VORTICITY IN HYPERBOLIC HEAT CONDUCTION

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ABSTRACT

Transport of heat in a crystal can be formulated within the kinetic theory of phonons, phonon hydrodynamics, or within the usual Fourier heat conduction theory. These various levels of description are linked by thermodynamic reductions. While the reductions typically involve linearization in both reversible and irreversible parts of the evolution, we reduce the Poisson brackets (generating the reversible part) without any linearization. Then, we obtain a new phonon hydrodynamics that contains convective terms dependent on vorticity of the heat flux, missing in the standard theories of phonon hydrodynamics.

The new vorticity-dependent terms violate the alignment of the heat flux with the temperature gradient even in the stationary state, which is expressed by a Fourier-Crocco equation, illustrated in Figure 1. With those terms, temperature plays a similar role in heat transport as pressure does in aerodynamics. This is illustrated on numerical simulations of flow past a cylinder, Figure 2. The vorticity-dependent terms lead to a colder spot just behind the cylinder, and for high-enough Reynolds numbers they lead to the von Kármán vortex street, Figure 3. See [1] for more details.

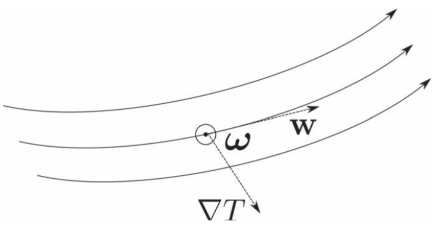

Figure 1: In the presence of heat flux vorticity $\omega = \nabla \times \mathbf{w}$, the heat flux $\mathbf{w}$ does not align with the temperature gradient ∇T.

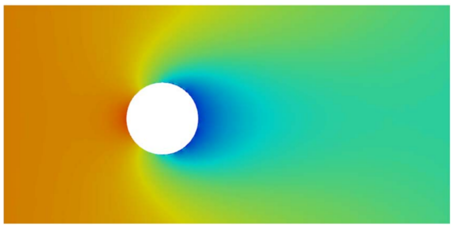

Figure 2: Temperature field when heat flows past an insulated cylinder.

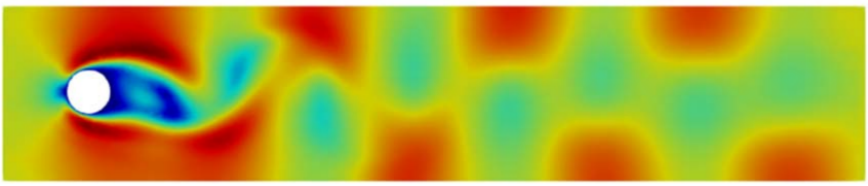

Figure 3: When the analogue of the Reynolds number grows (here $Re = 150$), the heat flux becomes unstable and produces an analogue of the von Kármán vortex street.

REFERENCES

[1] M. Sýkora, M. Pavelka, L. Restuccia, and D. Jou, Multiscale heat transport with inertia and thermal vortices, *Physica Scripta*, vol. 98(105234), 2023.