A Matheuristic Approach for the University Carpooling Problem

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1 Introduction

The carpooling problem consists of a shared use of private cars. Typically it is organized by a large company for encouraging its employees to pick up their colleagues minimizing the number of private cars travelling to/from the company site. The core of the efficient management of such a service is to find an optimal matching between the users and their preferred routing in such a way that spontaneous user matching is substituted by a solution found by means of an algorithmic approach [1, 6]. We consider the special case when users are the students of a university. This case differs from the carpooling problems considered in the literature mainly for the following characteristics: users (students) can have very different timetables (depending on the classes attended); users may indicate other users they would prefer to car-pool with (friends) or they don't want to (enemies). The objectives are to maximize the number of served users, minimize the total route length, maximize the satisfied friendship preferences, respecting the user time windows, and car capacities. The university carpooling problem has been proposed for the first time in [5] and tackled with a Monte Carlo algorithm in [2] and with a Variable Neighborhood Search (VNS) in [3]. In this work we propose a Mixed Integer Linear Programming (MILP) formulation of the university carpooling problem in order to apply a matheuristic based on VNS, namely the Variable Neighborhood Branching (VNB) [4].

2 Problem Formulation

Let D be the driver set, P the passenger set and M the set of users available to act both like drivers or passengers (mixed users). Let $\eta_{u'u''}$ be equal to 1 if users u' and u'' are friends, -1 if they are enemies. Let o_u and d_u be respectively the origin and the destination of user $u \in ALL = D \cup P \cup M$. Let V be the collection of all locations i.e. the origins and destinations of all users and let τ_{ij} the time needed to move by car from location i to location j. A time window $[T_u^{min}, T_u^{max}]$ is associated to each user u, where T_u^{min} represents the earliest departure time from o_u, and T_u^{max} represents the latest arrival time to d_u. Let

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t^*_u be the minimum travel time of user u to move by car from o_u to d_u and let $MaxLoS_u$ represent the maximum lengthening factor allowed travelling in carpooling. Let us introduce assignment binary variables w_{ku} equal to 1 if user $u \in P \cup M$ is assigned to the car of user $k \in D \cup M$, 0 otherwise. Let x_{ijk} be routing binary variables equal to 1 if user $k \in D \cup M$ visits location j immediately after location i, 0 otherwise. Let y_{ik} be counting variables to model the number of people on board in the car of user $k \in D \cup M$ when it leaves from $i \in V$. Let $\xi_{ku'u''}$ be binary variables equal to 1 if and only if both users u' and u'' are served by user k. Let dep_{ik} be the departure time from $i \in V$ of user $k \in D \cup M$ and arr_{ik} its arrival time in i. The university carpooling problem can be modeled by the following MILP:

$$\max \lambda_1 KB + \lambda_2 MC + \lambda_3 LOS + \lambda_4 PREF$$

$$\sum_{k \in D \cup M} w_{ku} \leq 1 \qquad \forall u \in P \cup M \quad (1)$$

$$\sum_{k \in D \cup M} w_{ku} \leq 1 - w_{up} \qquad \forall u \in M, \forall p \in P \cup M \quad (2)$$

$$w_{uk} = 0 \qquad \forall u \in P \cup M, \forall k \in D \cup M : \eta_{uk} = -1 \quad (3)$$

$$w_{u'k} + w_{u''k} \leq 1 \qquad \forall u', u'' \in P \cup M, \forall k \in D \cup M : \eta_{u'u''} = -1 \quad (4)$$

$$\sum_{j \in \delta^+(i)} x_{ijk} - \sum_{j \in \delta^-(i)} x_{jik} = 0 \qquad \forall k \in D \cup M, \forall i \in V \setminus \{o_k, d_k\} \quad (5)$$

$$\sum_{j \in \delta^+(o_k)} x_{o_k jk} - \sum_{i \in \delta^-(o_k)} x_{io_k k} = 1 \qquad \forall k \in D \quad (6)$$

$$\sum_{i \in \delta^-(d_k)} x_{id_k k} - \sum_{j \in \delta^+(d_k)} x_{d_k jk} = 1 \qquad \forall k \in D \quad (7)$$

$$\sum_{j \in \delta^+(o_u)} x_{o_u ju} = 1 - \sum_{k \in D \cup M} w_{ku} \qquad \forall u \in M \quad (8)$$

$$\sum_{i \in \delta^-(o_u)} x_{io_u u} \leq \sum_{k \in D \cup M} w_{ku} \qquad \forall u \in M \quad (9)$$

$$\sum_{i \in \delta^-(d_u)} x_{id_u u} = 1 - \sum_{k \in D \cup M} w_{ku} \qquad \forall u \in M \quad (10)$$

$$\sum_{j \in \delta^+(d_u)} x_{d_u jk} \leq \sum_{k \in D \cup M} w_{ku} \qquad \forall u \in M \quad (11)$$

$$x_{iju} \leq 1 - \sum_{k \in D \cup M} w_{ku} \qquad \forall u \in M \quad (12)$$

$$\sum_{j \in \delta^+(o_u)} x_{o_u jk} \geq w_{ku} \qquad \forall u \in P \cup M, \forall k \in D \cup M \quad (13)$$

$$\sum_{j \in \delta^-(d_u)} x_{id_u k} \geq w_{ku} \qquad \forall u \in P \cup M, \forall k \in D \cup M \quad (14)$$

$$y_{o_k k} = 0 \qquad \forall k \in D \cup M \quad (15)$$

$$y_{o_u k} \leq y_{ik}+1+q_k(2-w_{ku}-x_{io_u k}) \qquad \forall i\in V, \forall k\in D\cup M, \forall u\in P\cup M \quad (16)$$

$$y_{o_u k} \geq y_{ik}+1-(q_k+1)(2-w_{ku}-x_{io_u k}) \qquad \forall i\in V, \forall k\in D\cup M, \forall u\in P\cup M \quad (17)$$

$$y_{d_u k} \leq y_{ik}-1+(q_k+1)(2-w_{ku}-x_{id_u k}) \qquad \forall i\in V, \forall k\in D\cup M, \forall u\in P\cup M \quad (18)$$

$$y_{d_u k} \geq y_{ik}-1-q_k(2-w_{ku}-x_{id_u k}) \qquad \forall i\in V, \forall k\in D\cup M, \forall u\in P\cup M \quad (19)$$

$$arr_{jk} \geq dep_{ik}+\tau_{ij}-(\tau_{ij}+T_k^{max})(1-x_{ijk}) \qquad \forall i,j\in V, \forall k\in D\cup M \quad (20)$$

$$dep_{o_u k} \geq T_u^{min} - T_u^{min}(1-w_{uk}) \qquad \forall u\in P\cup M, \forall k\in D\cup M \quad (21)$$

$$arr_{d_u k} \leq T_u^{max} + T_k^{max}(1-w_{uk}) \qquad \forall u\in P\cup M, \forall k\in D\cup M \quad (22)$$

$$dep_{ik} \geq arr_{ik} \qquad \forall i\in V, \forall k\in D\cup M \quad (23)$$

$$arr_{d_u k}-dep_{o_u k} \leq t_u^* MaxLoS_u+T_k^{max}(1-w_{uk}) \quad \forall u\in P\cup M, \forall k\in D\cup M \quad (24)$$

$$arr_{d_k k} - dep_{o_k k} \leq t_k^* MaxLoS_k \qquad \forall k\in D\cup M \quad (25)$$

$$KB = \frac{1}{|D\cup M|}\frac{\sum_{k\in D\cup M}\frac{1}{q_k-1}\sum_{u\in P\cup M} t_u^* w_{uk} - arr_{d_k k} + dep_{o_k k}}{\max_{u\in P\cup M} t_u^*} \quad (26)$$

$$MC = \frac{1}{|P\cup M|}\sum_{k\in D\cup M}\sum_{u\in P\cup M} w_{ku} \quad (27)$$

$$LOS = \frac{1}{\sum_{u\in ALL}(MaxLoS_u-1)}\sum_{u\in ALL} z_u \quad (28)$$

$$z_u \leq MaxLoS_u - \frac{arr_{d_u k}-dep_{o_u k}}{t_u^*} + \max_{v\in ALL} T_v^{max}(1-w_{ku}) \quad \forall u\in P\cup M, \forall k\in D\cup M \quad (29)$$

$$z_k \leq MaxLoS_k - \frac{arr_{d_k k}-dep_{o_k k}}{t_k^*} \qquad \forall k\in D \quad (30)$$

$$z_u \leq \sum_{k\in D\cup M} w_{ku} \qquad \forall u\in P\cup M \quad (31)$$

$$z_k \leq \sum_{u\in P\cup M} w_{ku} \qquad \forall k\in D\cup M \quad (32)$$

$$PREF = \sum_{\substack{k\in D\cup M,\\ u'u''\in P\cup M:\eta_{u'u''}=1}} \xi_{ku'u''} \quad (33)$$

$$\xi_{ku'u''} \leq w_{ku'} \qquad \forall k\in D\cup M, \forall u',u''\in P\cup M : \eta_{u'u''}=1 \quad (34)$$

$$\xi_{ku'u''} \leq w_{ku''} \qquad \forall k\in D\cup M, \forall u',u''\in P\cup M : \eta_{u'u''}=1 \quad (35)$$

The objective function (o.f.) is the weighted sum with non negative weights λ_i of four utility functions: kilometer benefit (KB), i.e. the saving obtained on the total length covered by the carpooling users compared to the total length covered without carpooling; matching coefficient (MC), i.e. the fraction of carpooling requests satisfied; level of service (LOS) which measures the average path lengthening of all the users served compared to their minimum paths; user preferences (PREF) which measures the friendship preferences of the users. The components of the o.f. are ruled by constraints (26)-(33). Constraints (1) impose that each passenger or mixed user is served at most by one driver or mixed user. Constraints (2) prevent a served mixed user to act as a driver. (3) and

(4) prevent enemies being in the same pool. Constraints (5)-(7) guarantee that each driver follows a path from its origin to its destination. Constraints (8)-(11) impose the same thing to mixed users that act like drivers. While (12) set to 0 the routing variables of mixed users that do not act like drivers. Constraints (13) and (14) impose respectively that the origin and the destination of each user $u \in P \cup M$ is visited by the driver which u is assigned to. (15)-(19) increment or decrement the counter variables y_{ik} each time a user gets in or out the car of user k. Constraints (20) increase variables dep_{ik} according to the travel times, while (21)-(22) and (24)-(25) enforce user time windows and $MaxLoS$, respectively.

3 Some numerical results and conclusion

The proposed formulation is a special case of 0-1 Mixed Integer Program (0-1 MIP). Therefore, it is possible to use it in 0-1 MIP solution methods (matheuristics). Among the tested methods, VNB [4] performed the best. VNB adds constraints to the original problem by changing the neighborhoods in a systematic manner, according to the rules of the general VNS algorithm. In the table 1 we present some preliminary comparison results between VNB and CPLEX11.2 commercial solver within the same CPU time limit (300 s). "min time" denotes the minimum CPU time needed to detect the best solution, while "tot.time" denotes the time needed by CPLEX to prove the optimality (both times are in seconds). As can be seen, VNB can provide high quality solutions (in this case always the optima) requiring less time than CPLEX in 3 tested instances.

Table 1. Computational results – small test instances

Inst.		CPLEX			VNB	
	$\|ALL\|$	obj.val.	min.time	tot.time	obj.val.	min.time
1	4	0.484925	33.97	89.404	0.484925	1.27
2	5	1.495063	34.42	74.072	1.495063	129.64
3	6	0.770238	64.63	154.932	0.770238	29.26
4	10	2.02930	1.57	2.38564	2.02930	8.04
5	10	3.09217	53.19	129.43100	3.09217	12.33

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