

COUPLING OF NON-FOURIER THERMAL AND MECHANICAL EFFECTS IN 2D ISOTROPIC MATERIALS

Carmelo Filippo Munafò^{1,*}, Patrizia Rogolino¹, Ròbert Kovács^{2,3,4}

¹Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences, University of Messina, Viale F. Stagno d'Alcontres 31, Messina, 98166, Italy.

²Department of Energy Engineering, Faculty of Mechanical Engineering, Budapest, Hungary.

³Department of Theoretical Physics, Wigner Research Centre for Physics, Institute for Particle and Nuclear Physics, Budapest, Hungary.

⁴Montavid Thermodynamic Research Group, Budapest, Hungary.

*carmelofilippo.munafò@unime.it

ABSTRACT

In this work, we investigate the mechanical effects in two-dimensional isotropic thermo-elastic materials, focusing on the interaction between the thermal and mechanical fields [1]. We present a model that analyzes the thermo-mechanical behavior by coupling the generalized hyperbolic heat equation for the heat flux—known as the Maxwell–Cattaneo–Vernotte (MCV) equation—with a mechanical model that accounts for thermal expansion and stress in the material. This approach improves upon the classical model, where, in the case of the Duhamel-Neumann body, thermal expansion appears only in the pressure tensor. Here, we strengthen the coupling by adding mechanical effects to the internal energy, so that mechanical motion affects the temperature field, creating a two-way (strong) coupling between the thermal and mechanical fields. However, this classical approach still does not account for the second sound phenomenon.

To overcome this limitation, the Maxwell-Cattaneo-Vernotte equation is introduced as the first generalization of Fourier's law [2; 3; 4]. This equation addresses the shortcomings of the classical approach, and by incorporating temperature-dependent coefficients, it becomes necessary to include the effects of thermal expansion and, consequently, the coupling with mechanical effects. This is why we aim to study the effects of this mechanical coupling on the temperature distribution during a thermal pulse experiment and investigate whether it is possible to reproduce the longitudinal waves observed in McNelly's experiments [5; 6; 7; 8].

Using this model, we can more accurately predict the material's response under varying thermal conditions. To solve the governing equations for the heat pulse problem, we have developed a numerical solution using the staggered finite difference method and/or finite elements, ensuring precise results for the coupled thermo-mechanical interactions [9; 10].

Keywords: Thermoelasticity, Thermal expansion, Mechanical effects, Duhamel-Neumann, Cattaneo equation, Non-Fourier heat conduction.

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