MODELLING ANOMALOUS DYNAMICS IN CLASSICAL AND QUANTUM SYSTEMS

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ABSTRACT

Anomalous dynamics, characterized by deviations from standard Brownian motion, play a crucial role in understanding transport processes in complex classical and quantum systems. In this study, we investigate how non-local interactions influence anomalous diffusion and quantum wave propagation by incorporating fractional calculus and memory effects into classical and quantum models.

In the classical domain, we analyze a generalized diffusion equation with a memory kernel, Lévy-type superdiffusion and a non-local term [1], which accounts for interactions over extended spatial and temporal scales. We derive analytical solutions and explore their impact on transport properties. The results reveal how non-local effects modify the diffusion behavior and influence the probability distribution.

In the quantum domain, we extend our analysis to two non-local modifications of the Schrödinger equation: (1) A fractional Schrödinger equation with a memory kernel, capturing long-range correlations in quantum evolution [2]; (2) A three-dimensional Schrödinger equation incorporating a non-local potential constrained by a Dirac delta function, simulating heterogeneity of the system [3].

Using Green's function methods, we obtain exact solutions and analyze the impact of non-locality on wave packet dynamics and quantum transport. Our findings demonstrate that non-local interactions can significantly alter coherence properties, leading to modified localization effects and anomalous spreading of wave functions [4].

The interplay between fractional diffusion, memory effects, and non-local interactions provides a unifying theoretical framework for describing transport in systems that exhibit deviations from standard diffusive and wave-like behavior. Fractional diffusion equations generalize classical transport laws by incorporating memory and non-locality, allowing for a more comprehensive description of systems with long-range correlations. Memory effects introduce non-Markovian behavior, which plays a fundamental role in systems with strong temporal correlations, such as those governed by spatial geometrical constraints. Meanwhile, spatial non-locality enables the description of transport mechanisms where interactions extend beyond nearest neighbors, leading to correlated motion that cannot be captured by traditional local models. By bringing together these concepts, our approach provides a generalized perspective on transport phenomena, encompassing a broad spectrum of behaviors ranging from constrained subdiffusion to Lévy-type superdiffusion and non-local quantum evolution.

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KEYWORDS: Anomalous diffusion, non-locality, Lévy flights

PACS NUMBERS: 05.10.Gg, 05.30.Pr