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INVESTIGATION OF LEVELS OF THE BELGRADE VERTICAL CIRCLE

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SUMMARY /Two levels attached to the Belgrade Vertical Circle have been investigated both on the examinator and on the instrument itself. Data obtained by both methods are analysed. It could be found that the results provided by the measurements on the instrument itself, using the mercury horizon, were notably more realistic than those obtained with the examinator under laboratory conditions. In addition to the accounting for the temperature and the bubble length effect on the level division value determination and accounting for the systematic effects of the irregulatiries in the inner sliding surface and its graduation is suggested.

1. INTRODUCTION

The level and the graduated declination circle constitute two measuring applicances of the Vertical Circle (VC) whose intrinsinc features enter on essentially equal terms into the measured zenith distance, i.e. declination. Through these two appliences, therefore, the accuracy of determination of the absolute declinations is importantly conditioned in both random and systematical sense. The effect of the levels is commonly more pronounced with the larger instruments (the VC of the Belgrade Observatory belongs to this category) as these are often distinguished by a stronger inclination instability of their vertical axes.

The level investigation has hitherto been implemented chiefly according to Wanach and Vassilev methods under laboratory conditions, using an examinator. The investigation provided, as a result, the mean angular value of the level division including a qualitative–quantitative assessment of the inner sliding surface of the level tube. (The latter investigations, under roughly equal conditions, are known to often yield contradictory results). The analysis of a series of investigations was principally aimed at determining the dependence of the division value on the air temperature and the bubble length. Other possible reasons of the division value variability have earlier been rarely searched for owing, on one hand, to an apriori awarness of their being essentially petty, and on another, the lack of electronic computers entailed their determination and their subsequent utilization to be highly cumbersome.

Numerous investigations confirm the presence of the simillar and larger irregularities in the sliding surfaces inside the level tubes and in their graduations. Habitually, these errors may, conditionally, be divided into **random** and **systematic** ones. The effects of the former on the observing results (zenith distances, declinations, latitudes) may to a considerable degree be reduced. Namely, the vertical circles and zenith telescopes are commonly provided with a pair of levels and the measured inclinations, the bubble lengths, as well as the utilized level graduations are different from one stars to next during the same night. This is all the more true of different nights when the inclinations may differ even in sign. Even the effect of the systematic irregularities just reffered to on the mean measuring results is reduced, although not comletely removed. These effects keep being present since the mean inclination from a series of mesurements of the same star is not necessarily close to zero nor the same in the north and south stars and, equally, the level bubble positions are not necessarily symetrical with respect to the middle of the graduation.

With this in mind we dedicated the present paper to finding out the ways of determining the systematic irregularities in both sliding surfaces and in graduations of our levels and, more broadly, to the problems connected with the calibration of the angular value of the level division.

2. INVESTIGATION OF LEVELS OF THE BELGRADE VC WITH THE EXAMINATOR

Discussion of all the past laboratory investigations of levels of our VC, implemented according to Wanach's and Vassilev's methods, was performed by Mijatov and Trajkovska (1984). The two authors established the division value variability with time, temperature and the bubble length.

In contrast to them we separated in the present paper the latest, relatively large group of laboratory measurements, those from 1981, having processed them in a somewhat different way.

In the period from 24. January to 25. February 1981 there have been effectuated on each of our levels 18 sets of measurements adjusted to the handling according to the Wanach method. The temperature run through the interval $+1^o$ to $+15^oC$, while the bubble length varied from 16 to 22 divisions (our levels' graduations embrace 40 divisions). Depending on the bubble length, individual sets of measurements include from 16 to 22 meaned positions of the bubble middles. Only 16 positions, roughly symetrically distributed with respect to the graduation middle, were processed in our analysis. We acted so for several reasons:

a) This conditions the values of the mean positions of the bubble middles to be evenly distributed within the interval 11th to 29th division irrespective of the bubble length.
b) It is on extremely rare occasions only that the bubble ends come near to the graduation ends during the regular work with our VC and that only when the bubble lengths happen to be uncommonly large.
c) There is always the same number of measurements entering the calculus irrespecitve of the bubble length.

Data handling for each one of the levels was carried out in the following fashion. We first determined by the least square method, for each of the sets j of measures, the coefficients a_j and b_j in the linear set of 16 equations

$(i - 8.5) \cdot E_j = a_j + b_j \cdot (S_{ji} - 20)$ $(i = 1, 2, \ldots, 16; j = 1, 2, \ldots, 18)$ $E_j = 0.99983 + 0.00013 \cdot (T_j - 13.8)$

where:

i – ordinal number of the measurement – zero position on the examinator's disk. This equating is used in order to simplify the calculus without its results being affected, since any following zero position of the examinator screw differed from the preceding one by one division on its disk. To be sure, in order to minimize the effects of errors in the examinator's screw use has been made of different screw's turns and disk's sections.

E_j – Angular value of the examinator disk's division for the temperature during the investigation (Mijatov, Sadžakov, 1968).

a_j – correction to the coordinates' zero, i.e. to the adopted mean position of the disk ($8.5 \times E_j$) in the set j of measurements.

b_j – Mean value of the level division in the particular investigation.

S_{ji} – Mean position of the bubble middle from two measurements at the same disk position i (bubble s' displacement from left to right and vice versa).

20 – the middle of our levels' graduation.

With the coefficients a_j thus determined one made the coordinates' zeros be mutually conforming in all the sets of measurements, forming thereater new sets of $j \times i = 288$ equations for each of the levels

$$Y_{ji} = [A_o + A_1 (T_j - 8) + A_2 (B_j - 19) + A_3 (S_{ji} - 20) + A_4 (S_{ji} - 20)^2 + A_5 (S_{ji} - 20)^4](S_{ji} - 20) \quad (1)$$

$$Y_{ji} = [A_o + A_1 (T_j - 8) + A_2 (B_j - 19)](S_{ji} - 20) \quad (2)$$

$$Y_{ji} = (i - 8.5) \cdot E_j - a_j$$

where T_j and B_j – examinator's temperature and the bubble length, respectively, in the particular investigation j, 8 and 19 being their means from the totality of sets.

Using the method of least squares one determined the values of the unknown coefficients in the formulae (1) and (2). Table I summarizes the coefficients values obtained as well as their rms errors for the upper (U) and lower (L) levels (according to their position on VC).

As apparent, these results reveal both of our levels as having virtually the same dependence of their division values on temperature and bubble length (coefficients A_1 and A_2).

At variance with these, the coefficients A_3, A_4 and A_5, typifying the systematic irregularities in the level sliding surfaces and in their graduations, have opposite signs, implying their effects on the final results to be mostly comparatively slight (below $0''.1$). It is on extremely rare occasions that these features of our levels make themselves felt, when the levels happen to be mutually or otherwise poorly adjusted.

Attention is to be drawn to the notable difference of the coefficient values A_o in the formulae (1) and (2) in both levels, even though they follow from the same observational material. This difference is a consequence, in a way, of different meaning of these coefficients. Specifically, the coefficient A_o in (2) embodies the mean value of the level divisions resulting from the whole of the investigated graduation, while that in (1) is the mean division value as it results from the graduation around its middle. It is precisely this value that is mostly needed in the everyday practice.

For illustration we formed from 18 sets of measurements, for different combinations of instrument's inclinations, the mean differences examinator – level readings, computed by way of (1) and (2) using the coefficients from Table I.

$$O-C = \frac{1}{18} \sum_j [(k-i) \cdot E_j - (Y'_k - Y'_i)_j]$$

$$(i = 1, 2, \ldots, 15; \quad k = i+1, i+2, \ldots, 16)$$

These departures in hundredths of second of arc are listed in Tables II, III, IV and V.

Table I: Coefficients and their rms errors delivered by (1) and (2) for the upper (U) and the lower (L) level.

Level	Set	A_0	A_1	A_2	A_3	A_4	A_5	ϵ
U	1	0″.982 ±.005	.0028 ±.0003	.0070 ±.0007	−.0018 ±.0003	−.00082 ±.00027	.0000061 ±.0000031	±0.107
	2	0″.959 ±.002	.0029 ±.0003	.0068 ±.0007	–	–	–	±0.116
L	1	0″.920 ±.006	.0022 ±.0003	.0080 ±.0009	.0009 ±.0003	.00176 ±.00031	−.0000212 ±.0000037	±0.122
	2	0″.948 ±.002	.0025 ±.0003	.0084 ±.0009	–	–	–	±0.127

Table II: The (O−C) values in 0″.01 for the upper level computed according to (1.)

i \ k	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
1	0	0	0	1	2	0	2	−1	−3	−5	−3	7	2	3	−1
2	1	1	1	3	4	1	4	0	−1	−3	−1	9	4	5	
3	−3	−3	−3	−1	0	−3	0	−4	−7	−8	−6	4	0		
4	−2	−3	−3	−1	0	−2	0	−4	−6	−7	−6	4			
5	−7	−7	−8	−6	−4	−7	−4	−8	−11	−12	−10				
6	3	3	2	4	5	3	6	2	0	−1					
7	4	4	4	6	7	4	7	3	1						
8	3	3	3	5	6	3	6	2							
9	1	1	0	2	3	1	4								
10	−3	−3	−3	−1	0	−2									
11	0	0	0	1	2										
12	−2	−2	−3	−1											
13	−1	−1	−1												
14	0	0													
15	0														

Table III: The (O−C) values in 0″.01 for the upper level computed according to (2).

i \ k	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
1	−7	−2	1	8	12	10	13	7	3	0	0	8	3	3	−1
2	−5	−1	3	9	13	12	14	9	4	0	0	9	4	4	
3	−10	−6	−1	4	9	7	9	−4	0	−3	−4	4	0		
4	−10	−6	−1	5	9	7	10	4	0	−3	−4	5			
5	−15	−11	−6	0	4	2	5	0	−5	−8	−9				
6	−6	−1	2	9	13	11	14	8	4	0					
7	−6	−2	2	8	12	11	13	8	3						
8	−10	−6	−1	5	9	7	10	4							
9	−15	−10	−5	0	4	3	5								
10	−20	−16	−11	−5	0	−2									
11	−18	−13	−9	−2	1										
12	−19	−15	−10	−4											
13	−15	−11	−6												
14	−9	−4													
15	−4														

Table IV: The (O–C) values in 0."01 for the lower level computed according to (1).

i \ k	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
1	0	1	1	2	0	−1	−2	−4	−1	2	1	0	−1	1	1
2	−2	0	0	0	−1	−3	−4	−6	−3	0	0	−2	−3	0	
3	−1	0	0	0	0	−3	−4	−6	−3	0	0	−2	−3		
4	1	3	3	3	2	0	−1	−3	0	4	2	0			
5	0	2	2	2	1	0	−2	−3	−1	3	1				
6	−1	0	0	0	0	−2	−4	−5	−3	1					
7	−2	0	0	0	−1	−3	−5	−7	−4						
8	1	3	3	3.	2	0	−1	−2							
9	4	6	6	6	5	3	1								
10	2	4	4	4	3	1									
11	1	2	3	3	2										
12	−1	0	0	1											
13	−2	0	0												
14	−1	0													
15	−1														

Table V: The (O–C) values in 0."01 for the lower level computed according to (2).

i \ k	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2
1	−4	4	3	0	−5	−10	−11	−12	−6	0	0	−2	−5	−4	−5
2	0	9	8	4	0	−5	−6	−7	−1	5	5	2	0	0	
3	0	9	8	4	0	−5	−7	−7	−1	5	5	2	0		
4	0	9	8	5	0	−4	−6	−6	0	5	5	3			
5	−2	6	5	1	−3	−8	−9	−10	−4	2	2				
6	−5	4	2	0	−6	−10	−12	−12	−6	0					
7	−4	4	3	0	−6	−10	−12	−12	−6						
8	1	10	9	6	0	−3	−5	−5							
9	7	16	15	11	6	1	0								
10	7	16	15	11	6	1									
11	5	14	13	9	4										
12	1	10	9	5											
13	−4	4	3												
14	−8	1													
15	−9														

The O–C values for an inclination roughly equal to k-i ($E_j \approx 1''$) appear at the intersections of the k–th columns and the i–th rows.

The comparison of Table II and III (for the upper level)as well as Tables IV and V (for the lower level) reveal that the terms with the coefficients A_3, A_4 and A_5 in (1) represent fairly well the systematic irregularities in our levels. The values in Tables II and IV come already close to the random errors in determination, $\frac{\epsilon\sqrt{2}}{\sqrt{18}} = \pm\ 0.''04$. One might conclude that (1) yields markedly better results than the well–nigh classical formula (2).

In concluding this Section let it be noted that we analysed these laboratory measurements by other functional dependences as well, but the coclusion was reached that (1) was the best in representing **both** of our levels.

3. INVESTIGATION OF LEVELS ON THE INSTRUMENT

The applicability problem of the laboratory results is well known and often very acute in the astrometric practice. A settling of this problem is essential when it comes to the absolute star position determination. This is all the more true of the levels in view of their sensitivity and capriciousness on one hand, and the important difference between the conditions prevailing about them during observations with the instrument and those under which they are investigated on the examinator, on the other. Specifically, the heterogeneity of those conditions is reflected by:

a) On the instrument, the illumination devices are beneath the levels, being steadily kept on during observation, the bubble ends being read off in the small mirrors placed above the levels. On the examinator the levels are illuminated by a battery lamp held by the observer, the bubble ends are read off directly (without intermediary of the level mirror).
b) At working with the instrument there takes place, between two successive readings of the bubble ends, the shaking of liquid in the level tubes, their bubbles crossing over from one end of the graduation to another before returning to their proper positions. On the examinator, however, the bubble slowly glides from one position to its next following gentle displacements of the examinator's disk.
c) On the instrument, the levels are exposed to generally harsher external influences.

The solving of this and similar problems is usually sought in investigations, performed wherever possible under conditions made as close as possible to those prevailing during the regular observation with the instrument. In our particular case this would imply: under conditions prevailing in the instrument's pavilion in the course of standard work, without dislodging the levels from their place. That is just the kind of investigation we decided on.

In carrying out measurements necessary for clibrating the angular value of the level division on the instrument itself use was made of two mercury horizons in the nadiral direction. The illumination of threads in the eye–piece micrometer was accomplished by Lj. Paunović's, rather than with the Gaussian, eye–piece. This because the illumination of micrometer threads provided by Paunović's eye–piece proved incomparable superior to the one offered by that of Gauss.

These measurements were performed from time to time, usually following regular astronomical observations. The preparation run roughly according to the following scheme: Paunović's eye–piece was mounted on the eye–piece micrometer and the instrument turned toward nadir. If necessary, the mercury mirror horizons were cleaned up. The cardboard cylinders protecting the mirror horizons against the air disturbances were put on, covering the room between the mercury horizons' cicular priphery and the instrument's dew cap turned downwards. By suitable gentle pushing the instrument one achieved the coincidence of the micrometer moving thread with its image in the mercury horizon, that position being fixed by the instrument's clamps. Thereupon the threads of the I and IV microscope–micrometers were set in succession upon the junior and the senior division lines in order to periodically verify the instrument's position with respect to the microscope–micrometers. Then followed the reversing of instrument, the same procedure, after two minute time, being once again performed.

Necessary measurements were caried out in the following manner. First, the movable thread was brought to coincide five times with its image, its position being read off each time. Then followed readings of the left and right bubble ends in the upper and the lower level. Then the instrument was reversed and the same measurements were taken up after about two minute time ect.

The measurements were undertaken mostly with the inclination nearly the same as it was during the astronomical observation, provided it has not been less than 2 divisions. Usually after about ten of these measurements, somewhere about the middle of the procedure, one changed the inclination's sign. The change of inclination proceeded relatively simply since one of the legs of the pilar's support, the one lying in the meridian, is leaned against a metalic lever, which is easily lifted or lowered by means of a screw. The inclination's sign is changed in order to enfeeble possible systematic personal error of the observer in the micrometer measurements, being given that his using one auxiliary staircase caused him to be continually to the south or to the north of the instrument. The reversing prism was not used.

In the period from 18 September 1980 to 15 March 1984 one accomplished 47 sets of measurements. The temperature was confined between –8.2 and +23.3°C, while the bubble length run from 15.3 to 25.1 divisions. At the beginning of this experimenting (18 sets) only one mercury horizon was used. The measurements were mostly made in accordance with the usual method as the one applied on the examinator, the inclination being varied all the while. In the later 29 sets of measurements, in order that the measuring procedure be likened as much as possible to that followed in the star observations, we started using both mercury horizons and reversing the instrument between two measurements. As no perceptible difference could be noted between the results of the two procedures, all the measurements were processed in the same manner.

The data processing proceeded in such a way that the same micrometer readings were compared once with the reading of the upper, then with that of the lower level. In helping ourselves with the method of least squares we derived the mean coefficients in the equations of conditions of the form:

$$(M_W-M_E)_i = \{ (S_E-S_W) \cdot [A'_0 + A'_1 (T_j-13) + A'_2 (B_j-19)] + A'_3 [(S_E-20)^2 - (S_W-20)^2] + A'_4 [(S_E-20)^3 - (S_W-20)^3] + A'_5 [(S_E-20)^5 - (S_W-20)^5] \}_i (i = 1, 2, .., 444; j = 1, 2, ..., 47) \quad (3)$$

The above formula is obtained as a difference of two formulae (1) applied to each two measurements, the notations therein being:

M_W-M_E – the difference of the mean micrometer readings of nadir at two opposite clamps (E and W). In the measurements carried out on one instrument clamp, the inclination alone having been varied, this in most instances is the difference between two contiguous readings with the eye–piece micrometer.

i – the ordinal number of the pair of any contiguous measurements.

S_E-S_W – the difference of the bubble middles at two instrument clamps of one of the investigated levels.

T_j, B_j – the mean air temperature and the bubble length, respectively, in some of the sets of measurements.

In Table VI are displayed the coefficients values thus obtained as well as their rms errors for both levels. These results, delivered by measurements on the instrument itself, reveal the angular value of the level division to be only weakly dependent on temperature and the bubble length (coefficients A_1' and A_2') but more noticeably (the upper level in particular) in what portions of the graduation one performed the measurements (coefficients A'_3, A'_4 and A'_5).

4. DISCUSSION AND DERIVATION OF THE FINAL VALUES

One realizes from Table I (results from the examinator) and Table VI (results from the instrument itself) that the values obtained are more or less differing. This is in a way understandable on considering the diversity of the purposes, modes and conditions of measurements.

The investigation with the aid of examinator is organized in such a way (even distribution of the bubble middle positions, nearly constant temperature but varied bubble length) that the possible division value dependence on temperature, bubble length and irregularities in the sliding surface and the graduation is brought out rather dependably. Owing to reasons cited in Section 3 the applicability of the mean division value A_0 obtained with the examinator to measuremens made with the instrument is highly questionable. The applicability issue as far as the rest of the coefficients is concerned, is not so acute considering their values and effect on the measurements.

The objective of the level investigation on the spot, i.e. on the VC itself, was deducing the division value under conditions as close as possible to those prevailing during the reggular astronomical observations. As one had essentially to deal with what one had caught, the distribution of the bubble positions along the levele graduation within individual sets of measurements, and throughout, was found far from being an perfect one for a trustworthy derivation of the coefficients A_3', A_4' and A_5' appearing in (3). Hence we take these values rather as a proof of the presence of the systematic irregularities in the level tube sliding surfaces and in the level graduations and also of the adequacy of their representation. As apparent, for the case of the upper level, the coherence of the three coefficients is fairly well. With the lower level the coherence is considerably poorer. Concidering the temperature and the bubble length coefficients, the results obtained are in fact contradictory. The investigation on the examinator showed the levels as bieng strongly, and those on the VC as only weekly, dependent on temperature and bubble length. This disparity is apparently a consequence in the first place of the fact that the bubble length, in the regular work with the VC, is adjusted practically seazonally (i.e. four times a year). As the temperature and the bubble length are known to be correlated quantities, this adjustment entails the coefficients A'_1 and A'_2 to be inconclusively determined from the measurement on the instrument. For the same reason one cannot accept the mean division values A'_0 either, albeit deduced from the VC measurements.

Table VI: Values of coefficients in (3) and their rms errors for the upper and lower level

Level	A'_0	A_1'	A'_2	A'_3	A'_4	A'_5	ϵ'
U	**0.949** ±.005	**–.0008** ±.0003	**–.0027** ±.0011	**–.0021** ±.0004	**–.00150** ±.00021	**.0000067** ±.0000017	±.289
L	**0.923** ±.006	**.0007** ±.0003	**.0028** ±.0013	**.0019** ±.0005	**.00021** ±.00025	**–.0000057** ±.0000021	±.327

The matter was settled in the following way. On considering the reasons quoted above one accepted as more realistic the coefficient values, specifying the angular division variability, obtained on the examinator. The measurements performed on the VC were thereafter corrected by the values of these effects, thereby those only which were obtained when the instrument was reversed during investigation (29 sets of measurements involving 296 measurings of inclination). Then one determined, using the least square method, the mean angular division value. There resulted for the upper level $A_{oU} = 0''.914 \pm 0''.0027$ ($\pm 0''.332$) and for the lower one $A_{oL} = 0''.900 \pm 0''.0030$ ($\pm 0''.360$). In addition, on having introduced corrections for only temperature and the bubble length effects as obtained on the examinator, one determined the mean division value for this particular case. For the upper level there followed $A_{oU} = 0''.899 \pm 0''.0029$ ($\pm 0''.354$) and for the lower one $A_{oL} = 0''.931 \pm 0''.0033$ ($\pm 0''.377$). The rms errors are given in the brackets.

These mean values of the level divisions and the coefficients in Table I yield the following expressions for calculating corrections to the circle readings due to the presence of the VC vertical axis' inclination for any of our levels

$$C_{Ui} = \mp(S_i-20)[0''.914+0.0028(T-15) + 0.007(B-19) - 0.0018\,(S_i-20) - 0.00082\,(S_i-20)^2 + 0.000006(S_i-20)^4]$$

$$C_{Li} = \mp(S_i-20)[0''.900+0.0022(T-15) + 0.008(B-19) + 0.0009(S_i-20) + 0.00176\,(S_i-20)^2 - 0.000021(S_i-20)^4] \qquad (4)$$

$$i = E, W$$

$$C_{Ui} = \mp(S_i-20)[0''.899+0.0028(T-15) + 0.007(B-19)]$$

$$C_{Li} = \mp(S_i-20)[0''.931+0.0022(T-15) + 0.008(B-19)] \qquad (5)$$

As apparent, the angular division values thus obtained are lower than those resulting from the investigations on the examinator (Table I). This is particularly plain with the upper level. In order to verify how these results for the two levels were mutually harmonizing, the following test was carried out.

From the 1983 and 1984 observations we selected 51 nights on which the observing conditions differed among themselves at the most. From each of the nights one picked out 8 star observations. Being given that at any particular observation the VC occupies one inclination, measured by both upper and lower levels, $I_U = (S_E - S_W)_U \cdot A_U$, $I_L = (S_E - S_W)_L \cdot A_L$, the measured inclinations should be equal among themeselves apart from their random errors, provided the level division values A_U and A_L have been correctly determined. We therefore formed the differences of inclinations as supplied by the upper and the lower levels for any individual observation. In order to obviate these differences being dependent on the inclination's magnitude and to make sure they depended solely on error in the adopted division values, we divided them by mean inclination $\Delta A_i = ((I_U - I_L)/I)_i$, $I_i = ((I_U + I_L)/2)_i$. In order, further, to minimize the effect of the random errors we averaged the values obtained $\Delta A_j = \frac{1}{8}\sum_i \Delta A_i$, $i = 1, 2, \ldots, 8$. Thereafter one determined the mean value from all the nights and the rms of individual values

$$\Delta A = \frac{1}{51}\sum_j \Delta A_j \;, \quad \epsilon_j = [(\Delta A_j - \Delta A)^2/50]^{\frac{1}{2}} \;,$$

$$j = 1, 2, \ldots, 51$$

The actual calculations according to the procedure just laid out was implemented using four different division values. The results obtained are summarized in Table VII.

Table VII: Mutual agreement of the division values obtained by the method applied

Set	ΔA	ϵ_j
1	0.065	±0.068
2	0.065	±0.077
4	0.015	±0.067
5	0.015	±0.077

In the above Table VII 1 and 2 denote the values used from Table I (results supplied by the investigations with the examinator). 4 and 5 indicate that the computations have been performed according to (4) and (5) using coefficients therein of which A_o – mean angular division values are obtained from the investigation made on VC.

Being given that the division values of our levels are roughly about 1" we find $\Delta A \approx \Delta A_{oU} - \Delta A_{oL}$. The agreement of the results obtained on the instrument itself – in Table VII $\Delta A = 0.015$ – points to the possible errors in the division values in both levels being very small, or if larger vitually equal. In our view the former case is true as no larger systematic departure was noted in the more recent star observations whose origin could be attributed to the level division values. The unimportant value 0.015 might be neglected, as it is on the very limit of the measuring accuracy ($\epsilon_j/\sqrt{51}$). Moreover, the method itself, being but an approximate one, involves errors of that order. The reason of so highly concordant division values acquired by the measurements on VC itself lies in the fact that the same micrometer readings have been used in both levels, the latter being simultaneusly under the external conditions nearly identical with those prevailing during the star observation. In contrast to them the mean division values, resulting from the investigation on the examinator, display considerably poorer mutual agreement. The value 0.065 derives chiefly from the upper level. The same value 0.065 of the divergence of the two systems 1 and 2 as well as 0.015 in the systems 4 and 5 is understandable for two reasons:

a) The mean division value used in the present test are a result of different processing of the same observing material alike on the examinator and the VC;
b) The sample formed from inclinations during the star observation an 51 nights did not substantally affect the test results thanks to its having been selected rather well both in respect to the magnitude of inclination and in respect to symetry of measurements with respect to the level graduation middle.

As apparent from Table VII the use of the coefficients A_3, A_4 andA_5, typifying the systematic irregularities in the sliding surfaces of level tubes and in the level graduation, in the systems 1 and 4 results in a diminishing of the errors ϵ_j by about 13% with respect to those in the systems 2 and 5, which is another proof of their reality and the legitimacy of their use.

We would like at this place to pass a few words about the accuracy of measurements on both the examinator and the VC proceding from the rms errors ϵ and ϵ' in them (see Tables I and VI). One may claim both procedures to be of about equal accuracy. Namely, considering that the values S_i in (1) are obtained from two bubble middle positions (involving the bubble motion from left to right and vice versa) and that in (3) one is dealing with the position differences ($S_E - S_W$), we have $\epsilon = \pm\ 0.11$ and $\epsilon'/2 = \pm\ 0.15$. This small difference is due principally to the circumstance that the measurements on VC are affected also by the error in the difference of the eye–piece micrometer readings. This difference is ceratainly twice as great as the error in the mean of two installings of the examinator disk on some particular division. (The reading of some position is always less precise than the installing into that position).

In closing this Section let us note that in the actual employment of the formula (4) which is recommended, or eventually of the formula (4), there might emerge minor difficulties owing to the accounting of the cited dependence of the division values on temperature. This dependence is deduced from the measurements on the examinator under conditions enabling the level to assume the temperature shown on the examinator. At carrying out measurements on he VC one reads off the air temperature in the pavilion which sometimes may differ considerably from the true temperature of the level. One thing is certain: the range of level temperature fluctations during the year is undoubtedly narrower than the one of the air temperature fluctuations in the VC pavilion. Therefore, unless in the future work data on the level temperature are secured, the values specifying the level division dependence on temepratue is perhaps to be somewhat scaled down (by about 0.7 times in our free estimate). In addition to the need of being clear about this temperature matter one is advised to perform, in the future too, measurements designed for keepeing under control the value of the level division. The intricate nature of our levels, brought to light by these investigations, makes it necessary in the future work that our measurements be evenly distributed along the entire working range of the level graduation.

5. CONCLUSIONS

1. From the present work it appears that the level division as determined from the measurements on the examinator may differ considerably from the one furnished by the measurements on the instrument itself, involving therewith appreciable – depending on the amount of instrument's inclination – random and systematic errors in the star zenith distances, i.e. in star declinations and local latitudes. Moreover, the investigation on the examinator is a delicate and laborious undertaking (Teleki et al., 1968) only rarely carried through. Nevertheless, the measurements on the examinator are capable or rendering good sevices in investigating and studying diverse sources conditioning the angular division value to be variable.
2. It is demonstrated by our investigations that the level division may successfully be determined on the instrument itself and that, moreover, level examination in general is thus practicable. One should thereby adjust more rigorously than we did, the organization of measurements to the desired purpose (investigation – one clamp position of the instrument but even distribution of the bubble middles along the working range of the level graduation; calibration of the angular value of the level division – reversing (E – W) the instrument with the use of two mercury horizons).
3. An old rule has once again been reaffirmed by this determination of the angular value of the level division on the instrument itself: by an investigation

under conditions close to those under which regular star observations are carried out, quite fitting results are obtained.

4. Being given that both levels in such investigations on the instrument itself are sumultaneously treated, the furnished division values are mutually harmonizing, which implies that their possible errors are essentially equal. This has its bearing in the case a systematic error has been found in a prolonged observing run, having its origin in the levels, i.e. in the adopted division value. It may then easier be deduced and the results of observation corrected for its amount thanks to its affecting both levels equally.
5. It is our view that the actual measurements are fairly well representable by formulae (1) and (4). The same formulae allow these measurements to be corrected for effects of the systematic irregularities in the ampule sliding surface and the level graduation. The complexity of these formulae does not at the present time involve any trouble in veiw of the modern computing facilities, concerning both the determination of the coefficients and the reduction of the actual astronomical observations.
6. Striving after keeping the inclination as small as possible by properly adjusting the instrument is to be continued. The mean values of the inclinations from several observations of the same star at least should be close to zero.

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REFERENCES

Mijatov M., Sadžakov S.: 1968, *Publ. Obs. Aston. Belgrade,* **14**, 81.

Mijatov M., Trajkovska V.: 1984, *Bull. Obs. Astron. Belgrade,* **134**, 9.

Teleki G., Sadžakov S. Mijatov M.: 1968, *Publ. Obs. Astron. Belgrade,* **14**, 41.