NONLINEAR EFFECTS IN THE MICROSTRUCTURE-INDUCED FINITE SPEED HEAT PROPAGATION

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ABSTRACT

In condensed matter, the time variation of active microstructures that directly depend on temperature may induce finite speed heat propagation. Such a property is independent of the type of microstructures [1], so that the picture of the phenomenon can be considered in this sense universal.

The adjective ‘active’ refers to bodies with microstructure endowed with inner (but observable) degrees of freedom with pertinent interactions not directly associated with those kinematic mechanisms that determine the standard stress (namely, the crowding and shearing of material elements merely considered as black boxes). A physically significant case is the one of materials showing pyroelectric effects. They occur in different phases, even those in which strain can be negligible.

With respect to this last circumstance (one that could be ideally referred to rigid bodies with active microstructure), we can derive a hyperbolic equation for the temperature propagation, although we accept the Fourier law for the heat flux [1]. Its linear form can be solved in closed form at least in one-dimensional case; the result show wave-type temperature propagation with velocity that is obviously *different* from the thermal conductivity [2]. Here and always in one-dimensional setting, we discuss the first nonlinear version of that equation, namely

$$c_v\theta_t + \zeta\theta_{xt} - \kappa\theta_{xx} + \gamma\lambda^2(\theta_t\theta_x)_x = 0\,,$$

where c_v is the specific heat, κ the conductivity coefficient, while ζ, γ and λ are constitutive coefficients. Finally, the subscript indicate derivatives with respect to the space variable x and time t.

For Neumann’s type problems (prescribed heat flux at the boundary) we find a closed form solution with propagation velocity

$$\theta_t = \frac{\kappa q_1}{c_v + \gamma\lambda^2 q_1}$$

when we consider an external heat flux q_1 applied to the boundary, besides, considering, we repeat, the Fourier law within the body. We also discuss relevant numerical schemes and simulations.

REFERENCES

[1] P. M. Mariano (2017), Finite-Speed Heat Propagation as a Consequence of Microstructural Events, *Cont. Mech. Thermodyn.*, vol. 29, 1241-1248, 2017.
[2] P. M. Mariano and M. Spadini, Sources of Finite Speed Temperature Propagation, *J. Non-Equil. Thermodyn.*, vol. 47, 165-178.
[3] D. La Pegna and P. M. Mariano, Finite Speed Heat Propagation Due to Microstructural Effects, *provisional title, in preparation*, 2025.