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ANALYSIS OF SOME CHARACTERISTICS OF THE LEVELS OF THE BELGRADE VERTICAL CIRCLE

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SUMMARY: On the basis of laboratory examinations of the levels of the Belgrade Vertical Circle, executed in the period 1963 to 1981, mean division values, as well as their dependence on time, air temperature and bubble length, are derived. The formulae (4) and (5) are proposed for use in the reduction of astronomical observations. The level division values, as derived by laboratory measurements, proved to fit in the regular astronomical observations made with our Vertical Circle.

1. ORGANIZATION OF LABORATORY INVESTIGATIONS

Account is given in the present paper of the results of investigation of two levels of the Belgrade Large Verical Circle (Askania, N° 80118), with which the inclination of the LVC vertical axis is being determined. The objective of the investigation performed was the determination of the level division values to be applied in the reduction of the astronomical observations carried out with this instrument. The level investigation started in 1963, as noted in Tables I and II. The lower level (L) on the instrument has been examined 63 times, while the number of investigations of the upper level (U) amounts to 48. The method employed at first was that of Vassiley, but alter on we passed to the Wanach method. The measurements have, overwhemingly, been executed in a special box (thus. in an insulated medium), whereby several investigators have taken part. A minor part of examinations has been performed in the Geodetic –Geophysical Laboratory of the Hungarian Academy of Sciences (Sopron, Hungary) with various level triers, under various conditions, including artificial heating. Yet, these additional examinations failed to affect, to any appreciable degree, the results originally derived at our observatory.

The examinations have been carried out at various temperatures: they ranged from $-3^\circ\!.5$ C up to $+27^\circ\!.0$ C with the L level and from $-3^\circ\!.6$ C up to $+27^\circ\!.0$ for the U level. The L level bubble lenghts have been varied from 16.2 to 30.0 divisions and those of the U level from 16.7 to 30.0 divisions. In effecting these investigations we benefited from earlier experiences of Teleki and Grujić (1982).

Two level triers have been used in these examinations: „Askania" (N° 630348) as long as Wanach method was applied, and the level trier „Bamberg" (N° 630348) since our switching to the Vassilev method. In the reduction of our measurements the values of the level trier divisions, as furnished by the expressions below, have been used, respectively:

$$p\,(\text{Askania}) = 0''\!.99983 + 0''\!.00013\,(t_i - 13^\circ\!.8)$$
$$p\,(\text{Bamberg}) = 0''\!.99302 + 0''\!.00006\,(t_i - 14^\circ\!.7)$$

As evident, the level trier division values are subject to the temperature t_i effects. The level trier divided circle was moved by one division in the examinations under the Vanach method, while the motion from 2 to 3 divisions has been applied with the Vassilev method. The positions of the level bubble ends have been read up at 1.5 to 2 minutes intervals. The air temperature changes during any one of the sets of measurements did not exceed $0^\circ\!.2$ C.

2. ANALYSIS OF MEASUREMENTS MADE IN THE PERIOD 1963 to 1981

The level division values obtained (Tables I and II) have been submited to analysis with respect to three factors: temperature, bubble length and time. The aim we thereby had in view was the establishing of the best possible relation for the calculation of the division values. Three forms of such relation were alternatively adopted:

$$\lambda_i = \lambda_{01} + \alpha_1\,(T_i - T_o) + \beta_1\,(t_i - t_o) + \gamma_1\,(l_i - l_o) \tag{1}$$

$$\lambda_i = \lambda_{02} + \alpha_2\,(T_i - T_o) + \beta_2\,(t_i - t_o) + \gamma_2(l_i - l_o) + \delta\,(t_i - t_o)^2 + \xi\,(l_i - l_o)^2 \tag{2}$$

$$\lambda_i = \lambda_{03}\,\exp\,[\alpha_3\,(T_i - T_o) + \beta_3\,(t_i - t_o) + \gamma_3\,(l_i - l_o)] \tag{3}$$

Table I. Results of the L level investigation in the period 1963–1981.

N°	Date in fractions of year	Observers	Method	t	*l*m	Mean division value	Quality mark	Locality of measurement
1	1963.57	MM	Va	25°.6C	20.7p	1″.1810		Students' pavilion
2	.58	MM	Va	25.4	20.4	1.1669		Students' pavilion
3	.64	SS	Va	17.6	20.0	1.1084		Main building
4	.64	SS	Va	17.8	26.0	1.1082		Main building
5	1964.07	MM	Va	−0.5	25.5	1.0319		Geodetic pavilion
6	.08	SS	Va	−3.5	21.1	1.0746		Geodetic pavilion
7	1963.74	GT	Vn	18.7	20.0	1.1073	3	Sopron
8	.74	GA	Vn	18.5	20.0	1.1288	3	"
9	.74	GT	Vn	19.0	25.0	1.1317	1	"
10	.74	GA	Vn	19.0	25.0	1.1454	1	"
11	.75	GT	Vn	17.4	30.0	1.1207	1	"
12	.75	GA	Vn	17.5	30.0	1.0953	1	"
13	.75	photograph.	Vn	15.0	20.0	1.0908	3	"
14	.75	"	Vn	15.0	25.0	1.0760	3	"
15	.75	photograph.	Vn	15.0	30.0	1.0554	1	Sopron
16	1964.76	GT	Vn	18.0	25.0	1.1228	2	Sopron (with heating)
17	.76	GA	Vn	18.0	25.0	1.1214	1	"
18	.76	GT	Vn	18.0	25.0	1.1141	1	"
19	.76	GA	Vn	18.0	25.0	1.1212	1	Sopron (with heating)
20	.79	MM	Vn	17.1	22.1	1.1090	2	Levels' box
21	.79	SS	Vn	15.9	22.8	1.1167	2	"
22	1965.95	MM	Vn	15.1	23.7	1.1295	2	"
23	.95	SS	Vn	15.5	23.6	1.1882	4	"
24	.96	SS	Vn	15.7	23.1	1.0412	3	"
25	.96	MM	Vn	16.0	23.1	1.1475	3	"
26	.96	MM	Vn	15.1	24.0	1.1213	2	"
27	.96	SS	Vn	15.5	23.5	1.1029	4	"
28	.99	MM	Vn	15.6	22.4	1.1181	2	"
29	.99	SS	Vn	15.9	22.3	1.1229	3	"
30	1966.01	MM	Vn	12.8	22.9	1.0798	3	"
31	.01	SS	Vn	13.6	22.8	1.1295	3	"
32	.11	MM	Vn	15.4	23.0	1.1915	3	"
33	.11	SS	Vn	16.1	23.0	1.0289	1	"
34	.15	MM	Vn	16.9	21.2	1.1800	2	"
35	.15	SS	Vn	17.1	21.2	0.9960	1	"
36	.82	SS	Vn	17.4	23.2	1.1537	1	"
37	.82	MM	Vn	17.0	23.3	1.1302	2	"
38	.83	SS	Vn	16.2	23.4	1.0811	2	"
39	1967.89	MM	Vn	13.5	23.8	1.1419	1	"
40	.89	SS	Vn	12.6	24.3	1.1651	1	"
41	1968.78	MM	Vn	12.9	22.5	1.1122	3	"
42	.78	BK	Vn	13.1	22.4	1.1463	2	"
43	1976.16	MM	Vn	4.6	21.6	1.0010	4	"
44	.16	MM	Vn	4.8	21.7	0.9820	2	"
45	.19	GT	Vn	2.0	23.2	0.9923	1	"
46	1981.07	DB	Vn	2.5	18.6	0.9106	2	"
47	.09	MM	Vn	3.6	21.4	1.0217	1	"
48	.09	DB	Vn	3.4	22.0	0.9750	2	"
49	.09	MM	Vn	2.8	22.0	0.9420	1	"
50	.09	MM	Vn	12.1	20.9	0.9807	1	"
51	.09	DB	Vn	12.3	20.9	0.9671	1	"
52	.11	MM	Vn	10.1	20.3	0.9476	1	"
53	.11	VT	Vn	10.8	20.2	0.9528	1	"
54	.11	VT	Vn	12.4	17.4	0.9464	1	"
55	.11	MM	Vn	12.7	17.2	0.9458	2	"
56	.12	DB	Vn	11.7	17.8	0.9644	2	"
57	.12	MM	Vn	14.4	16.2	0.9330	2	"
58	.13	VT	Vn	1.6	18.3	0.9493	2	"
59	.13	MM	Vn	2.2	18.2	0.9330	2	"
60	.13	VT	Vn	2.6	18.0	0.9167	3	"
61	.13	DB	Vn	2.7	18.2	0.9251	2	"
62	.15	VT	Vn	2.3	20.1	0.9191	2	"
63	1981.15	MM	Vn	3.7	19.9	0.9175	1	Levels' box

Table II. Results of the U level investigation in the period 1963–1981. For notations see explanation in Table 1

N°	Date in fractions of year	Observers	Method	t_i	l_m	Mean division value	Quality mark	Locality of measurement
1	1963.67	MM	Va	26°2 C	19.4p	1".1024		Students' pavilion
2	.67	MM	Va	20.0	25.1	1.1274		Students'pavilion
3	.73	SS	Va	17.8	20.0	1.0825		Main building
4	.73	SS	Va	17.8	26.6	1.0891		Main building
5	1964.04	SS	Va	−2.6	20.3	1.0280		Geodetic pavilion
6	.04	MM	Va	−3.6	25.4	1.0465		Geodetic pavilion
7	1963.75	GA	Vn	13.8	20.0	1.0862	3	Sopron
8	.75	GT	Vn	18.7	25.0	1.0856	1	"
9	.75	GA	Vn	18.8	25.0	1.0880	1	"
10	.76	GA	Vn	17.8	30.0	1.0831	3	"
11	.76	GT	Vn	17.8	30.0	1.0807	1	"
12	.76	GT	Vn	18.0	20.0	1.1189	3	"
13	.76	photograph.	Vn	18.0	20.0	1.0289	1	"
14	.76	"	Vn	18.0	25.0	1.0583	2	"
15	.76	photograph.	Vn	18.0	30.0	1.0172	1	Sopron
16	1964.81	MM	Vn	14.7	23.1	1.1200	2	Levels' box
17	.81	SS	Vn	15.0	23.1	1.1156	1	"
18	.81	MM	Vn	14.9	22.3	1.0855	1	"
19	.81	SS	Vn	15.4	22.2	1.0756	1	"
20	1966.13	MM	Vn	18.0	24.0	1.1216	2	"
21	.13	SS	Vn	17.7	24.4	1.1202	1	"
22	.84	SS	Vn	14.0	23.0	1.0899	1	"
23	.84	MM	Vn	14.0	23.1	1.1225	1	"
24	1967.86	MM	Vn	14.2	24.1	1.1307	2	"
25	.86	SS	Vn	13.9	24.2	1.1266	2	"
26	1968.90	MM	Vn	15.0	22.3	1.1472	2	"
27	.91	BK	Vn	14.2	22.9	1.1232	2	"
28	1976.16	MM	Vn	3.8	22.2	0.8329	2	"
29	.16	MM	Vn	4.1	22.4	0.9062	1	"
30	.17	GT	Vn	2.1	21.9	0.8922	2	"
31	1981.09	MM	Vn	3.0	21.2	1.0006	4	"
32	.09	DB	Vn	3.2	21.5	0.9725	1	"
33	.09	MM	Vn	3.2	21.3	0.9645	1	"
34	.09	MM	Vn	10.2	22.0	0.9905	1	"
35	.09	DB	Vn	11.2	21.7	0.9941	1	"
36	.11	VT	Vn	11.5	20.3	0.9629	1	"
37	.11	MM	Vn	11.4	20.4	0.9773	2	"
38	.11	VT	Vn	11.5	17.0	0.9516	2	"
39	.11	MM	Vn	12.2	16.7	0.9591	2	"
40	.12	DB	Vn	13.5	17.7	0.9604	2	"
41	.12	MM	Vn	14.0	17.2	0.9762	2	"
42	.13	VT	Vn	2.4	17.1	0.9545	3	"
43	.13	MM	Vn	2.7	17.0	0.9458	2	"
44	.13	VT	Vn	1.6	17.5	0.9257	2	"
45	.13	DB	Vn	2.5	17.3	0.9364	1	"
46	.15	DB	Vn	2.9	17.8	0.9199	2	"
47	.15	VT	Vn	2.8	20.1	0.9135	2	"
48	1981.15	MM	Vn	3.0	20.1	0.9365	1	Levels' box

Labels of the Tables I and II:

Observers: MM – M. Mijatov, SS – S. Sadžakov, GT – G. Teleki, GA – G. Alpar, BK – B. Kubičela, DB – Dj. Bozhichkovich, VT – V. Trajkovska.
Method: Va – Vassilev, Vn – Wanach
Quality marks: 1 – very good, 2 – good, 3 – fair, 4 – bad (these marks are provided by the Wanach scale and relate solely to this method).

where:

λ_i – division value (in seconds of arc), resulting from the i-th measurement
λ_{ok} – most probable division value (in seconds of arc) at T_o, t_o and l_o defined by:

$$T_o = \frac{\Sigma T_i}{n}, \quad t_o = \frac{\Sigma t_i}{n}, \quad l_o = \frac{\Sigma l_i}{n};$$

k = 1, 2 or 3 are indices in expressions (1), (2) and (3)
n – number of measurements
T_i – time of the i-th measurements (in fractions of year)
t_i – temperature with the i-th measurement
l_i – level bubble length with the i-th measurement (in fractions of the level division)
α_k – time coefficient
β_k – temperature coefficient
γ_k – bubble length coefficient
δ and ξ – coefficients with the second order terms.

No quadratic term in T appears in (2), for the preliminary graphic illustration of the division values as a function of time disclosed an unmistakable linear dependence.

The solution of 63 equations relating to the level L and of 48 equations pertaining to the level U by the least square method furnished the most probable mean division values of both levels valid for $t' = +12^\circ\!.0$ C, l'_o = 22.0 divisions and T'_o = 1970.0 (these are rounded up figures of t_o, l_o and T_o), the coefficients as well as the determination errors. The results are summarized in Table III.

undergoes changes, a consequence, in all probability, of the non–adequate inlay of the ampulla in the level body (see, for instance, Tarczy–Hornoch's paper, 1959). This feature of our levels should, therefore, be pursued in the future also.

An special analysis has been performed of the (O–C) residuals of the division values as a function of temperature and the bubble length. Thus it was found that the temperature effects scattering with higher temperatures was conspicuously lesser than the one stated with the low temperature. This is, in our view, a result of decreasing, with higher temperatures, of both the surface tension and the viscosity (Sadžakov, Mijatov, 1968). The behaviour of the (O–C) values dependent on the bubble length, is typified by higher scattering with the growing bubble length. This might be an indication of the existence of some other effects, not accounted for in the present analysis.

Normal distribution test of the (O–C) deviations did not yield any reliable results, due to the low number of intervals and low frequences within these intervals.

Since no significant differences could be stated between the forms (1), (2) and (3), we adopted, for practical resons, the linear form for the calculation of the division values of both levels. Accordingly, we propose, for the effective use, the following expressions:

$$\lambda_L = 1''\!.0615 - 0.0083\,(T - 1970.0) + 0.0029\,(t - 12^\circ\!.0) + 0.0007\,(l - 22.0) \qquad (4)$$

$$\lambda_U = 1''\!.0408 - 0.0063\,(T - 1970.0) + 0.0037\,(t - 12^\circ\!.0) - 0.0004\,(l - 22.0) \qquad (5)$$

Table III. Results of analysis of the totality of level examinations in the period 1963–1981. λ_{ok}, α_k, β_k, γ_k, δ, ξ, k correspond to the expressions (1) – (3); r – the correlation coefficient; ϵ_{reg} is the mean regresion error; ϵ_{res} – mean error of residuals

Level	k	λ_{ok}	α_k	β_k	γ_k	δ	ξ	r	ϵ_{reg}	ϵ_{res}
	1	1″.0615	−0″.0083	0″.0029	0″.0007			0.89	0.12	0.002
L	2	1.0700	−0.0068	0.0036	0.0050	0″.00002	−0″.0013	0.91	0.08	0.001
	3	1.0580	−0.0080	0.0027	0.0008			0.90	0.11	0.001
	1	1″.0408	−0″.0063	0″.0037	−0″.0004			0.85	0.08	0.002
U	2	1.0567	−0.0063	0.0033	0.0010	−0″.0001	−0″.0008	0.87	0.05	0.002
	3	1.0378	−0.0062	0.0037	−0.0004			0.85	0.07	0.002

High correlation can be noted in all three cases. The errors, relating to both levels, are of the same order of magnitude in all three cases considered. The time coefficients α_k, by their assuming considerably values, as well as the „ageing" of the levels (at least in the period 1963 to 1981), produce the diminishing of the division values ($\alpha_k < 0$). The character of the level „ageing"

3. THE ANALYSIS OF MEASUREMENTS MADE IN 1981.

During 1981 the opportunity presented itself of carrying out a greater number of level examinations, following the conclusion of works on an absolute catalogue of declinations of bright stars in the zone $+65^\circ$ to $+90^\circ$

Table IV. Results of analysis of the laboratory level examinations performed in 1981. Notations the same as in Table 3.

Level	k	λ_{ok}	β_k	γ_k	δ	ξ	r	ϵ_{reg}	ϵ_{res}
	1	0\."9850	0\."0025	0\."0094			0.64	0.003	0.0005
L	2	0.9881	−0.0048	0.0221	−0\."0007	0\."0029	0.67	0.002	0.0005
	3	0.9852	0.0026	0.0098			0.64	0.003	0.0006
	1	0\."9914	0\."0027	0\."0066			0.72	0.003	0.0003
U	2	1.0142	0.0071	0.0404	0\."0006	0\."0058	0.86	0.002	0.0002
	3	0.9916	0.0029	0.0069			0.72	0.003	0.0004

Table V. Results of analysis of the laboratory level examinations performed in the period 1963–1981, omitting the level bubble lengths above 24.0 divisions. Notations as in Table 3.

Level	k	λ_{ok}	α_k	β_k	γ_k	δ	ξ	r	ϵ_{reg}	ϵ_{res}
	1	1\."0713	−0\."0075	0\."0029	0\."0100			0.92	0.11	0.001
L	2	1.0647	0.0070	0.0036	0.0135	0\."0001	0\."0008	0.92	0.07	0.002
	3	1.0682	0.0072	0.0027	0.0099			0.92	0.10	0.001
	1	1\."0470	−0\."0052	0\."0048	0\."0051			0.86	0.06	0.002
U	2	1.0488	0.0056	0.0043	0.0137	−0\."0001	0\."0022	0.87	0.04	0.002
	3	1.0433	−0.0051	0.0049	0.0046			0.85	0.06	0.002

declination. We, therefore, analysed separately, 18 sets of measurements effected in 1981. The expressions used were of the form (1), (2) and (3), the condition being $\alpha_k = 0$. The results of these calculations are presented in Table IV.

The deviations (O–C) of these measurements are by a whole order of magnitude lesser than those resulting from the totality of measurements.

The differences between the mean division values obtained from the totality of measurements and the corresponding values provided in 1981, are given by both linear and quadratic forms:

For L level: $-0\overset{''}{.}016$ and $+0\overset{''}{.}006$
For U level: $-0\overset{''}{.}021$ and $-0\overset{''}{.}028$

We assumed the differences of results to be due to the fact that the examinations in 1981 were carried out with the bubble lenght ranging from 16 to 22 divisions, while those covering the whole of the period were executed with the bubble lengths between 16 to 30 divisions (mostly above 22 divisions).

4. PROCESSING OF MEASUREMENTS MADE IN 1963 TO 1981 WITH THE BUBBLE LENGTHS FROM 16 TO 24 DIVISIONS

In view of the above asumption those measurements were taken apart from the whole of the material, which were performed with the bubble lengths between 16 and 24 divisions, i.e. the bubble lengths used in almost all the astronomical observations with the LVC.

The same kind of analysis, as the one previously described, was accomplished here, too. The results obtained are listed in Table V.

The linear dependence furnishes:

$$\lambda_L = 1\overset{''}{.}0713 - 0.0075\,(T - 1970.0) + 0.0029\,(t - 12\overset{\circ}{.}0) + 0.0100\,(l - 22.0) \quad (4')$$

$$\lambda_U = 1\overset{''}{.}0470 - 0.0052\,(T - 1970.0) + 0.0048\,(t - 12\overset{\circ}{.}0) + 0.0051\,(l - 22.0) \quad (5')$$

On comparing the values of β and γ, contained in Table V, and those comprised by Table IV, it becomes evident that there exists a good accordance between them. The difference, referred to in Section 3 is a consequence of the considerable divergence of γ values in Tables III and V. It follows that the level division values are greatly affected by larger bubble lengths and this fact should be given full consideration in the future. Similarly, the finding, stated in Section (3), about the scattering (O–C) being larger with growing bubble lengths, must be taken into account.

There arose the question of whether to use, in the reductions of observations with LVC, the expressions (4) and (5), or else, (4') and (5'). However, we found that the accuracy of the declination determination remained practically the same irrespective of what group of equations was used. That is why we propose the use of the expressions (4) and (5), obtained from the totality of laboratory measurements, considering the fact that there have been, with some of the observations, bubble lengths above 24 divisions.

Care must be taken in the future observations that the bubble length is not above 24 divisions.

5. COMPARISON OF THE RESULTS AND CONCLUSIONS

In the second stage of our study we used the LVC observational data for verifying the reality of the mean division values of both levels, obtained by the laboratory examinations. To this end 37 nights were picked up, with different temperatures and bubble lengths, the condition being, however, that no less than 10 stars have been observed. Mean inclination for each individual night has been applied, derived separately from readings of both U and L levels. As the number of the observed stars on individual nights was different, corresponding weights have been attached to each particular observation.

In the analysis of this material we proceeded from the formulae of Bozhichkovich (1978), wherein the dependencies on temperature t_i and the bubble length l_i are combined:

$$\frac{i''_U - i''_L}{\frac{i_U - i_L}{2}} = (\Delta\lambda_L - \Delta\lambda_U) + (\beta_L - \beta_U)(t_i - t_o) + (\gamma_L - \gamma_U)(l_i - l_o) \tag{6}$$

$$\frac{i_U - i_L}{i_U + i_L} \cdot k'' = (\lambda_{oL} - \lambda_{oU}) + (\beta_L - \beta_U)(t_i - t_o) + (\gamma_L - \gamma_U)(l_i - l_o) \tag{7}$$

where:

i_U and i_L denote the measured inclinations by the upper and lower levels, expressed in divisions;

i''_U and i''_L – measured inclinations by the upper and lower levels, expressed in seconds of arc;

$\Delta\lambda_U$ and $\Delta\lambda_L$ – corrections to the level divisions in seconds of arc;

λ_{oU} and λ_{oL} – most probable mean division value of the upper and lower levels in seconds of arc;

β_U and β_L – temperature coefficients

γ_U and γ_L – bubble lengths coefficients;

k'' – constant (in seconds of arc), obtained by the expressions (4) and (5) as the sum of the division values of the upper and lower levels, corresponding to the mean temperature $t_o = +10.0$C and the mean bubble length $l_o = 22.0$ divisions, reduced to the mean moment 1978.0.

In applying these realtions to the observational material we obtained by the method of least squares, weights being attached (number of stars), the following values:

Table VI. Results of analyses according to equations (6) and (7)

	Values by the equation (6)	Values by the equation (7)
$\lambda_{oL} - \lambda_{oU}$	–	0.0116
$\Delta\lambda_L - \Delta\lambda_U$	−0.1436	–
$\beta_L - \beta_U$	0.0008	0.0009
$\gamma_L - \gamma_U$	0.0113	0.0226
r	0.28	0.17
$\epsilon_{reg.}$	0.15	0.70
$\epsilon_{res.}$	0.005	0.07

Under the second variant the quadratic terms, depending on temperature and the bubble length, with the coefficients $\delta_L - \delta_U$ and $\xi_L - \xi_U$, have been added in (6) and (7). The results are presented in Table VII.

Table VII. Results of analysis according to eqs. (6) and (7) extended by the second order terms

	Values by the equation (6) (quadratic term included)	Values by the equation (7) (quadratic term included)
$\lambda_{oL} - \lambda_{oU}$	–	0.0294
$\Delta\lambda_L - \Delta\lambda_U$	−0.1457	–
$\beta_L - \beta_U$	0.006	0.0066
$\gamma_L - \gamma_U$	0.0118	0.0206
$\delta_L - \delta_U$	−0.0001	−0.0000
$\xi_L - \xi_U$	0.0015	−0.0053
r	0.30	0.20
$\epsilon_{reg.}$	0.09	0.45
$\epsilon_{res.}$	0.005	0.06

In the reduction of our observations the following level division values have been used: $\lambda_L = 0''.992$ and $\lambda_U = 0''.876$, whose difference $\lambda_L - \lambda_U = 0''.116$. Now, let us see what one is getting by performing the analysis with the eqs. (6) and (7). By introducing the relation

$$\lambda_L + \Delta\lambda_L = \lambda_{oL}$$
$$\lambda_U + \Delta\lambda_U = \lambda_{oU}$$

for the same T, t and *l*, using thereby the data from Table VI, we have

$$\lambda_L - \lambda_U = (\lambda_{oL} - \Delta\lambda_L) - (\lambda_{oU} - \Delta\lambda_U) =$$
$$= (\lambda_{oL} - \lambda_{oU}) - (\Delta\lambda_L - \Delta\lambda_U) = 0''.155$$

whereas from Table VII

$$\lambda_L - \lambda_U = 0''.175$$

Thus, the algorithm based on (6) and (7), though approximate one, yields rather accordant results.

By reducing the values from Tables VI and VII to 1976.0 i.e. to the instant for which the values λ_L and λ_U are deduced, the difference $\lambda_L - \lambda_U$ amounts to $0''.154$ if (1) is used and to $0''.156$ if (2) is used. These values can be considered as being comparable with $\lambda_L - \lambda_U = 0''.116$.

On the other hand, from the laboratory measurements (reduced to 1976.0) taken together, we obtain

$$\lambda_{oL} - \lambda_{oU} = 0''.015 \qquad \text{for the form (1)}$$

$$\lambda_{oL} - \lambda_{oU} = 0''.016 \qquad \text{for the form (2)}$$

i.e. the values which are in fair harmony with those in Tables VI and VII ($0''.012$ and $0''.029$).

On inspecting other data too in Tables VI and VII one may well conclude that the observational data of the LVC confirm the reality of the mean level division values as obtained by the laboratory method.

We are induced to state on the present occasion too that no substantial difference whatever is found between the results furnished by (1) and (2). Therefore, the linear form (1) should be accepted in future work, a suggestion that was put foreward in Section 2.

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