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MILP FOR INTEGRATED BERTH ALLOCATION AND CRANE OPERATIONS SCHEDULING IN CONTAINER TERMINALS

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Abstract: *In this paper, we consider an integrated model that addresses three key operations in container terminals: the berth allocation problem (BAP), the quay crane assignment problem (QCAP), and the quay crane scheduling problem (QCSP). A Mixed-Integer Linear Programming (MILP) formulation is developed and utilized with the Gurobi exact solver to obtain (near) optimal solutions for small-sized, randomly generated instances.*

Keywords: *Port operations, quay cranes, rectangle packing, operations scheduling, total cost minimization*

1. INTRODUCTION

Optimization problems in the field of container terminal (CT) operations have been gaining increasing attention in recent scientific literature [4, 5]. The quality of service provided by ports, the satisfaction of shipping companies as service users, and the economic viability and efficient utilization of port resources all hinge on the effective integration of seaside operations at container terminals. These operations encompass a wide range of activities and optimization challenges, most notably the berth allocation problem (BAP), the quay crane assignment problem (QCAP), and the quay crane scheduling problem (QCSP). Both BAP and QCSP are known to be NP-hard [7, 9]. In the literature, the integrated problem that combines BAP, QCAP, and QCSP is referred to as BACASP.

The study in [6] investigates BACASP with disaggregated quay crane (QC) tasks and various operational policies, demonstrating significant reductions in service times through the proposed models based on real-world data from a container terminal in Abu Dhabi. In [1], the authors address a variant with heterogeneous QCs by formulating a Mixed Integer Programming (MIP) model enhanced with valid inequalities and implementing a Branch-and-Cut method, supplemented by a rolling horizon heuristic, for experimental validation on real data. An initial Integer Programming (IP) formulation for BACAP is developed in [3] and extended to BACASP by adding necessary and sufficient conditions along with a cutting plane algorithm to iteratively refine and solve the problem. In [11], time-variant QC assignment and scheduling are considered, where the authors propose a bi-level MIP and a decomposition-based cutting plane algorithm. The authors in [10] introduce a MIP model for time-variant QC assignments aimed at minimizing vessel tardiness and berthing costs, using CPLEX for small instances and a Genetic Algorithm (GA) for larger ones. The study described in [2] addresses QC movement time in a MIP model, complementing it with a modified GA for large instances. In [8], the authors present an IP model targeting the minimization of total vessel waiting and processing times, incorporating water depth and tide conditions, and employing Random Topology Particle Swarm Optimization (RTPSO) for large instances. Collectively, these studies highlight the complex, integrated nature of BACASP and the diverse methodological approaches—ranging from exact methods to heuristics and metaheuristics—employed to tackle its inherent computational challenges and practical constraints.

In the relevant literature, BAP, QCAP and QCSP are usually considered separately or successively, one after the other. If the authors choose to integrate some of these problems, they usually combine two of them, due to the complexity of the underlying models. There are relatively few works that model and solve combinations of all three problems [5]. Our main aims are to contribute towards a comprehensive approach that involves the integration of all three problems and to simplify the model representation. We propose the deep integration of BAP, QCAP, and QCSP in the form of Dynamic Minimum Cost Hybrid Berth Allocation and Quay Crane Assignment and Specific Scheduling Problem (DMCHBACASSP). This problem is classified as $hybr|dyn|QCAP(specific),QCSP|\sum(w_1\,pos+w_2\,speed+w_3\,wait+w_4\,tard+w_5\,res)$, according to the notation from [4]. Our QCSP involves $bay, prmp|pos, move|cross, save|compl, move$ components as they are described in [5].

In Section 2, we present the MILP model for the DMCHBACASSP problem. Specifically, Subsection 2.1 offers a detailed description of the problem, including the assumptions and constraints. Subsection 2.2 outlines the input parameters of the problem instance. The MILP model itself is introduced in Subsection 2.3. Section 3 provides experimental results on small, randomly generated instances. The conclusion is presented in Section 4.

2. PROBLEM FORMULATION

In this section, we provide a description of the considered DMCHBACASSP, explain all input parameters, and provide the proposed Mixed Integer Linear Programming (MILP) model.

2.1. Description of the problem

Port can be modeled by a rectangle whose length is defined by the time horizon considered when planning operations on vessels and width equal to the length of the quay. DMCHBACASSP involves packing of small rectangles (representing vessels) into the large rectangle (port) while satisfying a number of constraints related to positioning of vessels and QCs, as well as the assignment and scheduling of QCs to vessels.

DMCHBACASSP is a variant of integrated optimization problems in port, namely BAP, QCAP and QCSP. It assumes the allocation of berth(s) and a berthing time to each vessel, and the assignment and scheduling of the appropriate number of QCs for vessel's processing, with an aim to minimize the selected objective function. More precisely, to each vessel, the following data need to be determined: (i) a reference point, defined by the lowest time index (denoting the beginning of its processing) and the lowest berth index (while the complete subset of assigned berths is determined by the vessel's size); (ii) Indices of QCs and their operating positions in each time interval the vessel is processed. The objective function includes the costs of positioning, waiting, and tardiness of completion for all vessels, the costs of positioning QCs and the costs associated with movements of QCs. The following assumptions are made in modeling this problem:

Time:
1. The planning horizon is divided into equal time units corresponding to minimum integer time interval required to control the movement of QCs.
2. Vessels may be processed only within the planning horizon.

Berths:
1. Berth may be assigned to only one vessel at a time, between its arrival and departure times.
2. Each berth is divided into an equal number of berth segments.
3. At most one QC can be positioned on each berth segment.

Vessels:
1. A vessel may be moored at any available berth, on its arbitrary berth segment.
2. Once moored, vessels may not change their position during processing.
3. Processing of the vessel begins immediately after its mooring and cannot be interrupted until all loading/unloading operations have been completed.
4. The processing time of the vessel depends on the number of assigned QCs. This number may vary during the processing of a vessel, however, must always be between the prespecified minimum and maximum number of QCs for each vessel.
5. The vessel is divided into bays (lanes of equal width) that correspond to the berth segments. Therefore, the width of bays and the width of berth segment has to be equal.

QCs:
1. QCs are initially positioned on their home berth segments.
2. All QCs have identical characteristics.
3. The total number of QCs assigned to the vessel can vary from one time interval to another, i.e., QC assignment is subject to the variable-in-time scenario.
4. A QC can only be assigned to one bay of the vessel at a time.
5. QCs can move along berths in two directions, but their paths cannot intersect.
6. QCs can move from one bay of the vessel to another bay of the same vessel.
7. QCs can be reallocated from one vessel to another.
8. The time for reallocation of a QC is not negligible and it increases both the vessel processing time and its processing cost.

2.2. Input parameters of a problem instance

The proposed MILP model is based on discretization on both location and time dimensions. The division of port into equidistant net is performed in preprocessing phase and depends on positions and lengths of vessel bays of all vessels considered by the problem instance. Preprocessing procedure is not presented here due to the lack of space. Here, we assume that all geometric parameters are expressed in number of *location units* indexed by the numbers $1, 2, \ldots, L$. Several locations units constitute a single berth in such a way that L locations units correspond to B berths. Similarly, we subdivide the considered time interval into small time segments that are called *time units*. We assume that considered time horizon is represented by T time units, with the corresponding time points denoted by $0, 1 \ldots, T$. Each problem instance considers V vessels and C quay cranes. The dimension of the problem instance is described by L, T, B, V and C.

In the most of previous related studies, vessels are located to predefined berth location along the port. Here, we relax this assumption by allowing vessels to be berthed at any location. However, our model can be easily extended with the set of constraints to ensure berthing vessels only to berth locations. To be comparable and compatible with previous studies, we retain the term of berth as there are still some operation limitations of cranes on vessels that are not berthed to its preferred berths. We divide an entire port (together with predefined berth locations) into small segments (location units) to track position and movement of QCs along vessel's berthing positions. By taking into account precise vessel geometry (Figure 2) we are able to ensure that QCs operate at exact bay locations of berthed vessels. Moreover, movement time of crane from one location to another is also taken into account. In other words, we track the movement of cranes in the port during the entire timeline. If crane is supposed to operate at particular location in particular time, it has to move to that location if it previously was at the different location. The time of movement is not neglected by the model.

For clarity, we introduce the notation $[N] = \{1, 2, 3, \ldots, N\}$ to represent a set of integers from 1 to N. If we consider also zero as a member of the set, the used notation is $[N]_0$. This notation is used throughout the model to denote various sets, such as the set of vessels $[V]$, the set of QCs $[C]$, and the set of time points $[T]$. In the remainder of this section, we provide the description of input parameters that should be specified for each considered DMCHBACASSP instance.

Port. We assume that the port (identified by the line of fixed width) is divided to L segments by $L+1$ location points denoted by $0, 1 \ldots L$. As we already mentioned, the length of the segment is called *location unit*. The port is described by set of berth locations $L_b = (\underline{l}_b, \overline{l}_b)$, where $\underline{l}_b < \overline{l}_b$, $\underline{l}_b, \overline{l}_b \in [L]_0$ represent start and end locations of the berth $b \in [B]$, where B is the number of berths (Figure 1).

Figure 1 Port representation

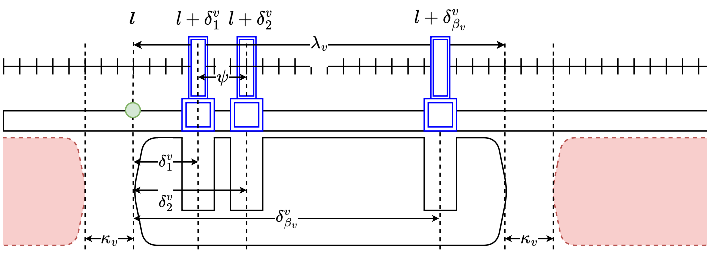

Figure 2 Vessel parameters

Vessel. There are several parameters defining a vessel (Figure 2). The length of the vessel $v \in [V]$ is denoted by $\lambda_v \in \mathbb{N}$. The left end of the vessel is considered as a reference point. If vessel v is moored to some location $l \in [L]$, then location $l \in [L]$ represents its reference point. Each vessel $v \in [V]$ has $\beta_v \in \mathbb{N}$ bays that are considered as touch point between vessels and QCs. Sometimes, we say that $\beta_v, \in [V]$ is a number of jobs that need to be completed on vessel v by QCs, i.e., we identify bays with jobs that need to be completed on vessel v at these bays. To ensure operable assignment of QCs to vessel's bays, we introduce parameter $\delta_j^v \in \mathbb{N}$ which represents the number of location units between reference point of vessel v and the middle of the entrance of bay j. The safe berthing of vessel v is ensured by introduction of the *clearance parameter* κ_v. The left most (reference point) and right most ends of the vessel v have to be at distance of at least κ_v location units from any other vessel v' that is moored during the stay of vessel v in the port.

A QC c, $c \in [C]$, is assigned to vessel $v \in [V]$ if c is located at $l + \delta_j^v$ in time t', for some $j \in [\beta_v]$ and $l \in [L]$, once v is berthed at l in time t'. It is required that at least $\theta_v^{min} \in \mathbb{N}_0$, and at most $\theta_v^{max} \in \mathbb{N}_0$, cranes are assigned to vessel v. Parameter $\gamma_j^v \in \mathbb{N}$ represents the number of time units required to complete the job j on vessel v.

Each vessel $v \in [V]$ has preferred berth b_v^*. If the vessel v is not located to the preferred berth, additional penalties $\pi_v^b \in \mathbb{R}_+$, $v \in [V]$ are imposed. The total value can be determined for each vessel v and for each of its possible reference points different from its preferred berth during the preprocessing phase. If vessel v is located at some particular location $l \in [L - \lambda_v]_0$ (Figure 3), the value of the corresponding penalty is calculated as follows:

$$\pi_{vl}^b = \pi_v^b \left((\underline{l}_{b_v^*} - l)^+ + (l + \lambda_v - \bar{l}_{b_v^*})^+ \right). \tag{1}$$

In Eq. (1), $(a-b)^+$ is defined as $(a-b)^+ = \begin{cases} a-b\,, & \text{if } a > b\,, \\ 0\,, & \text{otherwise.} \end{cases}$

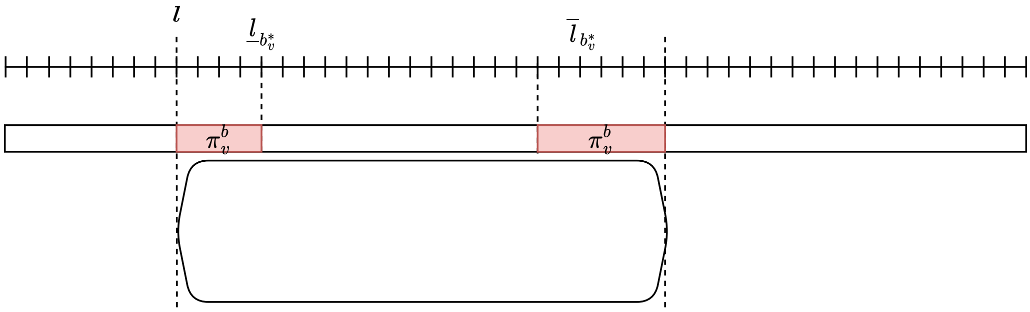

Figure 3 Penalty paid if vessel is not moored to its preferred berth

If vessel v is not moored to its preferred berth location, then the processing time of job j is extended by some parameters $\rho_j^v \in \mathbb{R}_+$, $v \in [V]$, $j \in [\beta_v]$, multiplied by the distance between the bay location and the preferred berth. The distance between bay j of vessel v and the preferred berth is denoted by σ_{jl}^v and calculated as follows:

$$\sigma_{jl}^v = (\underline{l}_{b_v^*} - l - \delta_j^v)^+ + (l + \delta_j^v - \bar{l}_{b_v^*})^+. \tag{2}$$

Now, we can calculate the extended number of time units required to complete job j if vessel is moored at location l, as follows:

$$\bar{\gamma}_{jl}^v = \gamma_j^v (1 + \sigma_{jl}^v \rho_j^v). \tag{3}$$

From previous calculations, it is obvious that time required to complete jobs increases if the distance from preferred berth increases. The rate of increase is given by the parameter ρ_j^v.

Our model also imposes several standard timing constraints. For each vessel v the earliest arrival time is specified by parameter $\varepsilon_v \in [T]_0$, while the latest departure time is defined by $\omega_v \in [T]_0$. The expected arrival time of vessel $v \in [V]$ is denoted by $\mu_v \in [T]_0$, while the schedule due time for departure is $\tau_v \in [T]_0$.

If vessel v is moored before its expected arrival time μ_v, then *earliness penalty* $\pi_v^e \in \mathbb{R}_+$ is paid for each time unit $t \in [T]_0$ between its actual arrival time and expected arrival time. As the total amount of this penalty depends on the arrival time of the vessel, we calculate earliness penalties for each particular time unit as follows:

$$\pi_{vt}^e = \pi_v^e (\mu_v - t)^+. \tag{4}$$

Similarly, we introduce *tardiness penalty* $\pi_v^t \in \mathbb{R}_+$, $v \in [V]$ that is paid for each time unit $t \in [T]_0$ between μ_v and its arrival time paid if vessel v is moored after its expected arrival time. If vessel v departs from the port after its scheduled due time, then *lateness penalty* $\pi_v^d \in \mathbb{R}_+$, $v \in [V]$ is paid for each time unit $t \in [T]_0$ between τ_v and vessel's v departure time. By the following equations, we explain the calculation of tardiness and lateness penalties for each time unit:

$$\pi_{vt}^t = \pi_v^t (t - \mu_v)^+, \tag{5}$$

$$\pi^d_{vt} = \pi^d_v (t - \tau_v)^+, \tag{6}$$

Crane. The initial position of crane $c \in [C]$ is given by parameter $l^0_c \in [L]_0$. The cost of crane operation per time unit is defined by the value of parameter $\alpha_c \in \mathbb{R}_+$, $c \in [C]$, while the movement cost per location unit is specified by $\phi_c \in \mathbb{R}_+$. If the crane $c \in [C]$ is assigned to vessel $v \in [V]$, then cost ξ_{cv} is paid. The distance between any two cranes $c', c'' \in [C]$ have to be at least $\psi \in \mathbb{N}_0$ location units (Figure 2). This distance constraint ensures the safe operation of cranes.

2.3. MILP model

Having all costs and penalties defined, we are ready to formulate MILP model for the considered problem. However, due to the lack of space space, objective function, constraints, and definitions of model variables are given in a separate file (supplementary material) that can be found at `http://www.mi.sanu.ac.rs/~tanjad/SYMOPIS2024-BACASSP-Model.pdf`.

3. EXPERIMENTAL EVALUATION

We implemented model using Gurobi as the `gurobipy` Python package with academic licence. The desktop computer with 11th Gen Intel(R) Core(TM) i7-11700 @ 2.50GHz processor with 8 cores, 32 GB RAM, and Windows 10 Professional Edition Operating System is used for computational experiments. As the hybrid model we propose is not considered previously in the literature, we conducted experiments on small set of randomly generated instances. We limited Gurobi solver to 6 CPU cores, 8 GB of RAM, and 2 hours running time. In Table 1: we present results of computational experiments.

T	L	V	C	B	$\max_{v\in[V]} \beta_v$	#Var	#Constr	Objective Value	Gap (%)	Time (s)
120	12	2	3	2	4	16388	50894	-35652.167	0.46	7200
120	12	2	3	2	4	16388	50894	-35661.667	0.52	7200
200	18	2	3	3	4	42910	150814	-59625.500	0.59	7200
120	18	2	3	3	4	25790	90494	-35613.500	0.95	7200
200	18	2	4	3	4	54934	201006	-	-	7200
200	18	2	5	3	4	67159	255401	-	-	7200

Table 1: Computational results on small instances

Only for 4 out of 6 instances feasible solutions are provided within time limit of two hours. The solution gap is below 1%. Number of constraints and variables suggest that model is difficult to solve to optimality even for a small instances with respect to number of time points, location points, vessels, cranes and berths. In Figure 4 we present the obtained feasible solution of first instance in Table 1:. Vessel 1 moored at expected arrival time 12 and departed before due time 108. Vessel 2 is moored at time unit 37, which is 13 units after its expected arrival time. In this case, tardiness penalty is paid. There are no other penalties, as vessel 2 departed before due date. Only crane 2 moved from location 5 to location 3 to complete job on vessel 2. This movement is obviously necessary as there are 4 jobs on vessel 2 and only 3 QCs available.

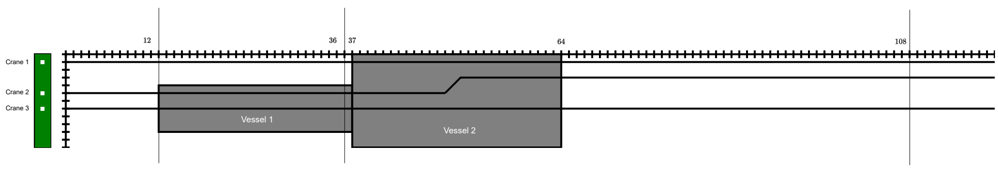

Figure 4 Example with 2 vessels, 3 cranes and 4 berths

4. CONCLUSION

We have presented a comprehensive Mixed-Integer Linear Programming (MILP) model for the integrated berth allocation, quay crane assignment, and quay crane scheduling problems in container terminals. Our model captures the complexities of port operations by considering the precise geometry of vessels, the dynamic assignment and movement of quay cranes, and the associated costs and penalties for non-preferred berths, early or late arrivals, and extended crane operations.

The model's comprehensive nature sets it apart from other models in the literature, as it integrates all critical aspects of berth and crane activities into a single framework. This integration provides a more realistic representation of port operations but also introduces significant computational challenges. The complexity of the model and the size of the instances require substantial solution times, making it difficult to obtain optimal or even first-feasible solutions within reasonable time frames.

Our computational results, conducted on a small set of generated instances using the Gurobi solver, indicate that the model's complexity leads to unsatisfactory performance. Specifically, only for a few instances near-optimal solutions could be obtained within the given time limit, while Gurobi failed to provide even the first feasible solution for the larger instances. This outcome was expected due to the model's depth and the inherent difficulties in solving such complex optimization problems. These results highlight the necessity for more sophisticated methods to find optimal, suboptimal, or near-optimal solutions efficiently. Heuristic and metaheuristic algorithms, such as Variable Neighborhood Search, Bee Colony Optimization, and other advanced techniques, could provide promising alternatives to exact methods. These approaches can offer significant improvements in solution times and quality, making them suitable for practical applications in large-scale and dynamic port environments. While our MILP model provides a comprehensive and detailed representation of integrated berth and crane operations, additional investigation is needed to develop and apply heuristic or metaheuristic methods to achieve practical and efficient solutions. This study sets the stage for such future work, aiming to enhance the operational efficiency and service quality of container terminals.

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