SYM-OP-IS-2016
XLIII симпозијум о операционим истраживањима

A NEW FORMULATION FOR MINIMUM COST HYBRID BERTH ALOCATION PROBLEM

JELENA JOVANOVIĆ
Faculty of Computer Science, John Naisbitt University, Beograd, jelena.jovanovic55@gmail.com

TATJANA DAVIDOVIĆ
Mathematical Institute of the Serbian Academy of Sciences and Arts, Beograd, tanjad@mi.sanu.ac.rs

Abstract: *The static minimum cost berth allocation problem (MCHBAP) is considered with an aim to minimize the total costs consisting of costs of positioning, speeding up or waiting, and tardiness of completion for all vessels. In addition, it is assumed that each berth is equipped with a crane, and therefore, the crane scheduling problem can be avoided. MCHBAP can be presented as the Mixed Integer Linear Program (MILP), and here we compare computational results obtained by CPLEX commercial solver for two different problem formulations, i.e., two different models. The first model already exists in the literature; it has a large number of 0-1 variables and a large number of constraints. The second model, proposed here, is much simpler since it offers a different approach: under the assumption that each berth is equipped with a crane, we can formulate this problem as two-dimensional bin–packing problem. It appears that the second model outperforms the first one by far: when using the second model CPLEX is able to find optimal solutions for all test instances very quickly, while with the first model given time limits are exceeded for medium size instances and the obtained solutions are far from the optimal ones.*
Keywords: *Container Vessel, Berthing, Mixed Integer Formulation, Two-Dimensional Bin-Packing.*

1. INTRODUCTION

A container terminal (CT) in a sea port can be described as open system of container flow with sea-side interface. This interface is quayside with loading and unloading of vessels. After arrival at the port, a container vessel is assigned to a berth equipped with quay cranes to load and unload containers. Various types of container vessels have to be served at the quayside. In order to compete in this environment, a CT should be organized efficiently. One issue of seaside operations planning is the assignment of quay space and service time to vessels that have to be unloaded and loaded at a CT [1], [9]. This problem is commonly referred to as the berth allocation problem (BAP).

BAPs can be classified as discrete or continuous, as well as static or dynamic [1]. In the discrete case, the quay is partitioned into a number of sections, called berths, and each berth can serve one vessel at a time. Moreover, time could also be partitioned into discrete units allowing using integer arithmetic for calculation of the objective function value. In the continuous case a calling vessel can be placed at any position, with the restriction to avoid overlapping with other vessels and time is also considered continuous. In a static BAP it is assumed that all vessels arrive to the container terminal in advance, namely before any berth becomes available. If the vessels can arrive at any time during the planning horizon (although we still have *a priori* knowledge of their arrivals), then we deal with dynamic BAP.

In recent years, an ever increasing number of papers on CTs considering BAP have appeared. In most of them crane resources were either ignored (assuming that each berth is equipped by a crane) [5], [9], [12], [15] or treated separately within the second stage of problem solving [11]. Moreover, different authors considered different objectives to be minimized within the solution of BAP. In some of the papers the total of waiting and handling times were minimized [1], [9], while in the others the minimization of total costs for waiting and handling as well as earliness or tardiness of completion was considered as the objective [6].

We consider the static minimum cost BAP (MCHBAP) in the case when crane resources are ignored (as said in the abstract, we assume that each berth is equipped with a crane). The first Mixed Integer Programming (MIP) formulation of this problem can be found in [3]. The new model (proposed here), which proved to be way more suitable, offers an alternative approach. Namely, we can consider this problem a special case of two-dimensional bin-packing problem [13], [14] (that is, we are given a single bin). This way the formulation of the problem becomes much simpler – it has less variables and less constraints (the objective function stays the same). We compared these two models using CPLEX commercial solver [7], [8].

The rest of this paper is organized as follows. MCHBAP is described and formulated as integer linear program (using our new model) in section 2. Section 3 contains experimental evaluation using CPLEX 11.2 optimization software [7], [8]. In this section we present evaluation results for both models on two sets of test examples. Section 4 concludes the paper.

2. MINIMUM COST HYBRID BERTH ALLOCATION PROBLEM

MCHBAP represents one of the major CT operations planning problem [1], [9]. It consists of assigning a berthing position and a berthing time to every vessel incoming to be served within a given planning horizon with and aim to minimize some objective. In this paper the minimization of berthing cost as well as the costs of positioning, speeding up or waiting, and tardiness of completion for all vessels is considered. The main assumption is that the number of cranes is equal to the number of berths and, therefore, crane scheduling problem can be avoided.

Typically, the decisions are made with respect to the different arrival times, lengths, and handling times of vessels. The handling (operation) times are usually assumed to be fixed and known in advance. As shown in Fig. 1, a solution to BAP can be depicted in a space-time-diagram. Both coordinates are assumed to be discrete (space is modeled by the berth indices while the time horizon is partitioned into segments in such a way that berthing time of each vessel is represented by an integer). The height of each of the rectangles corresponds to the length of a vessel (expressed by the number of berths) and the width corresponds to the needed handling time. The lower-left vertex of a rectangle gives the berthing position and berthing time of a vessel and it is referred to as *the reference point* of a vessel (marked by the index of vessel in Fig. 1). A berth plan is *feasible* if the rectangles do not overlap and all the vessels fit into the given space-time frame (see Fig. 1).

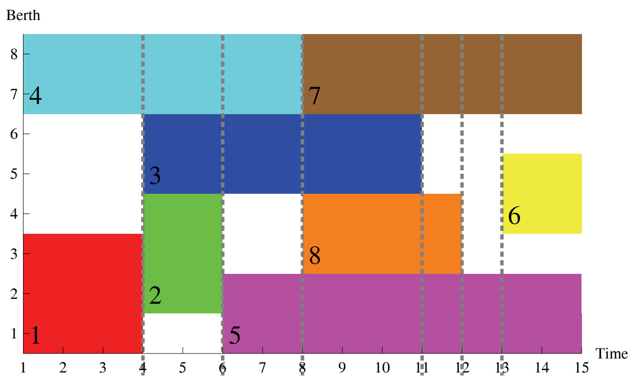

Fig. 1: *An example of BAP solution Source: Kordić, S. , PhD Thesis, 2016.*

The input parameters of MCHBAP are: The total number of vessels (l); The total number of different berthing positions (m); The total number of time segments (T); The expected time of arrival (ETA_k) of vessel k. *ETA* is a kind of agreement between carriers and the terminal operator regarding the arrival time of vessels. Berthing earlier than the promised berthing time causes the corresponding vessel to speed up, which in turn causes the extra consumption of fuel, while berthing later than the promised berthing time may incur complaints from carriers; The total operation time (a_k) of vessel k if only one crane operates on it during the berthing; The length (b_k) of vessel k expressed as the number of berths. Assuming that each berth is equipped with a crane, the time required to unload and load all the cargo for vessel k, *denoted by* H_k, equals $\lceil a_k / b_k \rceil$; The due time for the departure (d_k) of vessel k; The least-cost berthing location of the reference point (s_k) of vessel k; The container handling cost (c_{1k}) per unit distance of vessel k from the least-cost berthing location; The penalty cost (c_{2k}) of vessel k per unit time of arrival before ETA_k; The penalty cost (c_{3k}) of vessel k per unit time of arrival after ETA_k; The penalty cost (c_{4k}) of vessel k per unit time of delay beyond the due time d_k.

The goal is to minimize the total penalty cost which includes: the penalty induced by missing the least-cost (preferred) berthing location of the reference point; the penalties induced by the actual berthing earlier or later than the expected time of arrival and the penalty cost by the delay of the departure after the promised due time. The last three terms have impacts on the objective function provided that they are positive. More precisely, the objective function can be expressed in the following form:

$$\sum_{k=1}^{l} \{C_{1k}\sigma_k + C_{2k}(ETA_k - At_k)^+ + C_{3k}(At_k - ETA_k)^+ + C_{4k}(Dt_k - d_k)^+\}$$

where

$$\sigma_k = \sum_{t=1}^{T}\sum_{i=1}^{m} \{|\, i - s_k \,| : \text{vessel } k \text{ occupies possition } (j,i)\}$$

and

$$(a-b)^+ = \begin{cases} a-b, & \text{if } a > b \\ 0, & \text{otherwise} \end{cases}$$

Although containing some non-linearities (expressed by absolute values, positive components, conditional expressions), MCHBAP can be formulated as Mixed Integer Linear Program (MILP). Therefore, it is possible to apply optimization software (in our case, CPLEX 11.2, [7], [8]).

In order to develop MILP for this problem, let us introduce the following decision variables:

- integer variables:
  - At_k, the Berthing position of vessel k on time axis (taking values 1, 2,…, T).
  - Dt_k, The completion (departure) time of vessel k (taking values 1, 2,…, T).
  - P_k, the Berthing position of vessel k on berth axis (taking values 1, 2,…, m).

- binary variables:

$$z_{ijk} = \begin{cases} 1, & \text{if the reference point of vessel } k \text{ is } (j,i), \\ 0, & \text{otherwise} \end{cases}$$

$$l1_{k1k2} = \begin{cases} 1, & \text{if vessel } k1 \text{ is left to the vessel } k2 \text{ on time axis}, \\ 0, & \text{otherwise} \end{cases}$$

$$u_{k1k2} = \begin{cases} 1, & \text{if vessel } k1 \text{ is under the vessel } k2 \text{ on berth axis,} \\ 0, & \text{otherwise.} \end{cases}$$

Moreover, let us perform some preprocessing. First, it is obvious that we can calculate array $H_k = \lceil a_k / b_k \rceil$ in advance. In addition, we can introduce matrices $E1_{kj}$, $E2_{kj}$, $D1_{kj}$, Zb_{ki}, and F_{ijk} defined as follows:

$$E1_{kj} = \begin{cases} c_{2k}(ETA_k - j), & \text{if } ETA_k > j, \\ 0, & \text{otherwise} \end{cases}$$

$$E2_{kj} = \begin{cases} c3(j - ETA_k), & \text{if } j > ETA_k, \\ 0, & \text{otherwise} \end{cases}$$

$$D1_{kj} = \begin{cases} c_{4k}(j + H_k - d_k), & \text{if } j + H_k > \mathrm{d_k}, \\ 0, & \text{otherwise} \end{cases}$$

$$Zb_{ki} = \sum_{r=i}^{i+b_k-1} B_{kr}$$

where

$$B_{kr} = \begin{cases} H_k * c_{1k}(r - s_k) & \text{if } r > s_k, \\ H_k * c_{1k}(s_k - r) & \text{otherwise} \end{cases}$$

and

$F_{\mathrm{ijk}} = Zb_{ki} + E1_{kj} + E2_{kj} + D1_{kj}.$

In this way we extracted most of the problem non-linearities into preprocessing phase.

Based on the formulation of the Two-Dimensional Bin-Packing Problem given by Pisinger and Sigurd in [13], [14] and our preprocessing scheme, MCHBAP can be formulated as follows:

$$\min \sum_{k=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{T} z_{ijk} F_{ijk} \quad (1)$$

$$\text{s.t.} \sum_{i=1}^{m} \sum_{j=1}^{T} z_{ijk} = 1, \quad k = 1,2,\dots, l \quad (2)$$

$$\sum_{j=1}^{T} \sum_{i=1}^{m} j * z_{ijk} = At_k, \quad k = 1,2,\dots, l \quad (3)$$

$$At_k + H_k = Dt_k, \quad k = 1,2,\dots, l \quad (4)$$

$$\sum_{j=1}^{T} \sum_{i=1}^{m} i * z_{ijk} = P_k, \quad k = 1,2,\dots, l \quad (5)$$

$$l1_{k1k2} + l1_{k2k1} + u_{k1k2} + u_{k2k1} \geq 1, \quad k_1 = 1,2,\dots,l; \quad k_2 = 1,2,\dots,l \quad (k_1 < k_2); \quad (6)$$

$$At_{k1} - At_{k2} + T * l1_{k1k2} \leq T - H_{k1}, \quad k_1 = 1,2,\dots,l; \quad k_2 = 1,2,\dots,l; \quad (7)$$

$$P_{k1} - P_{k2} + m * u_{k1k2} \leq m - b_{k1}, \quad k_1 = 1,2,\dots,l; \quad k_2 = 1,2,\dots,l; \quad (8)$$

$$At_k \leq T - H_k + 1, \quad k = 1,2,\dots, l \quad (9)$$

$$P_k \leq m - b_k + 1, \quad k = 1,2,\dots, l \quad (10)$$

The objective function, given by equation (1), aims to minimize the weighted sum of the berthing cost components (the cost depending on the distance from the berthing location of a vessel to the preferred location, the penalty cost incurred by berthing earlier or later than the expected time of arrival (ETA), and the penalty cost incurred by the delay of the departure beyond the desired due time). Constraints (2) ensure that each vessel has exactly one reference point. Constraints (3), (4) and (5) define, respectively, the values of variables determining berthing time, total operation time and berthing position of each vessel. Constraints (6), (7) and (8) prevent overlapping of vessels. Constraints (9) and (10) ensure that all vessels fit in the given time-space frame.

3. EXPERIMENTAL EVALUATION

In this section we present computational results for both the old model as given in [3] and the new one as presented in the previous section. Results are obtained by CPLEX 11.2 optimization software [7], [8] on two sets of test instances: the first set contains small size artificially generated problems, while the second set is the set of real life instances as proposed in [2].

CPLEX is executed on Intel Core 2 Duo CPU E6750 on 2.66GHz with RAM=8Gb under Linux Slackware 12, Kernel: 2.6.21.5.

The Table 1 contains results for the examples from the first set. CPLEX solved these instances to optimality for both old and new model, but the new model required much less CPU time.

Table 1 *Computational results - artificial test problems: m=8, T=15*

l	OPT. COST	CPU Time	
		OLD MODEL	NEW MODEL
6	380	0.06	0.01
7	665	20.53	0.05
8	745	18.91	0.09
9	780	20.88	0.10
10	1070	35.19	1.56
11	1325	644.98	10.51
12	1375	129.76	39.93
13	1415	379.64	82.24
14	1485	635.40	123.87
15	1570	1788.80	392.62

Table 2 contains results for the examples from the second set. When using the old model, CPLEX CPU time required to solve these instances to optimality was too long, and therefore time limits were imposed in [3] as it is indicated in the fourth column of Table 2. In the first column of Table 2, the number of vessels is presented. The next two columns contain the solutions obtained by the old model within a given time limit. In the second column total cost is presented, while the third column contains the CPU time required to obtain that cost value. When using the new model, however, we obtained optimal solutions for all instances, execution times stayed very small.

Table 2 *Computational results – real life test problems:* m=12,T=54

l	OLD MODEL			NEW MODEL	
	COST	CPU Time	Time limit (sec.)	OPT. COST	CPU Time
21	24562	3698.41	3600	4779	1.31
22	16334	7434.44	7200	4983	2.02
23	96549	7404.73	7200	5193	3.59
24	6594	7429.48	7200	5643	3.14
25	13262	18709.60	18500	5953	5.47
26	26614	18716.10	18500	6298	12.97
27	26679	18638.50	21600	6478	15.22
28	8418	44530.70	43200	6980	91.21

It is obvious from the presented tables that the new model outperformed by far the old one, and therefore, it makes it possible to solve to the optimality much larger examples of MCHBAP. The MCHBAP can also be considered as the cutting problem, however, the guillotine cuts [4] cannot be applied. This may be the subject of further research on modelling MCHBAP.

5. CONCLUSION

We considered The Minimum Cost Hybrid Berth Allocation Problem (MCHBAP) with an aim to minimize the total costs for waiting and handling as well as speeding up or tardiness of completion for all vessels. The problem is formulated as a mix-integer linear program based on two models: the model given in [3] and the model presented in this paper, in which we consider this problem as a special case of two-dimensional bin-packing problem. For both models we performed experimental evaluation using CPLEX 11.2 commercial software, and the results suggest that our variation of the two-dimensional bin-packing problem is more suitable for solving MCHBAP. As the future work, we plan to examine the limits of the new model and to develop hybrid methods based on this model and (meta)heuristics that would be able to address large size real life problem instances.

Acknowledgment

This work has been partially supported by Serbian Ministry of Education, Science, and Technological Development, grant Nos. ON174010, ON174033 and a bilateral project from the Pavle Savić programme for year 2016/17 no. 451-03-39/2016/09/09.

REFERENCES

[1] Bierwirth, C., Meisel, F., A survey of berth allocation and quay crane scheduling problems in container terminals, *European Journal of Operational Research*, 202:615-627, 2010.

[2] Chang, D., Jiang, Z., Yan, W., He, J., Integrating berth allocation and quay crane assignments, *Transportation Research Part E*, 46(6):975-990, 2010.

[3] Davidovic, T., Lazic, J., Mladenovic, N., Kordic, S., Kovac, N., Dragovic, B., MIP-Heuristics for Minimum Cost Berth Allocation Problem, in *Proc. International Conference on Traffic and Transport Engineering*, *ICTTE 2012 Belgrade,* 21-28, 2012.

[4] Furini., F., Malaguti, E., Thomopulos, D., Modeling Two-Dimensional Guillotine Cutting Problems, *Informs Journal Of Computing, Informs,* (to apper).

[5] Guan, Y., Cheung, R. K., The berth allocation problem: models and solution methods, *OR Spectrum*, 26(1):75-92, 2004.

[6] Hansen, P., Oğuz, C., Mladenović, N, Variable neighborhood search for minimum cost berth allocation, *European Journal of Operational Research*, 191(3):636-649, 2008.

[7] ILOG Cplex 11.2 reference manual, 2008.

[8] ILOG AMPL CPLEX System, Ver. 11.0, User's guide, 2008.

[9] Imai, A.; Nishimura, E., Papadimitriou, S., Berth allocation with service priority, *Transportation Research Part B*, 37:437-457, 2003.

[10] Kim, K. H., Moon, K. C., Berth scheduling by simulated annealing, *Transportation Research Part B*, 37(6):541–560, 2003.

[11] Oğuz, C., Narin, Ő., Solving Berth Allocation Problem with Column Generation, In *Proceedings of the Multidisciplinary International Conference on Scheduling: Theory and Applications* (MISTA 2009), pages 744-747, Dublin, Ireland.

[12] Park, Y. M., Kim, K. H, A scheduling method for berth and quay cranes, *OR Spectrum*, 25(1):1-23, 2003.

[13] Pisinger, D., Sigurd, M., Using decomposition techniques and constraint programming for solving the two-dimensional bin-packing problem, *INFORMS Journal on Computing,* 19(1):36-51, 2007.

[14] Pisinger, D., Sigurd, M., The two-dimensional bin-packing problem with variable bin sizes and costs, *Discrete Optimization, Elsevier,* 2:154-167, 2005.

[15] Wang, F., Lim, A., A stochastic beam search for the berth allocation problem, *Decision Support Systems*, 42:2186-2196, 2007.