DOES THE FLUID-STATIC EQUILIBRIUM OF A SELF-GRAVITATING ISOTHERMAL SPHERE OF VAN DER WAALS' GAS PRESENT MULTIPLE SOLUTIONS?

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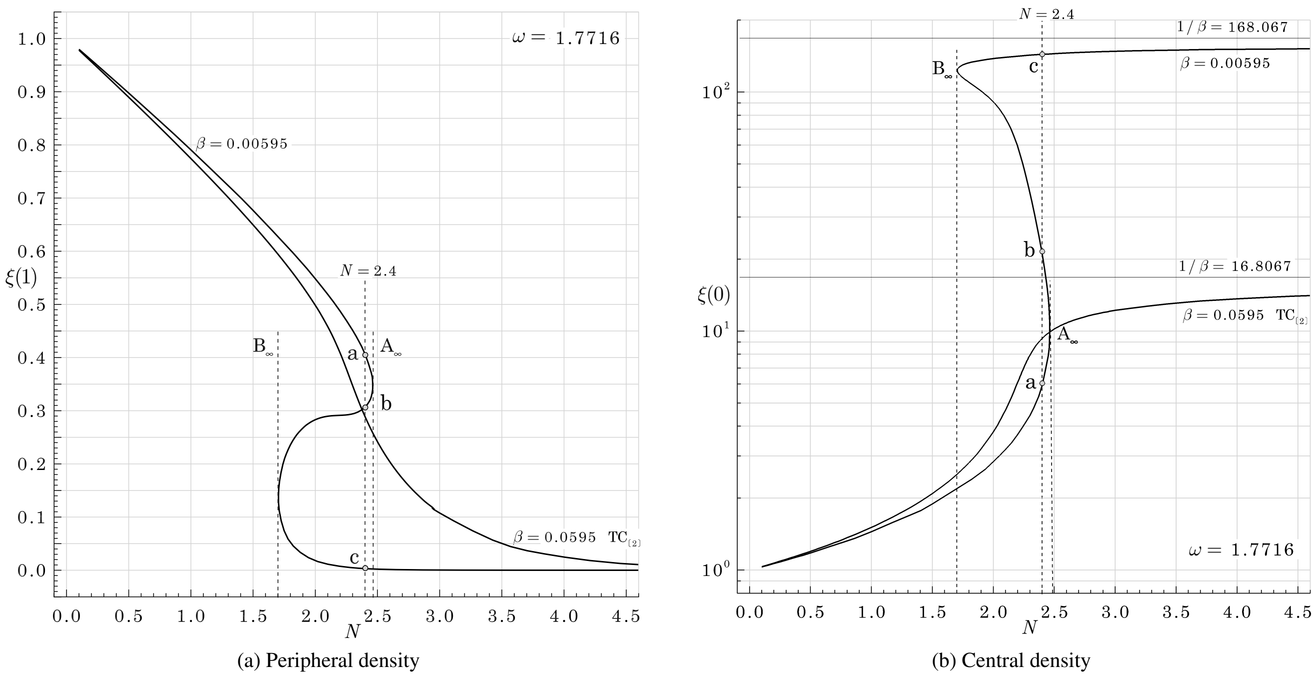
ABSTRACT

We describe the investigation we left in the future-work stack in Giordano *et al.* [1] in which we pointed out the obvious necessity to inquire about the existence or absence of values of the non-dimensional constants α and β of the van der Waals' equation of state in correspondence to which the perfect-gas model's self gravitational effects, namely, upper boundedness of the gravitational number, spiraling behavior of peripheral density, oscillating behavior of central density, and existence of multiple solutions corresponding to the same value of the gravitational number, appear also for the van der Waals' model.

The development of our investigation brings to the conversion of our M_2 scheme based on a second-order differential equation governing the fluidstatic equilibrium into an equivalent system of two first-order differential equations that incorporates Milne's homology invariant variables [2]. The converted scheme $1oM_2$ turns out to be much more efficacious than the M_2 scheme in terms of numerical calculations' easiness and richness of results.

We use the perfect-gas model as benchmark to test the $1oM_2$ scheme; we re-derive familiar results and put them in a more general and rational perspective that paves the way to deal with the van der Waals' gas model. We introduce variable transformations that turn out to be the key to study (almost) analytically the monotonicity of the peripheral density with respect to variations of the gravitational number.

The investigation brings to the proof that the gravitational number N is not constrained by upper boundedness, the peripheral density $\xi(1)$ does not spiral [Fig. 1(a)], and the central density $\xi(0)$ does not oscillate [Fig. 1(b)] for any couple of values assumed by the non-dimensional constants α and β; however, multiple solutions corresponding to the same value of the gravitational number can exist but their genesis is completely different from that of the perfect-gas model's multiple solutions. As crucial result of our investigation, we provide the boundary between the two regions of solution's uniqueness and multiplicity in the α, β plane.



(a) Peripheral density

(b) Central density

Figure 1: Effect of the characteristic number β on the peripheral/central density's dependence on the gravitational number and appearance of multiple solutions ($\omega = \alpha/\beta$).

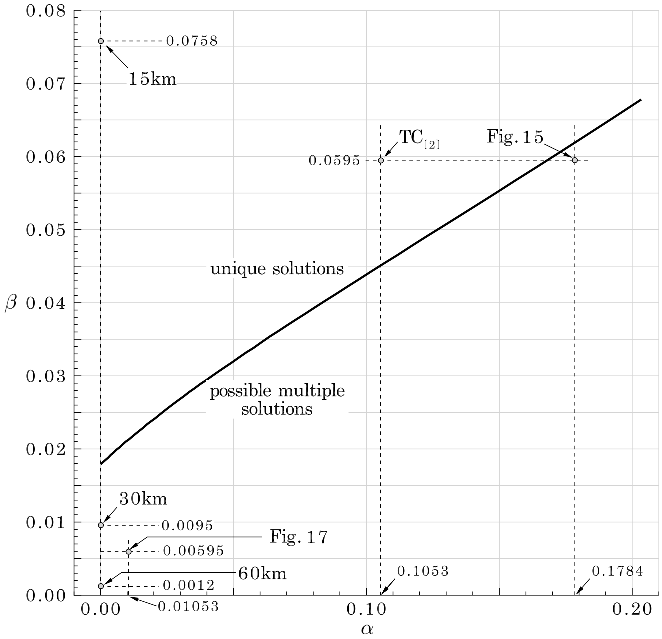


Figure 2: The $\beta - \alpha$ boundary separating the regions of uniqueness and possible multiplicity of the solutions; the cases dealt with in [3] together with the three cases investigated by Aronson and Hansen [4] ($\alpha = 0$) are explicitly indicated.

REFERENCES

[1] Domenico Giordano, Pierluigi Amodio, Felice Iavernaro, Francesca Mazzia, Péter Ván, and Mátyás Szücs. Fluid statics of a self-gravitating isothermal sphere of van der Waals' gas. *Physics of Fluids*, 36:056127 1–23, May 2024.

[2] E A Milne. The analysis of stellar structure. *Monthly Notices of the Royal Astronomical Society*, 92(7):610–643, May 1932.

[3] Domenico Giordano, Pierluigi Amodio, Felice Iavernaro, and Francesca Mazzia. Does the fluid statics equilibrium of a self-gravitating isothermal sphere of van der Waals' gas present multiple solutions? *Physics of Fluids*, 37:016138 1–20, January 2025.

[4] Elliot Aronson and Carl Hansen. Thermal equilibrium states of a classical system with gravitation. *The Astrophysical Journal*, 177:145–153, 1 October 1972.