A REVISITATION OF THE PRINCIPLE OF MATERIAL OBJECTIVITY: A LIE SYMMETRY APPROACH

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ABSTRACT

In continuum mechanics and thermodynamics, the principle of material frame-indifference, also referred to as the principle of material objectivity, is a fundamental postulate introduced by Truesdell and Noll in 1965 [1]. It represents the main requirement for assigning constitutive relations and identifying objective quantities.

Roughly speaking, the principle states that the constitutive relations must be invariant with respect to a change of the observer, namely by a transformation like

$$\mathbf{x}^\star = Q(t)\mathbf{x} + \mathbf{c}(t), \qquad t^\star = t + t_0$$

where Q is an arbitrary time dependent orthogonal matrix, and $\mathbf{c}$ an arbitrary time dependent vector. This principle implies, the invariance of scalar functions, and suitable transformations for vectors and tensors, say

$$f(\mathbf{x}^\star) = f(\mathbf{x}), \qquad \mathbf{v}(\mathbf{x}^\star) = Q\mathbf{v}(\mathbf{x}), \qquad T(\mathbf{x}^\star) = Q^T T(\mathbf{x})Q.$$

A long series of strong disputes accompanied this principle; a historical view (at least, until 2008) can be found in [2].

We exploit the transformations for scalar, vectorial and tensorial functions in order a *principle of material frame indifference* is fulfilled by using the infinitesimal invariance with respect to the so called fundamental group of mechanics [3], *i.e.*, the ten–parameter Lie goup made of isometries, time translation and Galilean transformations. So doing we can provide a suitable framework for identifying objective quantities, providing the desired transformation rules for scalars, vectors and tensors, and giving the representation for isotropic constitutive functions. This approach provides the same results obtainable by the Truesdell and Noll principle in most of the cases, but not all. The differences are discussed.

Keywords: Frame–indifference principle. Lie symmetries. Invariance.

Physics and Astronomy Classification Scheme: 05.70.-a; 11.30.-j.

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