ANALYTICAL SOLUTION OF MAXWELL-CATTANEO HEAT EQUATION WITH NON-HOMEGENEOUS BOUNDARY CONDITIONS

Patrizia Rogolino[1,*],Carmelo Filippo Munafò[1],

[1]Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences,University of Messina, Viale F. Stagno d'Alcontres 31, Messina, 98166, Italy.
[*]progolino@unime.it

ABSTRACT

The classical Fourier law is no longer applicable when studying thermal transient problems at the nanoscale or when describing heat transfer phenomena in heterogeneous materials. Non-Fourier models are of great interest in the engineering field, particularly due to the use of heat sources in many applications. In this paper, we provide an analitycal solution to the Maxwell-Cattaneo heat equation with homogeneous initial conditions and non-homogeneous time-dependent boundary conditions, restricting ourselves to the linear regime and a one-dimensional situation. Thus, the model solved analitycally is the following:

$$\rho c_v \partial_t T + \partial_x q = 0$$
$$\tau \partial_t q + q = -\lambda \partial_x T$$

with the following initial and boundary conditions

$$T(x=0) = 0; \qquad q(x,0) = 0$$
$$q(0,t) = f(t); \qquad q(1,t) = 0$$

respectively. In particular, by combining the temperature evolution equation and internal energy balance, an analytical solution of the heat wave-type equation is derived. This is achieved by substituting the time-dependent heating function acting on the boundary with a time- and space-dependent heat source. To solve the thermal equation, we apply Duhamel's theorem, considering the problem initially at zero temperature. An exact solution is obtained using the method of superposition, combining the homogeneous transient case and the inhomogeneous steady state.

Finally, the time evolution of the temperature history and the heat flux profile with time is represented for both the original problem and the one approximated with a space- and time-dependent heat source, thus validating the solution method.

Keywords: Maxwell-Cattaneo equation,, heat conduction, Analytical solution. *PACS*: 44.10.+i, 65.40.De, 05.70.Ln.

REFERENCES

[1] R. Kovàcs, P. Rogolino, *Numerical treatment of nonlinear Fourier and Maxwell- Cattaneo-Vernotte heat transport equations*, Int. J. Heat Mass Transf. **150** (2020), 119281.
[2] C. F. Munafó, R. Kovàcs, P. Rogolino, *Nonlinear thermal analysis of two-dimensional materials with memory*, Int. J. Heat Mass Transf. **219** (2024), 124847.
[3] D. W. Hahn, M. Necati, Ozisik, *Heat Conduction*, John Wiley and Sons, 2012.
[4] D. Jou, J. Casas-Vàzquez, G. Lebon, *Extended irreversible thermodynamics*, Springer, 1996.
[5] D. Jou, J. Casas-Vàzquez, G. Lebon, *Understanding non-equilibrium thermodynamics*, Springer, 2008.
[6] R. Kovàcs, *Transient non-Fourier behavior of large surface bodies*, International Journal of Heat and Mass Transfer **148** (2023), 107028.