NOETHER'S THEOREM IN IRREVERSIBLE THERMODYNAMICS

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ABSTRACT

After Helmholtz, the mechanical foundation of thermodynamics included the First Law $dE = \delta Q + \delta W$, and the first part of the Clausius heat theorem $\delta Qrev/T = dS$. The resulting invariance of the entropy S for quasistatic changes in thermally isolated systems invites a connection with Noether's theorem (only established later). In this quest, we continue an idea, first brought up by Wald in black hole thermodynamics and by Sasa et al. in various contexts. We follow both Lagrangian and Hamiltonian frameworks, and emphasize the role of Killing equations for deriving a First Law for thermodynamically consistent trajectories, to end up with an expression of "heat over temperature" as an exact differential of a Noether charge. We extend that scheme to a continuous symmetry for the dynamical fluctuations around (nonlinear) gradient flow. The latter connects macroscopic equilibrium conditions upon introducing a quasistatic protocol of control parameters. The entropy state function becomes the Noether charge.

Keywords: Noether theorem, gradient flow, Helmholtz theorem

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REFERENCES

[1] A. Beyen, C. Maes, *Entropy as Noether charge for quasistatic gradient flow. Journal of Non-Equilibrium Thermodynamics*, 2025 Feb 7(0).
[2] A. Beyen, C. Maes, *The first part of Clausius' heat theorem in terms of Noether's theorem*, Mathematics and Mechanics of Complex Systems, 2025 Feb 8;13(1):1-24.