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PRIMENA METAHEURISTIKA NA OPTIMIZACIJU DINAMIČKE IZDRŽLJIVOSTI I RADNOG VEKA KOD KUGLIČNIH LEŽAJA

META-HEURISTICS APPLICATION TO OPTIMISE BALL BEARINGS DYNAMICAL LOAD RATINGS AND RATING LIFE

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Rezime: *Primenom metaheuristika, razvijen je novi metod za optimizaciju dinamičke izdržljivosti i radnog veka kod ležaja u funkciji 10 različitih parametara. Cilj istraživanja je bio utvrđivanje parametara koji imaju najveći uticaj na postizanje maksimalnog radnog veka. Upoređeni su rezultati za tri metaheuristike i dobijena su poboljšanja u odnosu na vrednosti iz kataloga.*
Ključne reči: *Kontinualna optimizacija, Unutrašnja geometrija ležaja, ISO standard, Genetski algoritmi, Lokalno pretraživanje.*
Abstract: *A new method for optimisation of dynamical load ratings and rating life, as a function of 10 different parameters, of rolling bearings was developed using meta-heuristics. The aim of this research was to determine which parameters have the largest influence on achieving the maximum working life. The results obtained with three meta-heuristic methods are compared and the improvement, with respect to the catalogue values, is achieved.*
Keywords: *Continious optimisation, Inner geometry of bearings, ISO Standard, Genetic Algorithm, Local search.*

1. INTRODUCTION

A long fatigue life is one of the most important criteria in the optimum design of needle roller bearings (NRBs). Therefore, in (Waghole and Tiwari 2014) the dynamic capacity of the bearing was optimised. The non-linear optimisation model has been formulated and threaded with Artificial Bee Colony Algorithm (ABCA), Differential Search Algorithm (DSA), Grid Search Method (GSM) and Hybrid Method (HM, a combination of the ABCA/DSA and GSM). A total of four design variables corresponding to bearing geometry were considered. The dynamic capacity of optimised bearings was found better than those specified in bearing catalogues.

A constraint non-linear optimisation procedure based on GAs for designing rolling-element bearings has been developed in (Rao and Tiwari 2007). Based on maximum fatigue life, the objective function and associated kinematic constrains have been formulated. The design parameters include the bearing pitch diameter, the rolling element diameter, number of rolling elements and inner and outer-race groove curvature radii. The constraints contain unknown constants, which have been given ranges based of parametric studies through initial optimisation runs. In the final run of the optimisation, these constraint constants are also included as design parameters. The optimised design parameters yield better fatigue life as compared to those listed in standard catalogues. A convergence study has been performed to ensure that the optimised design variables do not suffer from local extremes.

Dynamical load ratings and rating life of a rolling bearing, based on (Standard ISO 281 and 76), depends on many factors, and it is obvious that the rating life can be extended by optimising the values of influential parameters. The goal of the optimisation was to find the optimal inner geometry of bearings based on the outer geometry. Meta-heuristics are the customary tools and in this study we used three well known meta-heuristic methods.

The optimisation is performed in the form of a numerical simulation. Apart from the formulas and procedures from (Standard ISO 281 and 76), the values of some specific parameters were varied in order to find the appropriate combination of geometry, rating factor (the value of which varies with a bearing type and design), static and dynamic radial load rating, the value of the parameter for mobility conditions, the dynamic radial and axial factor and the factor which depends on the geometry of the bearing components and the material.

In order to simplify the model, the radial component of the actual bearing load and the axial component of the actual bearing load are set as constant. Optimised parameters are mostly related to the bearings geometry, and the term geometry refers to an optimisation against the number of rolling elements in a single row bearing, the nominal ball diameter, the pitch diameter of the bearing and the radial contact angle of bearings. All of these factors, together with the factor of geometry and material, directly impact the calculation of basic dynamic load rating.

2. PROBLEM FORMULATION

The use of the term basic rating life L_{10} refers to the optimisation against the load rating and the equivalent load, which is formulated in the equation:

$$L_{10} = \frac{C_r^{\ 10/3}}{P_r} \tag{1}$$

Basic dynamic radial load rating for radial ball bearings is given by the following equations from (Standard ISO 281):

$$C_r = \begin{cases} b_m f_c (i\cos\alpha)^{0.7} Z^{2/3} D_b^{\ 1/8}, & D_b \le 25.4mm \\ 3{,}647 b_m f_c (i\cos\alpha)^{0.7} Z^{2/3} D_b^{\ 1/8}, & D_b > 25.4mm \end{cases} \tag{2}$$

Load rating P_r is given by the equation:

$$P_r = XF_r + YF_a \tag{3}$$

where X denotes the dynamic radial load factor and Y stands for the dynamic axial load factor. To find the optimum value of load rating P_r, it is necessary to find the optimal values of factors X and Y, which will be explained in the next sections.

Therefore, the optimisation function to be maximised is

$$L_{10}(S),$$

for $S = \{K_{D\min}, K_{D\max}, \varepsilon, m, \beta, Z, D_b, D_m, f_i, f_o\}$, with respect to the following constraints:

$$c_1(S) = \frac{\Phi}{2\arccos(D_b / D_m)} - Z + 1 \ge 0 \tag{4}$$

$$c_2(S) = 2D_b - K_{D\min}(D-d) \ge 0 \tag{5}$$

$$c_3(S) = K_{D\max}(D-d) - 2D_b \ge 0 \tag{6}$$

$$c_4(S) = D_m - (0{,}5-m)(D+d) \ge 0 \tag{7}$$

$$c_5(S) = (0{,}5+m)(D+d) - D_m \ge 0 \tag{8}$$

$$c_6(S) = \frac{d_i - d}{2} - \frac{D - d_o}{2} \ge 0 \tag{9}$$

$$c_7(S) = 0{,}5(D - D_m - D_b) - \varepsilon D_b \ge 0 \tag{10}$$

$$c_8(S) = \beta W - D_b \ge 0 \tag{11}$$

$$c_9(S) = f_i \ge 0{,}515 \tag{12}$$

$$c_{10}(S) = f_o \ge 0{,}515 \tag{13}$$

$$c_{11}(S)=\left[\frac{U^2+(D/2-T-D_b)^2-(d/2+T)^2}{2U(D/2-T-D_b)}\right]+1\geq 0 \tag{14}$$

$$c_{12}(S)=1-\left[\frac{U^2+(D/2-T-D_b)^2-(d/2+T)^2}{2U(D/2-T-D_b)}\right]\geq 0 \tag{15}$$

$$c_{13}(S)=(D_b/D_m)+1\geq 0 \tag{16}$$

$$c_{14}(S)=1-(D_b/D_m)\geq 0 \tag{17}$$

For the convenience of the bearing assembly, the number Z and the diameter D_b of balls should satisfy the requirement given by the relation (4), where Φ_o is the maximum tolerable assembly angle (Gupta et al., 2007), calculated by the following equation (see Fig. 1):

$$\Phi_o=2\pi-2\arccos\left[\frac{U^2+(D/2-T-D_b)^2-(d/2+T)^2}{2U(D/2-T-D_b)}\right]$$ where $T=(D-d-2D_b)/4$ and $U=(D-d)/2-3T$.

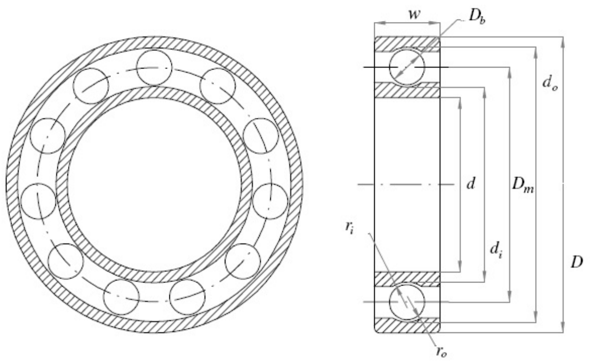

Figure 1: Radial ball bearing macro-geometries from (Gupta et al., 2007)

In addition to the presented constraints, some geometrical characteristics are expressed by the given lower and upper bounds: $3\leq Z\leq 200$, $1\leq D_m\leq 500$, $1\leq D_b\leq 500$, $0{,}6\leq K_{D\max}\leq 0{,}7$, $0{,}4\leq K_{D\min}\leq 0{,}5$, $0{,}3\leq\varepsilon\leq 0{,}35$, $0{,}03\leq m\leq 0{,}08$, $0{,}7\leq\beta\leq 0{,}85$.

3. ADDITIONAL PARAMETER SETTINGS

Values of b_m for radial ball bearings, are given by Table 1 from (ISO Standard 76). In this study $b_m=1{,}3$ has been adopted, since only radial contact ball bearings are investigated. The f_c can be calculated as follows:

$$f_d=37{,}91\left\{1+[1{,}04\,f_g^{\,1{,}72}\,f_{io}^{\,0{,}41}]^{10/3}\right\}^{-0{,}3}\left[\frac{\gamma^{0{,}3}(1-\gamma)^{1{,}39}}{(1+\gamma)^{1/3}}\right]\left[\frac{2f_i}{2f_i-1}\right]$$

where $\gamma=D_b\cos\alpha/D_m$, $f_g=\dfrac{1-\gamma}{1+\gamma}$, $f_{io}=\dfrac{f_i(2f_o-1)}{f_o(2f_i-1)}$, and α is the free contact angle that depends upon the type of a bearing. In order to simplify the model, the following working conditions are given: $F_a=100N$, $F_r=1500N$. For the purpose of this study, several bearing types are selected. They are described by the outer dimensions D,d,W as it is presented in the first four columns of Table 4, while the corresponding inner dimensions are obtained by optimisation methods. The selected bearing types belong to the class of the single-row-deep-groove ball bearings, and therefore it holds that $i=1$ and $\alpha=0^\circ$.

4. INTERPOLATION OF THREE-DIMENSIONAL DATA

To find a proper value of the dynamic radial load factor X and the dynamic axial load factor Y, it was necessary to perform an interpolation based on the relative axial load (given by the relations f_oF_a/C_{or} and

$F_a/(iZD_b^2)$) and e (the limiting value of F_a/F_r). Values of the appropriate dynamic radial and axial load factors, for the ranges of above-mentioned three values, are given in Table 3 from (Standard ISO 281). The relations f_oF_a/C_{or} and $F_a/(iZD_b^2)$ are input values of the separate interpolation module, and e is a result of calculations based on the working conditions F_a and F_r where $C_{or} = f_o iZD_b^2 \cos\alpha$.

5. META-HEURISTICS IMPLEMENTATION

The above described optimisation problem contains the continuous non-linear objective function, with linear continuous constraints. Therefore, various optimisation methods, designed for this case can be applied. In this study, the optimisation is performed using three different meta-heuristic methods: Genetic algorithms (GA), Multi-start Pattern search (MPS), Multi-start Fmicon (MF). Fmincon is the optimisation function from the MATLAB optimisation tool in (Venkataraman 2009). The proposed optimisation algorithm is presented in Fig 2.

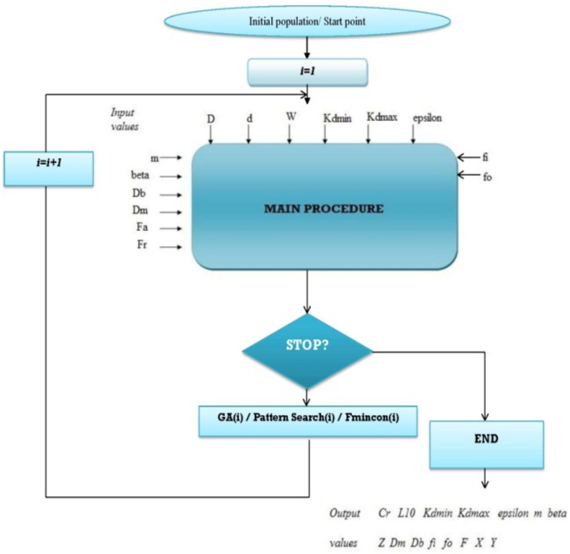

Figure 2: Optimisation algorithm

The MATLAB optimisation functions are used in order to implement all the proposed methods. The methods use the same main procedure and the same input values. Extensive experimental evaluation is performed in order to determine the best parameter settings for each of the used methods. The resulting settings are described in Tables 1, 2 and 3.

Table 1: The parameter settings in GA

Parameter	Settings
Poll method	GPS Positive basis 2N
Complete poll	On
Polling order	Success
Complete search	On
Search method	GPS Positive basis 2N
Mesh Initial size	1.0
Mesh Max size	Inf
Accelerator	On
Rotate	On
Scale	On
Expansion factor	2.0
Contraction factor	0.5
Initial penalty	10
Penalty factor	100
Bind tolerance	10^{-3}
Cache	On
Tolerance	Eps
Size	10^{4}

Table 2: The parameter settings in PS

Parameter	Settings
Population	Double vector
Population size	50
Initial population	Randomly generated
Scaling function	Proportional
Selection function	Tournament
Elite count	10
Crossover fraction	0.6
Mutation	Adaptive feasible
Crossover function	Two point
Number of generation	1000 generation
Stall time	50 sec

Table 3: The parameter settings in Fmincon

Parameter	Settings
Algorithm	Active set
Derivatives	Approximated by solver
Max iterations	400
Max function evaluations	1000
X tolerance	10^{-6}
Nonlinear constraint tolerance	10^{-6}
SQP constraint tolerance	10^{-6}
Function value check	None
User-supplied derivatives	None
Minimum pertubation	10^{-8}
Maximum pertubation	0.1
Type	Forward differences
Hessian	None
Typical X values	ones(10,1)

The stopping criterion is either of the two: the maximum number of generations or the maximum stall time. Since PS is an iterative (local search) heuristic method, the parameter settings (as presented in Table 2) are given for a single execution (without restarts). The stopping criterion is either of the six: Mesh tolerance (10^{-6}) or Max iterations (100*number of variables), or Max function evaluations (2000*number of variables), or X tolerance (10^{-6}), or Function tolerance (10^{-6}), or Nonlinear constraint tolerance (10^{-6}). The starting points are created randomly from the set of feasible solutions. The number of PS restarts is determined by reaching the GA stopping time.

Contrary to PS, Fmincon is a gradient-based method for non-linear constraint optimisation problems which is designed to work on problems where the objective and constraint functions are both continuous and have continuous first derivatives (Venkataraman, 2009). It is also an iterative (local search) heuristic method and, again, the parameter settings given in Table 3 correspond to a single execution. Fmincon attempts to find a constrained minimum of a scalar function of several variables starting from an initial estimate. The initial solutions for each restart and the number of restarts are determined in the same way as for MPS.

8. RESULTS

The simulation of dynamical load ratings of normal contact ball bearings is conducted by changing the various influential factors. As a result, the corresponding values of the rating life and dynamic capacity are measured and optimised by the applied meta-heuristic methods. At the same time, the extent to which specific factors influence the best value of the rating life is determined.

This optimisation problem is a non-linear, multi-objective problem with inequality constraints. Two functions, the dynamic load capacity and maximum working life (under a certain conditions), are optimised simultaneously. However, since the two functions are not conflicted, optimisation is conducted the same way as for single-objective problems.

The comparative results for three methods are given in tables 4 and 5. It appears that, by changing the number of balls in a ball bearing, it is possible to increase the dynamic load capacity with comparison to the available standards. Increasing of the dynamic load capacity leads to the increase in the value of the dynamic working life. The MF method requires less computing time with respect to GA and MPS. It generates the best values for the dynamic load capacity in six out of eight bearings type. GA gives the best results for working life in five out of eight cases. The recommended values for the number of balls are $Z=7$, $Z=7$, $Z=8$, $Z=8$, $Z=8$, $Z=9$, $Z=9$ and Z = 9, respectively, for the considered eight types of bearings in order to achieve the best values for the dynamic load capacity and the largest working life.

Table 4: Comparative results for three optimisation methods

Catalogue values from Shigley et al., 1989					Heuristic optimisation								
					GA results			PS results			Fmincon results		
Bear. type	D	d	W	Dyn. Cap	Dyn. Cap	Work. life	CPU time	Dyn. Cap	Work. life	CPU time	Dyn. Cap	Work. Life	CPU time
6200	30	10	9	5070	6842.4	**540.502**	43.4931	**6848.8**	531.169	41.746	6848.7	531.8595	**2.6832**
6201	32	12	10	6890	7223.7	**647.61**	21.2473	7223.7	627.242	83.772	**7238.6**	631.5472	**0.6084**
6202	35	15	11	7800	8079.6	940.601	18.3301	**8319.3**	**989.242**	8.5864	8263	971.6416	**0.39**
6203	40	17	12	9560	10636	**2351.4**	17.1445	10667	2181.8	87.719	**10703**	2186.7	**0.2808**
6204	47	20	14	12700	14211	**6178**	100.761	14243	5589.6	83.335	**14291**	5589.6	**0.234**
6205	52	25	15	14000	15307	7914.30	29.7962	15998	**8233.3**	41.543	**16085**	8232.6	**0.4212**
6206	62	30	16	19500	19974	19215	45.2101	21756	**22942**	42.370	**21878**	22941	**1.3416**
6207	72	35	17	25500	28241	**60961**	55.4272	28285	55027	83.850	**28447**	55028	**0.4056**

9. CONCLUSION

GA has proven to be a suitable technique in situations when it is necessary to deal with continuous optimization problems. However, it is always good to have comparative results obtained by other methods. Therefore, multi-start Pattern Search (PS) and multi-start Fmincon are applied to optimize the problem of dynamic load capacity and working life of ball bearings and the comparative results with GA are presented.

Table 5: Comparative results for three optimisation methods

Bear. type	Opt. met.	Design parameters										Calculated values		
		$K_{D\min}$	$K_{D\max}$	ε	m	β	Z	D_m	D_b	f_i	f_o	F	X	Y
6200	GA	0.431	0.699	0.301	0.047	0.848	7	6.999	18.772	0.515	0.515	4.015	0.56	2.02
6200	PS	0.4	0.7	0.3	0.066	0.85	7	6.999	18.8	0.515	0.515	4.015	0.56	2.02
6200	MPS	0.4	0.7	0.3	0.08	0.778	7	7.000	18.8	0.515	0.515	4.015	0.56	2.06
6201	GA	0.411	0.7	0.3	0.0483	0.7094	7	7.000	20.800	0.515	0.515	3.892	0.56	2.06
6201	PS	0.4	0.7	0.3	0.066	0.849	7	7.000	20.799	0.515	0.515	3.89	0.56	2.06
6201	MPS	0.4	0.7	0.3	0.038	0.85	7	7.000	20.8	0.515	0.515	3.892	0.56	2.06
6202	GA	0.401	0.700	0.302	0.067	0.819	8	6.853	24.000	0.515	0.515	3.732	0.56	2.06
6202	PS	0.4	0.7	0.3	0.066	0.85	8	7.000	23.799	0.515	0.515	3.7926	0.56	2.10
6202	MPS	0.4	0.7	0.3	0.08	0.85	8	7.000	23.1	0.515	0.515	3.76	0.56	2.06
6203	GA	0.487	0.700	0.3104	0.0583	0.7640	8	8.050	26.876	0.515	0.515	3.7705	0.56	2.22
6203	PS	0.4	0.7	0.3	0.0665	0.8484	8	8.050	27.11	0.515	0.515	3.7705	0.56	2.22
6203	MPS	0.4001	0.7	0.3	0.08	0.7	8	8.050	27.12	0.515	0.515	3.7705	0.56	2.22
6204	GA	0.4008	0.700	0.3001	0.0666	0.8096	8	9.4341	31.903	0.515	0.515	3.7668	0.56	2.3
6204	PS	0.4	0.7	0.3	0.0663	0.8484	8	9.450	31.879	0.515	0.515	3.7693	0.56	2.3
6204	MPS	0.4	0.7	0.3	0.08	0.85	8	9.450	31.88	0.515	0.515	3.7693	0.56	2.3
6205	GA	0.4037	0.6974	0.3051	0.0536	0.7909	9	9.1936	36.967	0.515	0.515	3.6268	0.56	2.3
6205	PS	0.4	0.7	0.3000	0.0665	0.8484	9	9.450	36.879	0.515	0.515	3.6585	0.56	2.3
6205	MPS	0.4	0.7	0.3	0.08	0.7	9	9.450	36.88	0.515	0.515	3.6585	0.56	2.3
6206	GA	0.499	0.7	0.3490	0.0645	0.8461	9	11.200	42.604	0.515	0.515	3.6529	0.56	2.3
6206	PS	0.4	0.7	0.3000	0.0663	0.8484	9	11.200	44.079	0.515	0.515	3.6529	0.56	2.3
6206	MPS	0.4	0.7	0.3	0.0318	0.85	9	11.200	44.08	0.515	0.515	3.6529	0.56	2.3
6207	GA	0.4197	0.699	0.3101	0.0535	0.8053	9	12.946	50.966	0.515	0.515	3.6486	0.56	2.3
6207	PS	0.4000	0.7	0.3000	0.0663	0.85	9	12.950	51.279	0.515	0.515	3.6489	0.56	2.3
6207	MPS	0.4	0.7	0.3	0.08	0.85	9	12.950	51.28	0.515	0.515	3.6489	0.56	2.3

The proposed optimization methods provide the increase in dynamic capacity with respect to the values form catalogue in all eight examples. The average percentage of the improvement is 9.4%, 12.2% and 12.6% for GA, PS and Fmincon respectively. Even assuming the producers left some degree of safety, the percentage of the improvement obtained by meta-heuristic optimization can be considered as significant. This preliminary results showed that single-solution meta-heuristic methods outperform GA and this phenomena maybe interesting for further evaluations.

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