ON THE POSSIBILITY OF SPATIAL INTERACTIONS IN HIGHER-GRADE MATERIALS

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ABSTRACT

An elastic material of grade N is a deformable continuous system in which, in order to model the effects of complex spatial interactions, the constitutive quantities are permitted to depend not only on the first gradient of the deformation, the strain, but also on all gradients of the deformation less than or equal to the integer N. A heat conducting elastic material of grade N is said thermoelastic material of grade N. More complex systems with respect to the thermoelastic ones are the elastic solids of grade N with heat conduction and viscosity, which are said thermo-viscoelastic solids of grade N. Classically, for thermo-viscoelastic solids of grade N, the higher-order gradients of the deformation represent the elastic properties, while the viscosity is accounted by the presence in the state space of the time derivative of the deformation tensor. A troubling aspect higher-grade materials is that they are, in general, incompatible with second law of thermodynamics [1], unless some modifications of the basic field equations of Rational Thermodynamics [2] is made. Accordingly to this last point of view, in 1986 Dunn and Serrin [1], postulated the following new form of the local balance of energy

$$\rho\dot{\varepsilon}_I + q_{I,I} - T_{iL}\dot{F}_{iL} - \rho r = u_{I,I}, \tag{1}$$

wherein ρ is the referential mass density, T_{iL} are the components of the first Piola-Kirchhoff stress tensor, ε is the referential specific internal energy, q_I are the components of the referential heat flux, r is the local heat supply, and u_I are the components of an additional flux of mechanical energy, the interstitial working, engendered by long-range spatial interactions [1]. In this way they succeeded in proving the thermodynamic compatibility of such materials, with some severe restrictions. In particular, they showed that second law of thermodynamics requires that for elastic materials of grade 3 the free energy, the entropy and the internal energy cannot depend on the second gradient of the deformation tensor. As a consequence, the materials for which such a dependency is evident from the experimental point of view, from a theoretical point of view are not compatible with thermodynamics. In a series of papers we proposed an alternative approach to the above problem that does not modify the basic thermodynamic laws, but generalizes the classical Coleman-Noll procedure for the exploitation of the entropy principle [2]. The basic idea is to consider as additional equations to be substituted into the dissipation inequality the spatial differential consequences (gradients) of the balance laws, up to the order of the gradients entering the state space [3; 4; 5; 6; 7]. Here we present a complete thermodynamic analysis of thermo-viscoelastic solids of grade 3 based on the above mathematical procedure, and show that such a class of materials is fully compatible with the classical formulations of the field equations and of the second law of thermodynamics postulated in [2]. For homogeneous and isotropic bodies, under the validity for the heat flux of a generalized Maxwell-Cattaneo equation [8] which depends on the deformation too, we study the propagation of small-amplitude thermomechanical waves [6].

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