JEAN-MARIE SOURIAU LIE GROUPS THERMODYNAMICS & KOSZUL INFORMATION GEOMETRY STRUCTURES FOR THERMODYNAMICS-INFORMED NEURAL NETWORKS (TINN) AND LIE GROUPS MACHINE LEARNING

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ABSTRACT

The symplectic model of statistical mechanics developed by Jean-Marie Souriau—termed the "Thermodynamics of Lie Groups"—extends the structures of information geometry to the realm of Lie groups. This framework enables the definition of Maximum Entropy Gibbs densities possessing the property of covariance under the action of the group operating on the system. Moreover, it generalises the Fisher-Rao-Fréchet metric to Lie groups, rendering it invariant under the group's action. Crucially, Shannon's axiomatic definition of entropy is supplanted by a purely geometric construction, wherein entropy emerges as a Casimir invariant function defined on the leaves of the foliation induced by coadjoint orbits through the moment map associated with the group action (the moment map being the geometric counterpart of Noether's theorem).

Souriau's thermodynamics of Lie groups introduces a web-like geometric structure composed of two transverse foliations: a symplectic foliation generated by coadjoint orbits (corresponding to the level sets of entropy) and a transverse Riemannian foliation (corresponding to the level sets of energy). The dynamics on each foliation make it possible to distinguish between non-dissipative phenomena (with constant entropy) and dissipative phenomena (with constant energy). This dynamic behaviour is governed by a metriplectic flow that encapsulates the first law of thermodynamics through Poisson bracket (quantitative conservation of energy) and the second law through metric flow bracket (qualitative degradation of energy and generation of entropy).

We shall explore the connections between TINNs (Thermodynamics-Informed Neural Networks), metriplectic flows, and the Lie groups thermodynamics. The overarching aim is for TINNs not merely to learn from data, but also to adhere to thermodynamic constraints, thereby enabling more accurate predictions and a deeper understanding of physical systems—particularly those characterised by dissipative phenomena.

Souriau Lie Groups Thermodynamics is studied in the framework of two european action, European CaLISTA COST action and European CaLIGOLA MSCA action.

Keywords: Thermodynamics, Transverse Symplectic Foliation, Lie Groups Machine Learning, Thermodynamics-Informad Machine Learning.
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