On Laplacian Spectrum of Nilpotent Graph over $\mathbb{Z}_n$

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Outline

1. Some Concepts of Ring Theory

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2. Introduction to Nilpotent and Zero Divisor Graphs

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3. Some Known Results for $\Gamma(R)$ and $\Gamma_N(R)$

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3. Some Known Results for $\Gamma(R)$ and $\Gamma_N(R)$
4. The nilpotent graph over $\mathbb{Z}_n$
5. On Laplacian Eigenvalues of Nilpotent Graph over $\mathbb{Z}_n$

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Introduction to Nilpotent and Zero Divisor Graphs

Definition

[Beck,1988] The (original) zero divisor graph of a ring R is a simple graph whose set of vertices consists of all elements of the ring, with an edge defined between a and b if and only if $ab = 0$.

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Example

$R = \mathbb{Z}_6$, $V(R) = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$

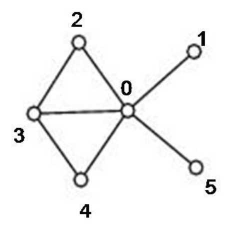

Figure:

Introduction to Nilpotent and Zero Divisor Graphs

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[Anderson,Livingston [4], 1999] The zero divisor graph of a ring R is a simple graph whose set of vertices consists of all (non-zero) zero divisors, with an edge defined between a and b if and only if $ab = 0$. It will be denoted by $\Gamma(R)$.

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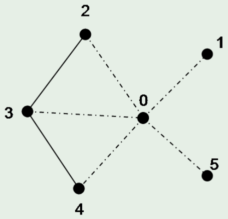
$R = \mathbb{Z}_6$, $V(R) = Z^*(R) = \{2, 3, 4\}$



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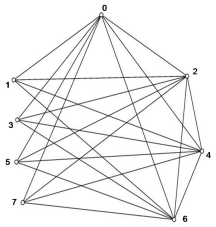
Example

If $R = \mathbb{Z}_8$, then $N(R) = \{0, 2, 4, 6\}$ and $V(R) = \{0, 1, 2, 3, 4, 5, 6, 7\}$



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Definition

[A.Li- Q.Li [6], 2010] The nilpotent graph of R, denoted by $\Gamma_N(R)$, is a graph with vertex set $V_N(R)^*$ and two distinct vertices x and y are adjacent if and only if $xy \in N(R)$, where $N(R)$ is the set of all nilpotent elements of R.

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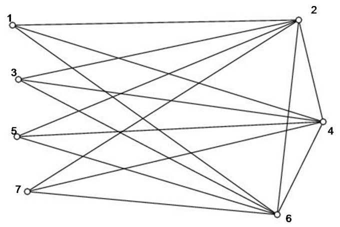

Some Known Results for $\Gamma(R)$ and $\Gamma_N(R)$

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Let R be a finite commutative ring. If $\Gamma(R)$ is complete, then either $R \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ or R is a local ring with $charR = p$ or p^2 and $|\Gamma(R)| = p^n - 1$, where p is prime and $n \geq 1$.

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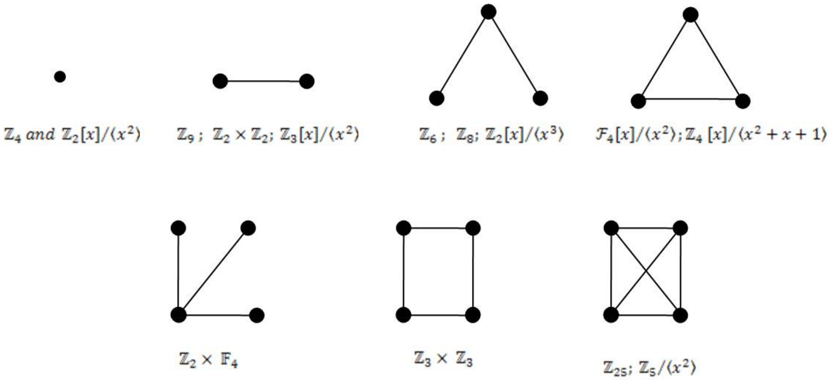

Figure: $\Gamma(R)$

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- a $\Gamma(R)$ *is finite if and only if either* R *is finite or* R *is integral domain.*
- b $\Gamma(R)$ *is connected and* $diam(\Gamma(R)) \leq 3$.
- c *If* $\Gamma(R)$ *contains a cycle, then* $gr(\Gamma(R)) \leq 4$.*(the length of the shortest cycle)*

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- a $gr(\Gamma(R)) \leq 4$.
- b $\Gamma(R)$ *is a star graph.*
- c $\Gamma(R)$ *which* $R \cong \mathbb{Z}_2 \times \mathbb{Z}_4$ *or* $R \cong \mathbb{Z}_2 \times \mathbb{Z}_2[x]/\langle x^2 \rangle$

Some Known Results for $\Gamma(R)$ and $\Gamma_N(R)$

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(Nikmerh, 2013) If $\mathbb{F}$ is a field and $n \geq 3$, then $diam(\Gamma_N(M_n(F))) = 2$.

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Lemma

(Nikmerh, 2013) *If* $\mathbb{F}$ *is a finite field and* $char(\mathbb{F}) = 2$*, then* $diam(\Gamma_N(M_2(F))) = 3$.

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The nilpotent graph over $\mathbb{Z}_n$

Theorem

(Patra-Begum, 2015) Let $\Gamma_N(\mathbb{Z}_{p^\alpha q})$ be the nilpotent graph of the commutative ring $\mathbb{Z}_{p^\alpha q}$, where p and q are two distinct primes and α is an even positive integer. Then the graph $\Gamma_N(\mathbb{Z}_{p^\alpha q})$ is

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a p^{3n}- *partite, if* $\alpha = 4n$, $n = 1, 2, 3, \ldots$

b $p^{3n+1} + 1$- *partite, if* $\alpha = 4n + 2$, $n = 0, 1, 2, \ldots$

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When given the ring $R = \mathbb{Z}_n$, it is well known that it has a nonzero nilpotent element if and only if n is divisible by the square of some primes.

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When given the ring $R = \mathbb{Z}_n$, it is well known that it has a nonzero nilpotent element if and only if n is divisible by the square of some primes. . From this fact, $\mathbb{Z}_n$ does not have any non-zero nilpotent element when n is prime number or $n = p_1 p_2 ... p_t$.

Remark

It is easily seen that

$$N(\mathbb{Z}_n) = \{\overline{0}, \overline{p}, 2\overline{p}, 3\overline{p}, \ldots, (p^{m-1} - 1)\overline{p}\} \tag{1}$$

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when $n = p^m$, $m > 1$ *and*

$$N(\mathbb{Z}_n) = \{\overline{(p_1 p_2 \ldots p_t)}, 2\overline{(p_1 p_2 \ldots p_t)}, \ldots, (\prod_{i=1}^{t} p_i^{s_i - 1} - 1)\overline{(p_1 p_2 \ldots p_t)}\} \quad (2)$$

when $n = \prod_{i=1}^{t} p_i^{s_i}$, $t \geq 2$.

Lemma

Let $\mathbb{Z}_n$ be integer ring, where $n = p^m$ and p is a prime number. Then, the vertex set of $\Gamma_N(\mathbb{Z}_n)$ is

$$V_N(\mathbb{Z}_{p^m})^* = \mathbb{Z}_{p^m}^* \quad (3)$$

Moreover, we have $d_{\bar{i}} = p^m - 2$ for $\bar{i} \in N(\mathbb{Z}_{p^m}^*)$ and $d_{\bar{i}} = p^{m-1} - 1$ for $\bar{i} \notin N(\mathbb{Z}_{p^m}^*)$.

Remark

By Lemma, we see that $\Gamma_N(\mathbb{Z}_{p^m})$ has two distinct degrees such that $\Delta = p^m - 2$ and $\delta = p^{m-1} - 1$.

Theorem

If p is a prime number then

$$S(\Gamma_N(\mathbb{Z}_{p^m})) = \left(0, (\delta)^{(\Delta-\delta)}, (\Delta+1)^{(\delta)}\right) \quad (4)$$

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Lemma

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(i) *If $s_i = 1$ for each i, then*

$$V_N(\mathbb{Z}_n)^* = \bigcup_{i \in I} S_{p_i} \tag{5}$$

where $S_{p_i} = \{p_i k : 1 \leq k \leq \frac{n}{p_i} - 1\}$ and $I = \{1, 2, \ldots, t\}$.

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(ii) *If $s_i \geq 2$ for at least i, then*

$$V_N(\mathbb{Z}_n)^* = \mathbb{Z}_n^* \tag{6}$$

Remark

In Lemma 2.6 (i), we can easily see that the number of vertices of $\Gamma_N(\mathbb{Z}_n)$ is $n-1-\tau(n)$, where $\tau(n)$ is the number of positive divisors of n.

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On Laplacian eigenvalues of nilpotent graph over $\mathbb{Z}_n$

Lemma

Let's consider the graph $\Gamma_N(\mathbb{Z}_n)$ *, where* $n = \prod_{i=1}^{t} p_i^{s_i}$, $t \geq 2$.

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(ii) *If* $\prod_{m=1}^{r} p_{l_m} \mid \bar{i}$ *for* $\bar{i} \in V_N(\mathbb{Z}_n)^*$ *such that* $l_1, \ldots, l_r \in A$ *for* $1 \leq r \leq k$ *and* $A = \{1, 2, \ldots, k\}$ *, then we get*

$$N_{\bar{i}} = \{z, 2z, \ldots, (\frac{n}{z} - 1)z\} \tag{7}$$

i.e.

$$d_{\bar{i}} = \frac{n}{z} - 1 \tag{8}$$

where $z = \prod_j p_j$ *for every* $\bar{j} \in A - \{l_1, \ldots, l_r\}$; d_i *and* $N_{\bar{i}}$ *are the degree of vertex* $\bar{i}$ *and the set of neighbors of* $\bar{i}$*, respectively.*

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On Laplacian eigenvalues of nilpotent graph over $\mathbb{Z}_n$

Lemma

(iii) *If* $(\bar{i}, p_k) = 1$ *for all* $1 \leq k \leq t$ *, then we get*

$$N_{\bar{i}} = N(\mathbb{Z}_n^*) = \{\overline{(p_1 p_2 \dots p_t)}, \dots, (\prod_{i=1}^{t} p_i^{s_i - 1} - 1)\overline{(p_1 p_2 \dots p_t)}\} \tag{9}$$

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Theorem

Let $\mathbb{Z}_n$ be a ring , where $n = p_1^{s_1} p_2^{s_2} \dots p_t^{s_t}$. Then some eigenvalues of the graph are the degree of vertices.

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Theorem

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Some Sample Graphs

Example

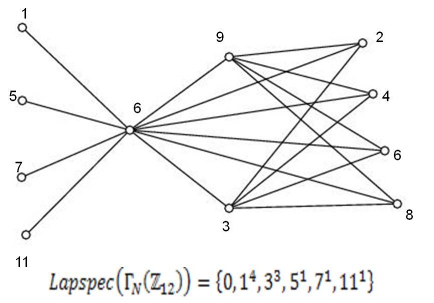

Figure: $\Gamma_N(\mathbb{Z}_{12})$

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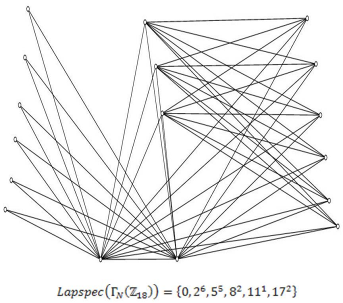

Figure: $\Gamma_N(\mathbb{Z}_{18})$

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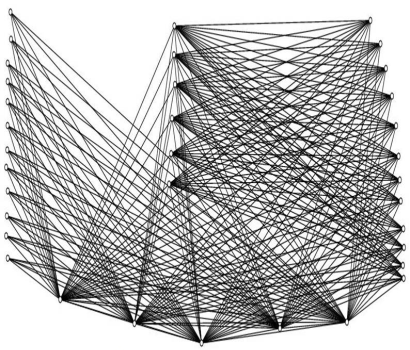

Figure: $\Gamma_N(\mathbb{Z}_{36})$

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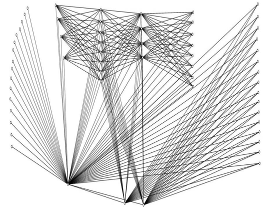

$Lapspec(\Gamma_N(\mathbb{Z}_{60})) = \{0, 1^{16}, 3^{15}, 5^7, 9^3, 11^7, 19^3, 29^1, 59^1, x, y, z, k, l\}$

Figure: $\Gamma_N(\mathbb{Z}_{60})$

Some Sample Graphs

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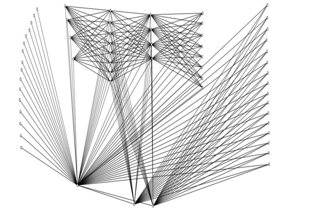

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Figure: $\Gamma_N(\mathbb{Z}_{60})$

Remark

In this graph, eigenvalues x, y, z, k, l is not integer eigenvalues with multiplicity 1.

References

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THANK YOU!