# THERMODYNAMICS OF BLACK HOLES: HAWKING-PAGE TEMPERATURES AND SECOND ORDER PHASE TRANSITIONS

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## ABSTRACT

Since black holes lack a straightforward notion of geometrical volume due to their event horizon structure and coordinate dependence [3], various approaches have been proposed to introduce a meaningful geometric and thermodynamic volume [2,4,5]. There is no volume in classical **black hole thermodynamics**, the nonextensivity of black holes is a consequence. Extensivity of such systems can be restored by introducing a new thermodynamic state variable, typically the volume, the mass or the number of particles. Smarr relations show that black hole thermodynamics is nonextensive. However, restored **extensivity** leads to a volume which corresponds to geometric concepts and is also meaningful from a physical point of view. The article examines various volume definitions in the context of Anti-de Sitter (AdS) black holes and their implications for phase transitions, focusing on the **Hawking — Page phase transition in Kerr — AdS** spacetime, including the Christodoulou — Rovelli volume [1]. The goal is to examine the feasibility of integrating volume as a meaningful variable in the thermodynamic framework of different types of black holes.

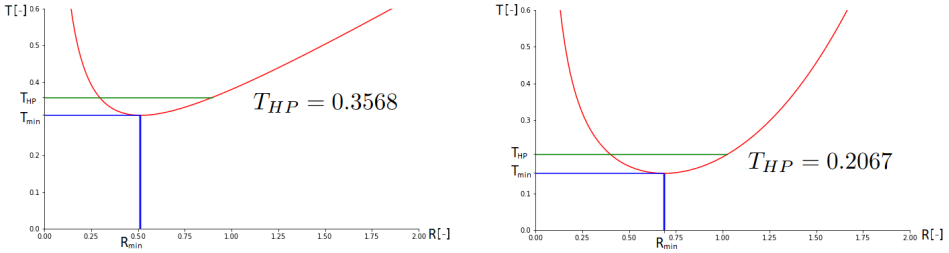


Figure 1: The dimensionless temperature $(T)$ expressed in terms of radius $(R)$ in case of $p = 0.15$. The local minimum remarked with blue and the Hawking – Page temperature with green. Left: For Hawking and Page; Right: with the Christodoulou – Rovelli volume.

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