Three of these microscope pairs are LVC's „own" ones and other two have been borowed from the neighbouring Large Meridian Circle (LMC). The LMC microscopes had to be fastened on the LVC by means of a special supporting construction, whereby the general stability of the measuring system was preserved. Let it be noted that only two microscope pairs, 90° apart, are used in the regular astronomical observations with the LVC.

2. METHOD APPLIED

In spite of its high efficiency the Nikolić method is little known; up to now it has nowhere, outside Yugoslavia, been actually applied. We shall, therefore, expose it in more details, in particular one of its versions which, even the author did not insist on, although it appeared to us as most suitable.

The implementation of the Nikolić's method involves k microscope pairs, whereby $k = 3$ is a minimum, $k = 5$ being the actual number of pairs used in our investigation. Higher accuracy is attained with microscope pairs distributed at different angles θ_i (Fig. 1.). For the programme to be fulfilled in respect to the desired number of determined corrections and their weight, it is necessary to have determined the quantity $W = 180^{\circ}/mn$, where m – the number of measuring series comprised by a programme and n – the number of circle displacements in a series, i.e. the number of readings of all microscopes at rotating the circle in one sense. As usual, the readings of diameters are executed at both direct and inverse circle displacements.

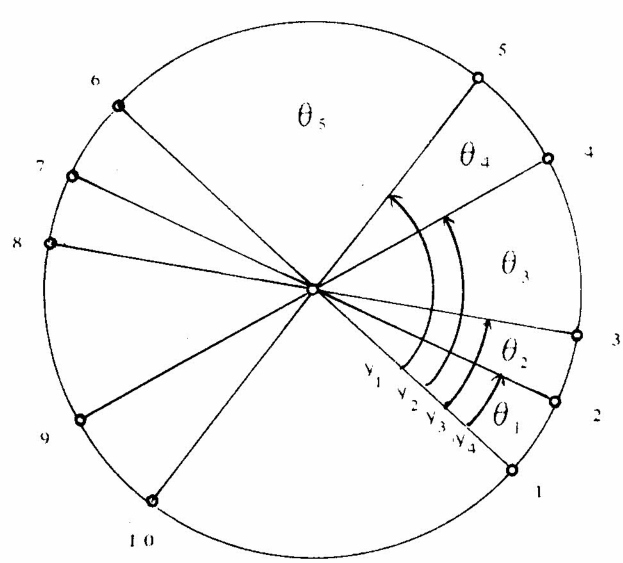

Fig. 1. Arrangement of microscope pairs in conformity with Nikolić's method.

The microscopes can be installed in such a way that the relation $Y_i/W = Q_i+L_i$ ($i = 1,2,.., k-1$), where Y_i – angles between the prime and the rest of microscope pairs (Fig. 1) complies with one of the following contingencies:

a. Q_i – integers, divisible by k, $L_i = 0$. Each of the required corrections is obtained k times in one programme.
b. Q_i – integers. The remainders L_i are evenly distributed over the interval W. The required corrections are determined but once in a given programme. (We were met by this case four times).
c. Q_i – integers. The remainders L_i are unevenly distributed over the interval W. The number of determinations of individual corrections is different.

Nikolić (1968a, 1968b), in his investigation of the LVC and LMC circles at 4^o spacing, whereby the second circle was supplementary, investigated at $0^o.5$ spacing, applied the version a.. We, as above indicated, applied the version b. in our determination of the 10' corrections of LVC circle diameters. We opted for the version b. because it seems as most convenient for detailed investigation of circles whose microscopes are read visually. Repeating the entire programmes enables the desired accuracy to be attained with all the required diameter corrections. Care should thereby be taken that any subsequent programme is started from the prime diameter in the first series of the first programme, increased by one L_i.

The measuring procedure unrolls in the following way: A particular diameter is first proclaimed reference diameter and brought under the prime microscop-micrometer. All of the microscop-micrometers are read. Next, the circle is rotated by the angle $180^o/n$, the microscopes being read again. Upon completing the n-th microscope reading, the circle is rotated in the inverse sense and the same readings are performed once more. This accomplished, the measurements under one series are completed. By adding the value W to the position of the initial diameter, the position of the initial diameter for the second series of measurements is obtained ect.

The basic treatment of the measuring results proceeds as follows: First, the means are formed of the readings of the same diameter obtained at both direct and reverse circle rotation. Denote by the mean values obtained x_{ji} ($j = 1,2,.., n$ – the ordinal number of measurement in the series, $i = 1,2., k$ – the ordinal number of the microscope pair with which the particular diameter has been measured). Let x_{ji} be expressed by

$$x_{ji} = X_{ji} + F_{ji} \tag{1}$$

where X_{ji} is unknown, exact, value of the mean reading x_{ji} of the diameter and $F_{ji} = f_{ji} + \epsilon_{ji}$, the measuring error in x_{ji} consisting of the looked for error f_{ji} of the measured diameter and of ϵ_{ji} – measuring error in that diameter. The mean value of the readings by k microscope pairs is:

$$\frac{1}{k}\sum_i x_{ji} = \frac{1}{k}\sum_i X_{ji} + \frac{1}{k}\sum_i F_{ji} \tag{2}$$

Form the differences $B_{ji} = (2) - (1)$:

$$B_{ji} = \frac{1}{k}\sum_i X_{ji} - X_{ji} + \frac{1}{k}\sum_i F_{ji} - F_{ji}$$

or

$$B_{ji} = C_{ji} + \frac{1}{k}\sum_i F_{ji} - F_{ji} \tag{3}$$

By summing (3) according to j and forming the means we have

$$B_i = C_i + \frac{1}{kn}\sum_j\sum_i F_{ji} - \frac{1}{n}\sum_j F_{ji} \tag{4}$$

where

$$C_i = \frac{1}{n}\sum_j C_{ji}$$

Having regard to the nature of the quantities concerned there will be, for the same i, $C_i = C_{ji}$.

Thus, the difference of the relations (3) and (4) gives:

$$E_{ji} = -F_{ji} + \frac{1}{k}\sum_i F_{ji} + \frac{1}{n}\sum_j F_{ji} - \frac{1}{kn}\sum_j\sum_i F_{ji} \tag{5}$$

In the b. version any one of m measuring series under some of p programmes will furnish nk values of E_{ji}. In other words, any of p programmes will supply $knm=N$ values in the form E_{ji}, i.e. there will be one value E_{ji} for any one of the diameters investigated. Their mean value E for any of the investigated diameters, according to (5) will be:

$$E = -F + \frac{1}{kp}\sum F + \frac{1}{n}\sum F - \frac{1}{kpn}\sum F \tag{6}$$

For the sake of clarity we omitted subscripts from this relation (four subscripts with each F), as well as the manyfold symbols Σ. Otherwise, close attention should be paid to these notations in the actual treating.

As evident, the quantities E with any of the of N investigated diameters consist of: – F – the required mean measured correction to the given diameter. The term $(1/pk)\Sigma F$ contains the measured errors of the *non–uniformly* distributed diameters of the investigated circle. The value of this term is determined by successive approximations (iterative procedure). All the values of this term lied within ± 0."30, but treated as accidental quantities, they amounted on the average to ± 0."11. The last term in (6) $(1/kpn)\Sigma F$ contains the measured errors of a large number of *uniformly* distributed diameters, its value being, accordingly, unconsequential and negiligible. The term $(1/n)\Sigma F$ contains mainly the errors of *n uniformly* distributed diameters. On account of the short period division errors, the value of this term. if the values of *n* are relatively small, while being small, is not necessarily negligible. However, this term cannot be determined from the above measurements alone and its values emerge, in some respect, as the systematic errors introduced by Nikolić's method as such. This neglecting should be taken care of when evaluating the accuracy degree of the errors obtained. With the more up to date circle division reading, as for instance the photographic and photoelectric reading, which afford measurements to proceed considerably faster, *n* can attain considerably greater values. By this very fact this error is becoming insignificant. With the measuring procedure suggested by Bozhichkovich (1981), which provides for the uniformly rotating circle and the photoelectrical registering of the division positions, the term under consideration would contain the errors of *all* the diameters, for all the diameters would have been read *k* times within one series. In that case the last two terms in (6), in addition to being trivial, would have been eliminated. All the necessary measurements would be executed by one or two series (depending on the accuracy aspired at) within about three hours.

As above indicated, the term $(1/pk)\Sigma F$ in (6) can be determined by successive approximations. In the first step we take, instead of the unknown values of F, the corresponding values E, thus

$$\begin{aligned} E_1 &= E + \frac{1}{pk}\,\Sigma E \\ E_2 &= E + \frac{1}{pk}\,\Sigma E_1 \\ &\dots\dots\dots\dots \\ E_i &= E + \frac{1}{pk}\,\Sigma E_{i-1} \end{aligned} \tag{7}$$

The operation is repeated until the term $(1/pk)\Sigma E_i$ has become practically constant, i.e. until $E_i \approx E_{i-1}$. This is usually arrived at by the second or third approximation.

In addition to the procedure just cited, of settling the question of the term $(1/pk)\Sigma F$, the following one has also been applied. The values of E (6), are miltiplied by $k/k-1$. Denoting by E_o the result obtained, there will be $E_o = (k/k-1)E$. Thus

$$E_o = -F + \frac{1}{p(k-1)}\Sigma F + \frac{1}{n}\,\Sigma F - \frac{1}{pn(k-1)}\,\Sigma F \tag{8}$$

The successive approximations proceed as above:

$$\begin{aligned} E_1 &= E_o + \frac{1}{p(k-1)}\,\Sigma E_o \\ E_2 &= E_o + \frac{1}{p(k-1)}\,\Sigma E_1 \\ &\dots\dots\dots\dots \\ E_i &= E_o + \frac{1}{p(k-1)}\,\Sigma E_{i-1} \end{aligned} \tag{9}$$

This procedure seems to be more correct if one keeps in mind the way of calculating E, the expressions (1) through (6) as well as the quantities appearing in the terms (6). Moreover, it supplies the final results somewhat faster. Following the third approximation the differences between the results, yielded by the two procedures, were reduced to ±0.015, i.e. the final E were practically equal.

In consideration of all above stated, the accuracy of visual measurements inclusive, the quantities expressed by (7) or (9) can even with E_1 be taken as the true corrections to the given circle diameter.

How close is the approximation of errors, derived in the manner just indicated, can be estimated by using the expression, deducible from (9), considering all the quantities in it as being independent. Thus we have

$$\epsilon_E^2 = \left(1 + \frac{1}{p(k-1)}\right)\epsilon_o^2 \tag{10}$$

By identical procedure the error ϵ_{E_o} in (8) is also determined. It is thereby assumed that we have removed completely all errors in the nonuniformly distributed diameters, appearing in the term $(1/p(k-1))\Sigma F$, and that the measuring errors in them were the only remaining. Therefore

$$\epsilon_{E_o}^2 = \left(\frac{1}{p} + \frac{1}{p(k-1)} + \frac{1}{pn} + \frac{1}{pn(k-1)}\right)\epsilon_x^2 + \left(\frac{1}{n} + \frac{1}{pn(k-1)}\right)e^2 \tag{11}$$

Here, by ϵ_x (previously by ϵ_{ji}) the measuring error of one diameter. The determination (evaluation) of ϵ_x is usually accomplished through measurements of the same diameters in both senses of circle motion within one series. The last term appears as a result of the neflectings, related to earlier, conditioned by the method applied. By e are denoted the true accidental errors in the circle lines positions, characteristic of workmanship quality of individual circles. Since we are dealing with the sums of errors of the uniformly distributed diameters, we can assume their systematic errors as mutually cancelled, except for the shortperiodic ones, whose effect should be taken into account at accuracy estimating. The errors e can otherwise be estimated by differences of corrections in neighbouring diameters.

3. CIRCLE INVESTIGATION

Correction determination of 1080 circle diameters (10' spacing) by Nikolić's method involved the mounting of k=5 visual micrometer pairs with the following angular spacing in reference to the prime microscope pair: $Y_1 = 17°20'$, $Y_2 = 36°20'$, $Y_3 = 85°10'$ and $Y_4 = 120°20'$. The measurements implied p = 4 programmes each comprising m = 12 series. Accordingly, 48 series in all, were produced. Any individual series involved n = 18 circle positions, at each 10° distance at the direct and as many at the retrograde circle rotating. The starting diameter was the one defined by the 130°0' division line, the instrument's tube occupying thereby a horizontal position. Since we had W = 50', the starting diameter in the second series was the one at 130°50' marking. The starting diameter in the remaing three programmes were those at 13°10', 130°20' and 130°30'. Unfortunately, we were denied the possibility of producing the fifth programme, which should have started at 130°40' marking due to our obligation of returning two microscope pairs borrowed from the LMC.

A half of the measurements (two programmes) was carried out by Dj. Bozhichkovich and the other half was executed jointly by two observers: Dj. Bozhichkovich and M. Mijatov. In those instances where two observers were at work, the former observer read off the first five microscopes and the latter the reamaining opposite five. Accordingly, the measurement of one diameter is constituted by the mean of readings by both observers. The measurements, loudly pronounced, were recorded on a magnetoscope tape, to be later, usually the next day, replayed and transcribed in the observer's notebook.

The measurements were produced in the afternoon and evening hours in the closed pavilion, mostly by cloudy or reiny weather. Most often, two series of measurements daily were realized. The measurements of the first series were performed usually by one observer and those in the second series by both observers.

The average series of measurements took about 2^h30^m if executed by one observer and 1^h45^m if performed jointly by both observers It took about 102 hours in all to accomplish the entire examination. The air temperature inside pavilion ranged from 0.5°C to 7.0°C, the average being 3.6°C. The maximum temperature variation during a series amounted to 1°C, its average being 0.3°C. The temperature circumstances, prevailing at our examinations, may therefore be termed as rather stable. The temperature inside instrument was not measured. As a precaution measure, the circle illumination was turned on 15 minutes before starting the examinations. When two measuring series in the same evening were produced, the circle illumination between the first and the second series was not interferred with.

As only diameters at 10'spacing were measured, the interpolation had to be performed for the go–between diametereres with 2' spacing. In order to reduce the effect of the accidental errors in the directly investigated diameters, on those interpolated ones, one diameter is understood as a mean of three consequtive division lines. The division lines 8', 10' and 2' were set upon. In placing the desired diameter under the prime microscope pair, care has been taken to get the 10' line as close to the microscope index as possible, in order to achieve the readings on all the microscopes to be approximately equal. In view of low eccentricity (about 5") of the LVC circle, this presented no difficulty. Hence, we even could dispense with the micrometer runs.

4. THE CORRECTIONS DEDUCED AND THEIR ACCURACY

The forming of reading means and their cheking for gross errors was performed in the observer's notebooks The diameter readings were transferred on punched cards for the processing on the WANG 2200B computer of the Belgrade Observatory.

Following three successive approximations, corrections were derived of 1080 diameters. These corrections are presented in Table 1. These corrections appear with three decimal places as a result of our computer failing to round up the figures. The above corrections are also illustrated in Fig. 3. As evident, the LVC circle diameter corrections lie all within ± 1".5 boundaries.

By means of the above corrections we computed:

$$\left(\frac{\Sigma E^2}{1079}\right)^{\frac{1}{2}} = \pm 0.50; \quad \left(\frac{\Sigma (E_i - E_{i+1})^2}{1079}\right)^{\frac{1}{2}} = \pm 0.25\,;$$

$$\frac{\Sigma\,|E|}{1080} = 0.40; \quad \frac{\Sigma\,|E_i - E_{i+1}|}{1080} = 0.20$$

Table 1. Corrections for 1080 diameters at 10′ spacing of the 2′ divided circle of the Belgrade LVC.

0°	−0″.214	0″.135	0″.243	−0″.390	0″.355	−0″.206
1	0.384	0.444	0.343	0.213	0.021	0.293
2	0.200	−0.135	−0.003	−0.342	−0.019	−0.061
3	−0.092	0.011	−0.159	−0.228	−0.050	−0.230
4	−0.117	0.114	0.027	−0.276	−0.100	0.050
5	−0.226	−0.439	−0.263	−0.147	−0.189	−0.206
6	−0.299	−0.546	−0.113	−0.315	−0.257	−0.712
7	−0.442	−0.684	−0.756	−0.476	−0.696	−0.394
8	−0.814	−0.747	−0.794	−0.791	−0.636	−0.985
9	−1.019	−0.814	−0.538	−0.748	−0.998	−0.575
10	−0.537	−0.822	−0.896	−0.671	−1.050	−0.838
11	−0.856	−0.950	−0.859	−0.904	−1.003	−0.717
12	−0.887	−0.821	−0.942	−1.017	−0.671	−0.675
13	−0.940	−0.851	−1.225	−0.865	−0.937	−1.010
14	−0.915	−0.819	−0.642	−0.965	−0.444	−0.832
15	−0.741	−0.411	−0.495	−0.599	−0.698	−0.181
16	−0.325	−0.850	−0.750	−0.837	−0.903	−0.644
17	−0.571	−0.448	−0.786	−0.806	−1.067	−0.869
18	−0.504	−0.623	−0.402	−0.369	−0.538	−0.316
19	−0.005	−0.540	−0.375	−0.352	−0.248	−0.216
20	−0.743	−0.735	−0.588	−0.169	0.157	0.098
21	0.224	−0.081	0.081	0.006	0.220	0.071
22	0.097	0.165	−0.195	−0.369	−0.175	−0.089
23	0.090	0.076	−0.546	0.192	−0.280	−0.215
24	−0.111	−0.399	−0.082	−0.229	0.013	−0.073
25	−0.361	−0.588	−0.053	−0.476	−0.502	−0.941
26	−0.141	−0.069	−0.086	0.347	−0.353	−0.300
27	−0.231	0.154	−0.266	−0.552	−0.167	−0.226
28	−0.405	−0.386	−0.335	−0.017	0.108	−0.112
29	0.046	0.026	0.015	0.116	−0.204	−0.005
30	−0.113	−0.350	−0.175	−0.442	−0.358	−0.792
31	−0.526	0.032	−0.466	−0.383	−0.398	−0.408
32	−0.167	−0.094	−0.376	−0.202	0.125	0.029
33	−0.083	0.074	0.346	−0.165	−0.113	−0.018
34	−0.367	−0.367	−0.101	−0.293	−0.217	−0.555
35	−0.253	−0.187	−0.236	−0.171	−0.217	−0.428
36	−0.584	−0.847	−0.586	−0.999	−0.884	−0.959
37	−0.643	−0.639	−0.667	−0.576	−0.886	−0.666
38	−0.027	0.448	0.635	0.072	0.672	0.356
39	0.016	0.207	0.034	0.356	−0.045	−0.033
40	0.047	0.078	−0.041	0.243	0.371	0.180
41	−0.129	0.008	−0.060	0.352	0.074	0.075
42	0.190	−0.616	−0.104	−0.191	−0.187	−0.257
43	−0.051	−0.352	−0.262	−0.374	−0.215	−0.455
44	−0.482	−0.293	−0.457	0.061	−0.444	−0.320
45	−0.377	0.085	−0.213	−0.211	−0.105	−0.016
46	−0.155	−0.573	−0.351	−0.546	−0.614	−0.019
47	−0.275	−0.140	−0.399	−0.379	−0.123	−0.405
48	−0.275	−0.461	0.148	−0.127	−0.296	−0.303
49	−0.039	−0.098	−0.265	−0.098	−0.078	−0.392
50	−0.248	−0.569	−0.384	−0.156	−0.332	−0.313
51	−0.396	−0.531	−0.528	−0.537	−0.211	−0.279
52	−0.234	−0.364	−0.324	−0.211	−0.307	−0.669
53	−0.489	−0.413	−0.298	−0.296	−0.363	−0.289
54	−0.176	−0.312	0.172	−0.135	−0.280	−0.181
55	−0.145	0.007	−0.216	−0.479	−0.176	−0.086
56	−0.569	−0.931	−0.630	−0.492	−0.570	−0.544
57	−0.388	−0.182	−0.389	−0.489	−0.490	−0.473
58	−0.594	−1.162	−0.880	−0.878	−1.384	−0.842
59	−1.441	−1.090	−1.107	−1.015	−0.911	−0.969
60	−0.742	−0.429	−0.605	−0.671	−0.502	−0.671
61	−0.641	−0.666	−0.304	−0.579	−0.443	−0.741
62	−0.491	−0.410	−0.343	−0.188	−0.491	−0.247
63	−0.754	−0.430	−0.434	−0.774	−0.709	−0.764

Table 1 (continued)

64°	−0″.586	−0″.297	−0″.507	−0″.127	−0″.445	−0″.778
65	−0.534	−0.083	−0.444	−0.884	−0.399	−0.475
66	−0.750	−0.593	−0.288	−0.638	−0.191	−0.592
67	−0.452	−0.632	−0.260	−0.136	0.049	−0.452
68	−0.346	−0.236	−0.285	−0.371	−0.145	−0.217
69	−0.319	−0.288	−0.451	−0.208	−0.162	−0.591
70	−0.178	0.024	−0.194	−0.169	−0.257	−0.025
71	−0.587	−0.275	−0.463	−0.378	−0.395	−0.447
72	−0.281	−0.415	−0.003	−0.194	−0.279	−0.004
73	−0.084	−0.297	−0.096	−0.351	−0.461	−0.233
74	−0.437	−0.191	−0.141	−0.249	−0.191	−0.343
75	−0.026	−0.050	−0.528	−0.305	−0.441	0.210
76	−0.326	−0.269	−0.337	−0.037	−0.048	−0.148
77	−0.058	−0.361	0.122	−0.146	−0.063	−0.065
78	0.046	0.191	0.313	−0.038	0.120	−0.119
79	0.153	−0.056	0.138	0.176	0.226	−0.141
80	0.530	0.260	0.705	0.333	0.195	0.224
81	0.162	0.222	0.054	0.302	−0.004	0.100
82	−0.044	0.003	0.002	−0.050	−0.049	0.157
83	0.014	−0.068	−0.254	−0.160	0.207	−0.285
84	0.004	−0.369	−0.496	−0.565	−0.626	−0.692
85	−0.327	−0.517	−0.415	−0.446	−0.259	−0.301
86	−0.577	−0.543	−0.341	−0.511	−0.083	−0.204
87	−0.518	−0.336	−0.103	−0.528	−0.417	0.006
88	−0.066	0.190	−0.406	−0.133	0.082	0.108
89	0.192	−0.074	−0.266	−0.009	−0.310	−0.021
90	0.056	−0.429	−0.294	−0.097	−0.690	−0.369
91	−0.327	−0.438	−0.256	−0.397	−0.278	−0.517
92	−0.415	0.125	−0.008	0.266	−0.301	−0.066
93	0.134	0.576	0.320	0.125	0.151	0.278
94	0.497	0.759	0.406	0.430	0.389	0.798
95	0.524	0.665	0.352	0.541	0.802	0.518
96	0.289	0.030	−0.142	−0.024	−0.249	−0.154
97	−0.147	−0.058	−0.028	0.196	0.086	−0.021
98	0.017	0.263	0.322	−0.050	−0.105	0.165
99	0.053	0.243	0.125	−0.147	0.389	0.218
100	0.202	−0.030	0.159	−0.145	−0.308	0.389
101	0.076	0.003	0.018	−0.062	−0.214	−0.133
102	−0.554	−0.542	−0.354	−0.391	−0.477	−0.804
103	−0.366	−0.629	−0.475	0.167	0.413	0.473
104	0.207	−0.320	−0.552	−0.639	−0.295	−0.015
105	0.151	−0.189	−0.044	0.393	0.455	0.413
106	0.591	1.005	1.021	1.019	1.312	1.075
107	0.962	0.840	1.185	1.261	1.084	0.752
108	0.635	0.831	0.879	0.814	0.831	1.042
109	1.091	0.930	0.764	0.786	1.069	0.997
110	0.633	0.538	0.532	0.645	0.695	0.840
111	0.546	0.625	0.703	0.841	0.505	0.432
112	0.584	0.847	1.153	1.177	0.943	0.882
113	1.343	0.849	C.929	0.948	1.187	0.914
114	0.886	0.993	0.991	1.138	0.855	1.175
115	1.103	1.049	1.270	1.106	0.871	0.858
116	0.861	1.092	1.205	1.128	1.265	1.211
117	1.213	0.876	0.945	0.932	1.102	1.239
118	0.951	1.158	0.738	0.588	0.571	1.205
119	1.130	1.063	0.754	0.656	0.894	0.718
120	0.631	0.614	0.849	0.632	1.123	0.861
121	1.196	1.027	0.893	1.379	1.072	1.022
122	1.014	1.149	0.861	1.226	1.116	1.046
123	1.025	1.348	1.307	1.263	1.022	1.454
124	1.186	0.472	0.605	0.684	0.446	0.830
125	0.770	0.453	0.636	0.492	0.712	0.631
126	0.467	0.591	0.531	0.542	0.444	0.560
127	0.237	0.577	0.730	0.357	0.514	0.220

Table 1 (continued)

128°	0".489	0".135	-0".008	0".464	0".382	0".385
129	0.409	0.340	0.174	0.460	0.327	0.350
130	0.121	0.303	0.381	0.758	0.344	0.136
131	0.400	0.238	0.412	0.165	0.295	0.345
132	0.129	0.394	0.207	0.237	0.308	0.309
133	0.063	0.094	0.096	0.167	0.228	0.121
134	0.043	0.087	0.346	0.153	0.500	0.256
135	0.048	0.040	−0.043	0.197	-0.261	−0.168
136	0.237	0.466	-0.146	0.328	0.363	0.456
137	0.072	0.181	0.193	0.513	0.595	0.466
138	0.319	0.076	0.241	0.465	0.244	−0.023
139	0.201	0.668	0.555	0.191	0.391	0.378
140	0.354	0.402	0.168	0.297	−0.164	0.317
141	0.464	0.081	0.447	-0.082	0.283	0.253
142	0.125	0.147	0.145	−0.162	−0.020	-0.125
143	0.078	-0.318	0.115	−0.064	−0.133	0.127
144	−0.179	0.344	-0.034	0.058	0.046	-0.000
145	0.148	−0.180	−0.040	0.207	−0.282	−0.145
146	0.215	0.307	0.004	0.150	−0.120	0.243
147	-0.019	0.037	−0.025	0.221	0.119	0.183
148	0.094	0.336	0.439	0.474	0.261	−0.054
149	0.084	0.031	0.386	0.085	0.136	0.239
150	0.404	0.711	0.176	0.305	0.617	0.117
151	0.403	0.375	0.374	0.340	0.337	0.342
152	0.394	0.823	0.439	0.684	0.581	0.575
153	0.589	0.487	0.814	0.543	0.255	0.359
154	0.582	0.444	0.220	0.449	0.507	0.217
155	0.025	0.255	0.426	0.306	0.218	0.206
156	0.240	0.636	0.296	0.595	0.304	0.209
157	0.342	0.168	0.364	0.314	0.200	0.155
158	0.390	0.179	−0.045	0.295	0.376	0.323
159	0.255	0.021	0.391	0.138	0.346	0.545
160	0.124	0.572	0.267	0.354	0.398	0.385
161	0.153	0.321	−0.119	0.330	0.123	0.488
162	0.180	0.032	0.095	0.087	0.264	0.051
163	-0.216	0.229	0.263	0.099	−0.057	0.176
164	0.315	0.222	0.142	-0.004	0.166	0.316
165	0.089	0.024	0.048	0.449	0.310	0.047
166	0.509	0.528	0.355	0.050	0.251	0.224
167	0.414	0.565	-0.074	0.281	−0.077	0.264
168	0.268	-0.218	−0.134	-0.351	−0.387	−0.505
169	-0.342	−0.386	-0.309	-0.258	−0.136	−0.330
170	−0.368	−0.293	−0.289	-0.687	−0.586	-0.334
171	−0.543	−0.427	−0.266	-0.598	0.010	−0.172
172	0.165	−0.279	-0.246	−0.342	−0.355	−0.036
173	−0.259	−0.379	−0.434	−0.215	−0.143	−0.406
174	−0.347	−0.064	0.111	0.519	0.131	0.149
175	0.136	0.071	0.263	0.038	0.164	0.078
176	0.312	0.572	0.372	0.236	0.346	0.292
177	0.517	0.085	0.225	0.014	0.246	0.286
178	−0.173	0.026	-0.423	−0.032	−0.154	−0.102
179	−0.470	−0.176	−0.029	−0.124	−0.679	-0.159

By making comparison of the old (Fig. 2) and the new (Fig. 3) circle corrections one realizes that our circle's division did not undergo any appreciable change in 16 years elapsed, even though some damaging is now noticeable, which previously was not present. No wonder then that the general features of the LVC circle corrections demonstrate close resemblance with those of the LMC corrections (Sadžakov, Šaletić, 1968; Trajkovs-

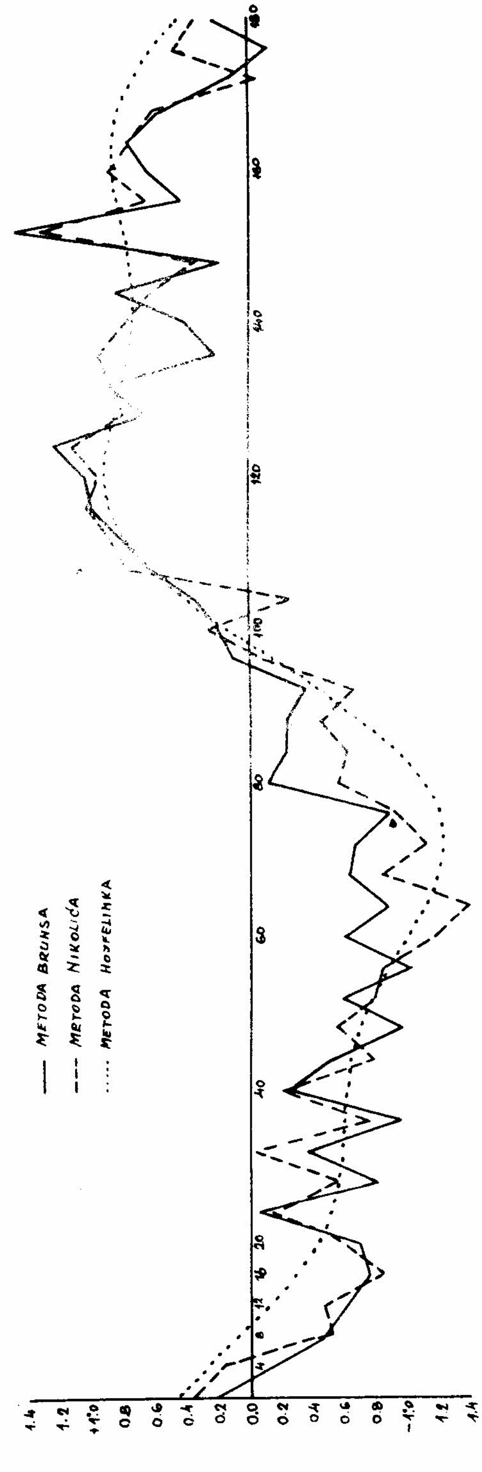

Fig. 2. Circle corrections of the LVC of the Belgrade Observatory determined in 1964. (Nikolić, 1968a)

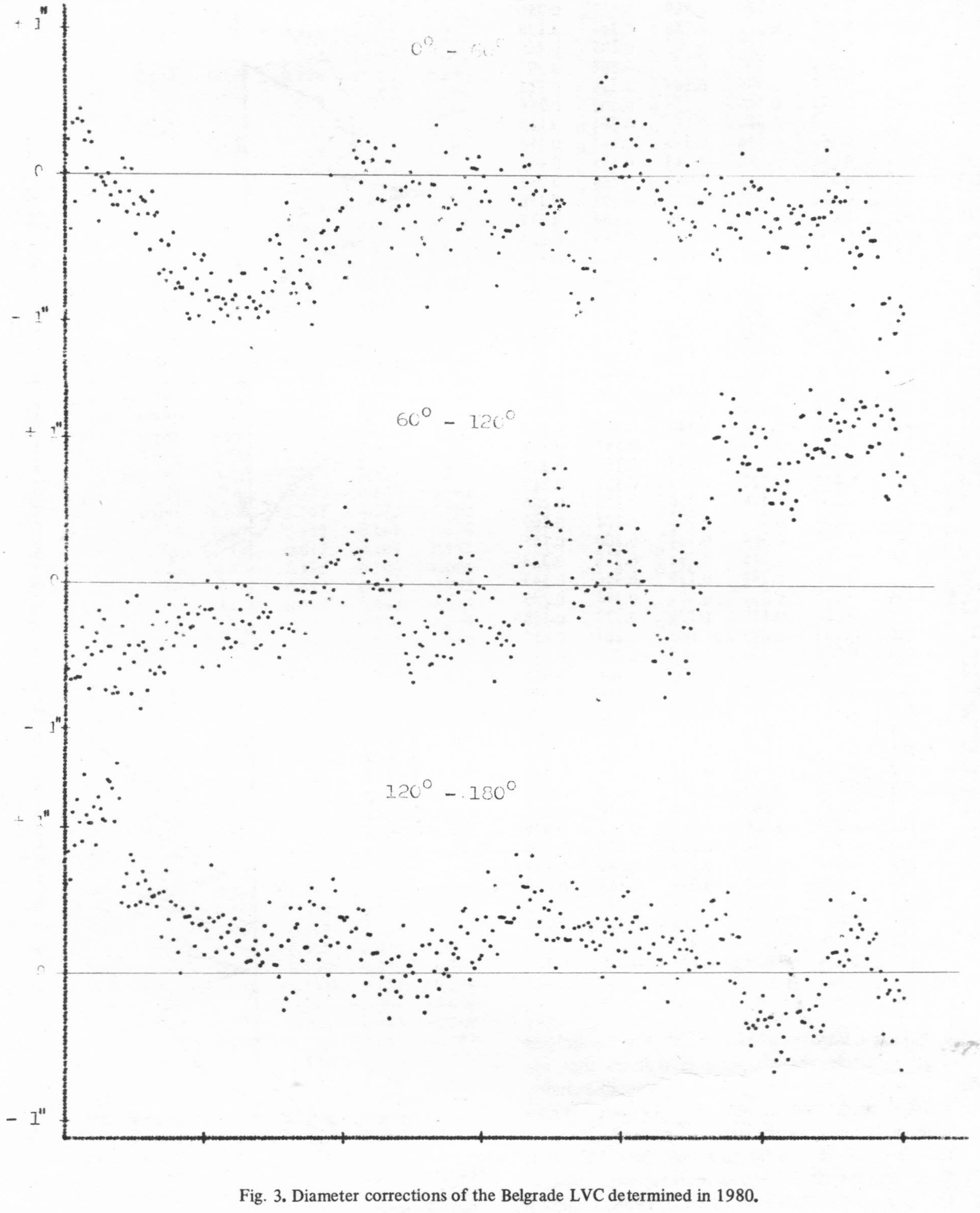

Fig. 3. Diameter corrections of the Belgrade LVC determined in 1980.

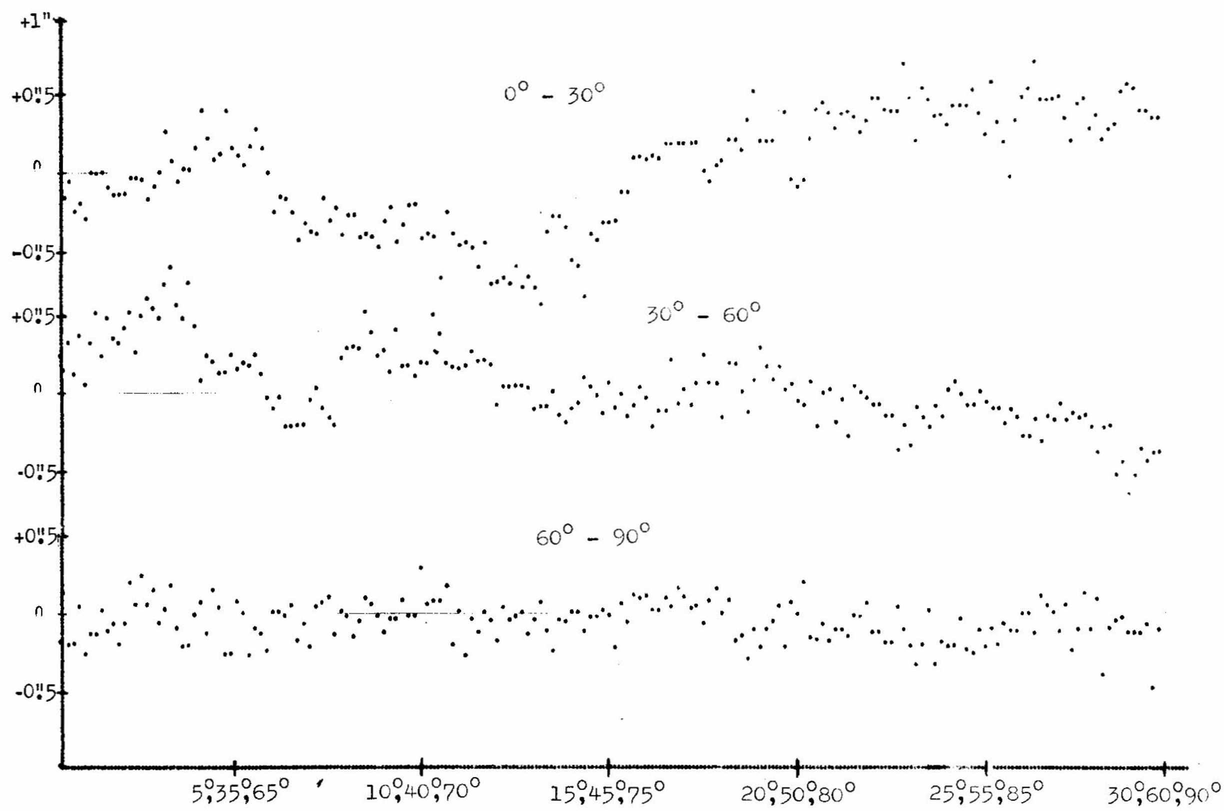

Fig. 4 Corrections to the mean readings of two perpendicular diameters.

ka, 1979). This may, in some way, be taken as providing confirmation of the corrections, brought out by this investigation, being real ones, as both circles have been manifactured at Askania at about the same time (early twenties).

The corrections to the mean readings of two diameters, lying at 90° to each other, are illustrated in Fig. 4. It can be seen that these mean corrections, denoted by $\bar{E}$, are all within the ± 0."85 limits. We calculated, as before

$$\left(\frac{\Sigma \bar{E}^2}{539}\right)^{\frac{1}{2}} = \pm 0.27; \quad \left(\frac{\Sigma (\bar{E}_i - \bar{E}_{i+1})^2}{539}\right)^{\frac{1}{2}} = \pm 0.17;$$

$$\frac{\Sigma |\bar{E}|}{540} = 0.21; \quad \frac{\Sigma |\bar{E}_i - \bar{E}_{i+1}|}{540} = 0.14$$

From the above numerical values, as well as from the curve illustrating the mean corrections Fig. 4, one realizes that they are not large.

The determination of the mean square error of the corrections to 1080 diameters at 10' spacing, procceds by inserting in the expresions (10) and (11), developed for the accuracy estimate of the correction determination by Nikolić's method: the number of programmes (p=4), number of microscope pairs (k=5), and the number of circle positions for one of rotation senses (n=18).

In order to determine ϵ_x – mean square error of the mean value of two readings the same diameter, the readings made at both direct and reverse circle rotation were analysed. In this, the measurements of the same diameters in the framework of 36 series (12 series had to be left unused on account of an unexpected technical difficulty) were emploed. The results was $\epsilon_x = \pm$ 0". 18.

Our measuring series lasted, on the average, four times 30^m – the usually admitted duration of a series. Hence, we tried to bring out, if possible, the diameter changes depending on time. It was demonstrated by analysis that some changes of the kind were present in the course of practically all the series. Yet, this dependence turned out to be mostly weak, the correlation coefficients being usually bellow 0.5. This was also confirmed by the error $\epsilon_x = \pm$ 0".155, deduced from the differences of reading the same diameter relieved of the time dependent effects.

There were, in the course of these measurements, seven days in a row on which no interference whatever with the microscopes has taken place (e.g. illumination adjustment, lamp bulbs replacements, microscope drum displacings accidental knocks against some from among the microscope ,,forest"), except for the focusing. Over this period the values B_i in (4), characterizing the microscope positions relative to the fictitious mean one, do not practically display any variation. This, in turn, lends a kind of confirmation of high stability of the microscopes, having once been fastened in their places. Hence our inclination to regard the slight displacements of the microscopes during the measurements as being due to the observers' fatigue. One should keep in mind that the mean reading of any diameter is formed from the readings made at both direct and reverse circle rotation, entailing approximately the same mean time for all mean diameter readings. Consequently, the slight variations stated in the diameter readings are largely conpensated and do not practically affect the mean readings. All things considered, the mean square error (in the diameter double readings) we adopt, is $\epsilon_x = \pm 0''.15$. It is obvious at once that it is practically equal to the conventional error of a *single* reading by visual microscopes, resulting from series four times as short (Zverev, 1954).

Owing to the neglectings, above indicated, entailed by the method as such, an error is comitted whose amount can be estimated in the following way. With regard to the uniform distribution of the diameters, their systematic error can be assumed as largely removed (save the short period ones). Consequently, the accidental diameter errors are the only ones left over. From the differences of the neighbouring corrections we find the accidental errors in our circle diameters $e = \pm 0''.18$, $e = \pm 0''.25/\sqrt{2}$. On inserting these values in the second term in (11), its value becomes $(\pm 0''.04)^2$. However, as the possible existence of short period division errors are disregarded by this way of treating, we scrutinized the values of the sums $(1/15)\Sigma E$ and $(1/20)\Sigma E$. The deduced values have further been treated as random quantities. Their possible disregarding would produce errors $\pm 0''.06$ and $\pm 0''.05$, respectively. Even though these values have, in their turn, been obtained from corrections affected by errors, it still seems to us that the value $(\pm 0''.06)^2$ of the second term in (11) is a fair representative of the error comitted by the said neglectings.

On inserting the above values in (10) and (11) we obtain $\epsilon_E = \pm 0''.11$ for the mean square error of our circle division corrections.

The trustworthiness of the mean square error just stated can be judged from the measurings executed in the first series, repeated twice, of the first programme. The second measuring tour of the first series differed from the first tour in that each circle line was set on twice. The necessary time for such a measuring series to be performed by a single observer amounts to 4^h30^m, which is far to long. We, therefore, abstained from this sort of measurements. The results of this lengthy measurement series were processed but were omitted from the actual derivation of corrections. Since the same diameters were measured in both series, 90 values according to (5) were furnished by each one of the series. From the differences of the corresponding values we deduced $\epsilon_{E_{ii}} = \pm 0''.24$. The fact being that the corrections are practically obtained as the mean value of four independent values (5) (one from each programme, for each diameter), we have $\epsilon_E = \pm 0''.12$, which agrees well with the adopted one.

The mean correction to two perpendicular diameters investigated were determined with the accuracy $\epsilon_E = \pm 0''.09$.

Considering that, at determining the zenith distance of the observed celestial object, two positions of the instrument and the circle (CE and CW) are made use of, the zenith distance can be assumed free from the systematic errors in the circle diameters involved, with an accuracy of $\pm 0''.06$.

5. CONCLUSIONS

From what has above been brought foreward the following may be stated:

1. The Nikolić's method of circle investigation proved once again highly efficient. It can, therefore, be recommended for such investigation, in defiance of some minor deficiencies. This applies in particular to investigations involving automatic circle reading. If the version b. of the method is used, as we ourselves did, it is desirable to accomplish k = 3 programmes. where k – the number of microscope pairs. For higher accuracy it is necessary, before starting *k* new programmes, to have microscope pairs redistributed, ect.
2. The investigation of the Belgrade LVC circle diameters at 10' spacing has been inplemented with the mean error $\epsilon_E = \pm 0''.11$. All the corrections are found within $\pm 1''.5$ limits, displaying a manifestly systematic character. There are no distinct jumpings from one division line to its next, so all the measurements can be assumed as having been executed without gross errors.
3. It follows from our investigations that LVC circle is of good quality, its main features having not undergone any noteworthy changes in 16 years elapsed since its first investigation.

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