2.4. MILP model

Having all costs and penalties defined, we are ready to formulate MILP model for the considered problem.

Variables. Variables for controlling vessel's arrival and departure at particular location and time, x_{vlt} and z_{vlt} for $v \in [V]$, $l \in [L]$ and $t \in [T]_0$, are introduced as follows:

$$x_{vlt} = \begin{cases} 1, & \text{vessel } v \text{ is arrived at location } l \text{ at time point } t, \\ 0, & \text{otherwise,} \end{cases} \tag{7}$$

and

$$z_{vlt} = \begin{cases} 1, & \text{vessel } v \text{ is departed from location } l \text{ at time point } t, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

On the other hand, we introduce variables y_{clt}, s_{clt} and r_{clt}, $c \in [C]$, $l \in [L]_0$ and $t \in [T]_0$, to control position, movement and work of QCs at time points:

$$y_{clt} = \begin{cases} 1, & \text{quay crane } c \text{ is located at location } l \text{ at time point } t, \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

$$s_{clt} = \begin{cases} 1, & \text{quay crane } c \text{ is at location } l \text{ at time points } t-1 \text{ and } t, \\ 0, & \text{otherwise,} \end{cases} \tag{10}$$

$$r_{clt} = \begin{cases} 1, & \text{quay crane } c \text{ is working at location } l \text{ from time point } t-1 \text{ to } t, \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Variables w_{cv}, for $c \in [C]$, $v \in [V]$, are introduced to control the number of QCs assigned to vessel v:

$$w_{cv} = \begin{cases} 1, & \text{quay crane } c \text{ is located at some bay location of vessel } v \text{ while } v \text{ is in the port,} \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

We also introduce several groups of auxiliary variables. To limit minimum distance between two cranes at any particular point of time, variables $d_{c'c''t} \in \mathbb{R}_+$, for $c' < c''$, $c', c'' \in C$, $t \in [T]_0$ represent distances between cranes c' and c'' at time t. Variables $u^1_{vv'}$, $u^2_{vv'}$, $u^3_{vv'}$ and $u^4_{vv'}$, for $v, v' \in [V]$ are introduced to ensure feasible berthing time and location of vessel v, i.e., to prevent overlapping of vessel v with any other vessel moored in the port at the same time:

$$u^1_{vv'} = \begin{cases} 0, & \text{vessel } v' \text{ departed before vessel } v \text{ arrived to the port} \\ 1, & \text{otherwise,} \end{cases} \tag{13}$$

$$u^2_{vv'} = \begin{cases} 0, & \text{vessel } v \text{ departed before vessel } v' \text{ arrived to the port} \\ 1, & \text{otherwise,} \end{cases} \tag{14}$$

$$u^3_{vv'} = \begin{cases} 0, & \text{vessel } v' \text{ berthed to the location left from vessel } v \\ 1, & \text{otherwise,} \end{cases} \tag{15}$$

and

$$u^4_{vv'} = \begin{cases} 0, & \text{vessel } v \text{ berthed to the location left from vessel } v' \\ 1, & \text{otherwise.} \end{cases} \tag{16}$$

Objective function. The objective is to minimize the total cost of processing vessels in the port. Our objective function consists of the following terms:

- first term penalizes mooring of the vessel to non-preferred berth (corresponds to $w_1\, pos$), arrival to the port before (corresponding to $w_2\, speed$) or after the expected arrival time (corresponding to $w_4\, tard$),
- second term penalizes departure of the vessel after scheduled due time (corresponding to $w_3\, wait$)

- third term represents the total cost of quay crane movements,
- fourth term represents the total operation cost of quay cranes,
- and fifth term represents the total cost of assignment of cranes to vessels.

Third, fourth, and fifth terms in the objective function correspond to $w_5\,res$. The negative sign associated with the total crane movement cost, ϕ_c for $c \in [C]$, contributes to maximizing crane stillness. The fifth term is a novel addition not found in previous approaches. It is introduced to penalize the assignment of cranes to vessels when they are not actively working but are still located in some bays of the vessel during its stay in the port.

$$\min \sum_{v=1}^{V}\sum_{t=0}^{T}\sum_{l=0}^{L-\lambda_v}(\pi^b_{vl}+\pi^e_{vt}+\pi^t_{vt})x_{vlt}+\sum_{v=1}^{V}\sum_{t=0}^{T}\sum_{l=0}^{L-\lambda_v}\pi^d_{vt}z_{vlt}-\sum_{c=1}^{C}\sum_{l=0}^{L}\sum_{t=0}^{T}\phi_c s_{clt}+\sum_{c=1}^{C}\sum_{l=0}^{L}\sum_{t=0}^{T}\alpha_c r_{clt}+\sum_{c=1}^{C}\sum_{v=1}^{V}\xi_{cv}w_{cv} \tag{17}$$

Constraints. We give the list of all constraints that can be divided into two groups. There are constraints controlling bert to vessel allocation, and constraints that control crane movement, crane to vessel assignment and crane to job scheduling and assignment.

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}x_{vlt}=1, v\in[V] \tag{18}$$

$$\sum_{t=0}^{T}x_{vlt}-\sum_{t=0}^{T}z_{vlt}=0, v\in[V], l\in[L-\lambda_v] \tag{19}$$

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tz_{vlt}-\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tx_{vlt}\geq 1, v\in[V] \tag{20}$$

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tx_{vlt}\geq \varepsilon_v, v\in[V] \tag{21}$$

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tz_{vlt}\leq \omega_v, v\in[V] \tag{22}$$

$$\sum_{l=0}^{L-\lambda_{v'}}\sum_{t=0}^{T}tz_{v'lt}-\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tx_{vlt}\leq T\cdot u^1_{vv'}, v<v'v,v'\in[V] \tag{23}$$

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}tz_{vlt}-\sum_{l=0}^{L-\lambda_{v'}}\sum_{t=0}^{T}tx_{v'lt}\leq T\cdot u^2_{vv'}, v<v'v,v'\in[V] \tag{24}$$

$$\sum_{l=0}^{L-\lambda'_v}\sum_{t=0}^{T}(l+\lambda_{v'})x_{v'lt}-\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}lx_{vlt}\leq L\cdot u^3_{vv'}, v<v'v,v'\in[V] \tag{25}$$

$$\sum_{l=0}^{L-\lambda_v}\sum_{t=0}^{T}(l+\lambda_v)x_{vlt}-\sum_{l=0}^{L-\lambda_{v'}}\sum_{t=0}^{T}lx_{v'lt}\leq L\cdot u^4_{vv'}, v<v'v,v'\in[V] \tag{26}$$

$$u^1_{vv'}+u^2_{vv'}+u^3_{vv'}+u^4_{vv'}\leq 3, v<v'v,v'\in[V] \tag{27}$$

$$y_{c'lt}+y_{c'(l+1)(t+1)}+y_{c''(l+1)(t)}-y_{c''l(t+1)}\leq 4, c',c''\in[C], c'\neq c'', l\in[L-1]_0, t\in[T-1]_0 \tag{28}$$

$$y_{clt}-y_{c(l-1)(t-1)}-y_{c(l+1)(t-1)}-y_{cl(t-1)}\leq 0, c\in[C], l\in[L-1], t\in[T] \tag{29}$$

$$y_{cLt}-y_{c(L-1)(t-1)}-y_{cL(t-1)}\leq 0, c\in[C], t\in[T] \tag{30}$$

$$y_{c0t}-y_{c1(t-1)}-y_{c0(t-1)}\leq 0, c\in[C], t\in[T] \tag{31}$$

$$y_{cl^0_c 0}=1, c\in[C] \tag{32}$$

$$\sum_{c=1}^{C}y_{clt}\leq 1, l\in[L]_0, t\in[T]_0 \tag{33}$$

$$\sum_{l=0}^{L}y_{clt}=1, c\in[C], t\in[T]_0 \tag{34}$$

$$s_{clt}-r_{clt}\geq 0, c\in[C], l\in[L]_0, t\in[T] \tag{35}$$

$$s_{clt}-y_{cl(t-1)}-y_{clt}\geq -1, c\in[C], l\in[L]_0, t\in[T] \tag{36}$$

$$s_{clt}-y_{cl(t-1)}\leq 0, c\in[C], l\in[L]_0, t\in[T] \tag{37}$$

$$s_{clt} - y_{clt} \leq 0, c \in [C], l \in [L]_0, t \in [T] \tag{38}$$

$$\sum_{c=1}^{C}\sum_{t=1}^{t'} r_{c(l+\delta_j^v)t} \geq \overline{\gamma}_{jl}^{v} z_{vlt'}, v \in [V], j \in [\beta_v], \\ l \in \{\kappa_v, \kappa_v+1, \ldots, L-\lambda_v-\kappa_v\}, t' \in [T] \tag{39}$$

$$\sum_{c=1}^{C}\sum_{t=t'+1}^{T} r_{c(l+\delta_j^v)t} \geq \overline{\gamma}_{jl}^{v} x_{vlt'}, v \in [V], j \in [\beta_v], \\ l \in \{\kappa_v, \kappa_v+1, \ldots, L-\lambda_v-\kappa_v\}, t' \in [T-1]_0 \tag{40}$$

$$\sum_{c=1}^{C}\sum_{j=1}^{\beta_v} r_{c(l+\delta_j^v)t} \geq \sum_{t=0}^{t'-1} x_{vlt} + \sum_{t=0}^{t'} z_{vlt} - 1, v \in [V], j \in [\beta_v], \\ l \in \{\kappa_v, \kappa_v+1, \ldots, L-\kappa_v\}, t' \in [T] \tag{41}$$

$$w_{cv} - r_{c(l+\delta_j^v)t'} - \sum_{t=0}^{t'-1} x_{vlt} - \sum_{t=t'}^{T} z_{vlt} \geq -2, c \in [C], v \in [V], j \in [\beta_v], \\ l \in \{\kappa_v, \kappa_v+1, \ldots, L-\kappa_v\}, t' \in [T] \tag{42}$$

$$\sum_{c=1}^{C} w_{cv} \leq \theta_v^{max}, v \in [V] \tag{43}$$

$$\sum_{c=1}^{C} w_{cv} \geq \theta_v^{min}, v \in [V] \tag{44}$$

$$d_{c'c''t} \geq \psi, t \in [T]_0, c' < c'', c', c'' \in [C] \tag{45}$$

$$d_{c'c''t} - \sum_{l=0}^{L} l y_{c'lt} + \sum_{l=0}^{L} l y_{c''lt} \geq 0, t \in [T]_0, c' < c'', c', c'' \in [C] \tag{46}$$

$$d_{c'c''t} - \sum_{l=0}^{L} l y_{c''lt} + \sum_{l=0}^{L} l y_{c'lt} \geq 0, t \in [T]_0, c' < c'', c', c'' \in [C] \tag{47}$$

Vessel constraints. The constraints in (18) ensure that each vessel must berth at exactly one port location. Similarly, the constraints in (19) guarantee that if a vessel arrives at a specific port location, it must depart from the same location. The constraints in (21) ensure that each vessel arrives no earlier than its earliest possible arrival time, while the constraints in (22) ensure that each vessel v departs no later than its latest departure time ω_v. Each vessel can be represented as as a rectangle in two dimensional coordinate system with location and time axis. Thus, safely mooring constraints are equivalent to the rectangle non-overlapping constraints. Constraints from (23) to (27) ensure that vessel v can safely moor to some some port location l taking into account arrival time and locations of other vessels.

Crane constraints. Constraint (28) ensures that cranes c' and c'' do not overlap in their movements. Specifically, it ensures that if crane c' is at location l at time t, and crane c'' is at location $l+1$ at time $t+1$, then crane c'' cannot be at location l at time $t+1$. This constraint effectively prevents cranes from occupying adjacent locations in consecutive time periods in a way that could cause operational conflicts. At time point t, crane c can be at location l only if at time $t-1$ it was at location $l-1$, l or $l+1$ (29). This constraints also must hold for right and the left boundary of the port (30), (31). In constraints (32) the initial positions of cranes are given, i.e., their positions at time 0. At most one crane can be located at location $l \in [L]_0$ and time $t \in [T]_0$ (33). Also, each crane is located at the exactly one location at any time point (34). We say that crane stays at location l from $t-1$ to t if it is located at l in time points $t-1$ and t. If crane operates at location l from $t-1$ to t, then it stay at the same location in the same time interval (35). Constraints (36), (37) and (38) ensure consistency between variables that control position and movement of cranes.

Crane scheduling and job assignment constraints. If the vessel v is departed from location l at time t', then all cranes (in total) must operate at least $\overline{\gamma}_{jl}^{v}$ times units at each location $l+\delta_j$, for $j \in [\beta_v]$, before time t', i.e. before the vessel departs from the port (39). Similarly, if vessel v arrives at location l at time t', then all cranes (in total) must operate at least $\overline{\gamma}_{jl}^{v}$ times units at each location $l+\delta_j$, for $j \in [\beta_v]$, after time t', i.e. after the vessel arrives at the port (40). At each time interval from $t'-1$ to t' while the vessel v is at the port at location l, there must be at least one crane that operates at one of the locations $l+\delta_j$ in time interval from $t'-1$ to t' (41). Crane c is assigned to vessel v if crane c operates at some of the bay locations of vessel v during the stay of the vessel in the port (42). Constraints in (44) and (43) ensure the minimum and maximum number of assigned cranes to the vessel $v \in [V]$, respectively. Any two cranes c' and c'' have to be at distance at least ψ location units at any point of time (45), (46) and (47).