BOOK OF ABSTRACTS

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Stanković soft

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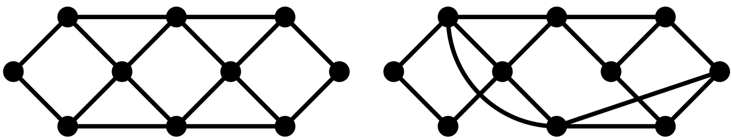

Design of the smallest pair of cospectral graphs
with cospectral decks of vertex-deleted subgraphs
by **Dragan Stevanović**

P R O G R A M

Sunday, June 9
Mathematical Institute, room 301F
Contributed talks

11.00 Baoyindureng Wu, On odd induced subgraphs of a graph
11.20 Goran Popivoda, On the σ-irregularity index
11.40 Open problem session

Monday, June 10
Serbian Academy of Sciences and Arts, Ceremonial hall
Invited talks

09.30 Opening ceremony
10.00 Huiqiu Lin, Spectral extremal results on H-minors

11.00 **Coffee break**

11.30 Stephan Wagner, Mean subtree order and related invariants
12.30 Klavdija Kutnar, Hamilton paths and cycles in vertex-transitive graphs

BREAK

15.00 Bo Ning, On a conjecture of Mohar about HL-index
16.00 Péter Csikvári, About the Merino-Welsh conjecture

17.00 **Cocktail in the club of the Academy** (mezzanine floor)

Tuesday, June 11
Serbian Academy of Sciences and Arts, 2nd floor
Contributed talks

09.30 Xueliang Li, Some results on Gallai-Ramsey type problem
09.50 Shenggui Zhang, The absence of monochromatic triangles implies various properly colored spanning trees
10.10 Đorđe Baralić, The chromatic number of the associahedron
10.30 Xia Zhang, Edge coloring of hypergraphs

10.50 **Coffee break**

11.20 Ligong Wang, The signless Laplacian spectral radius of $2K_3$-free graphs
11.40 Nino Bašić, On nut graphs with a given automorphism group
12.00 Jiangdong Ai, Graph operations and a unified method for kinds of Turán-type problems on paths, cycles and matchings
12.20 Maurizio Brunetti, Eigenvalues of complex unit gain graphs and gain regularity

BREAK

14.30 Vedran Krčadinac, On higher-dimensional Hadamard matrices and designs
14.50 Sumin Huang, An embedding technique in the study of word-representability of graphs
15.10 Erfei Yue, Results on Bollobás set-pair systems
15.30 Jie Hu, On generalizations of Dirac-type problems

15.50 **Coffee break**

16.20 Zhenzhen Lou, Some progress on minimum spectral radius of a graph
16.40 Irena Jovanović, On some spectral properties of graphs with self-loops
17.00 Shixin Wang, On 2-integral Cayley graphs
17.20 Tamás Réti, On the classification and characterization of graphs with two main eigenvalues

Wednesday, June 12
Serbian Academy of Sciences and Arts, Ceremonial hall
Invited talks

09.30 Vilmar Trevisan, Limit points of spectral radius of graphs

10.30 **Coffee break**

11.00 Genjiu Xu, Values for cooperative games with a prior unions and a communication graph based on combined effects

12.00 Zhao Zhang, Approximation algorithm for unrooted prize-collecting forest with multiple components

BREAK

Excursion (exact information available on the spot)

18.30 **Conference dinner, Mama Shelter hotel**

Thursday, June 13
Serbian Academy of Sciences and Arts, 2nd floor
Contributed talks

09.30 Francesco Belardo, Families of signed graphs with strong (anti-)reciprocal eigenvalue property
09.50 Zhouningxin Wang, Density of C_3^*-critical signed graphs
10.10 Tamara Koledin, On strongly regular signed graphs
10.30 Callum Huntington, Constructions of nonregular cospectral signed graphs

10.50 **Coffee break**

11.20 Dragan Stevanović, Reinforcement learning for graphs and beyond
11.40 Tatjana Davidović, Metaheuristics for finding threshold graphs with maximum spectral radius
12.00 Zorica Dražić, Connected graphs of fixed order and size with minimal spectral radius
12.20 Marko Milošević, On computational searches for borderenergetic graphs

BREAK

14.30 Liqiong Xu, Uniformly 3-connected graphs
14.50 Tobias Hoffman, Bounds on the largest eigenvalue and energy of the connectivity matrix
15.10 Heping Zhang, Minimum degree of minimal k-factor-critical graphs
15.30 Xiaowei Yu, A Ramsey-Turán theory for tilings in graphs

15.50 **Coffee break**

16.20 Tomislav Došlić, Evolution of geometric graphs of fullerene spectra
16.40 Snježana Majstorović-Ergotić, On Graovac-Ghorbani index of unicyclic graphs
17.00 Luka Podrug, Metallic cubes and beyond

Friday, June 14
Serbian Academy of Sciences and Arts, 2nd floor
Contributed talks

09.30 Ivan Damnjanović, Polynomial indicator of flat bands
09.50 Weigen Yan, On the edge reconstruction of six digraph polynomials
10.10 Xianan Jin, Twist polynomial as a weight system
10.30 Milica Anđelić, The Moore-Penrose inverse of a Laplacian matrix of a signed graph

10.50 **Coffee break**

11.20 Gašper Domen Romih, ℓ_1-embeddability of hypergraphs
11.40 Haiyan Chen, Hitting times for the simple random walk on cube-like networks
12.00 Liming Xiong, How does a necessary condition effect a graph to have hamiltonian property?
12.20 Jiaao Li, Some saturation problems for nowhere-zero flows

Closing remarks

ABSTRACTS
INVITED TALKS

Spectral extremal results on H-minors

Huiqiu Lin, Longfei Fang, Mingqing Zhai

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Minors play an important role in graph theory, and extremal problems on forbidding minors have attracted appreciable amount of interest in the past decades. In 1990, Cvetković and Rowlinson conjectured that $K_1 \vee P_{n-1}$ attains the maximum spectral radius over all n-vertex outer-planar graphs. Subsequently, Boots and Royle, and independently Cao and Vince, conjectured $K_2 \vee P_{n-2}$ attains the maximum spectral radius over all n-vertex planar graphs. In 2019, Tait proposed a conjecture on $K_{s,t}$-minor free graphs. In this talk, we first give a survey on spectral extremal problems and results for some special minors. Then we introduce stability method and absorbing method with their applications to deal with general H-minors.

Mean subtree order and related invariants

Stephan Wagner

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The mean subtree order of a tree is the average number of vertices in a randomly chosen subtree. Its study goes back to the work of Jamison in the 1980s. In spite of its simple definition, there are still many open questions around this invariant. In particular, while it is known that the minimum mean subtree order of an n-vertex tree is always attained by a path, there is no precise characterisation of the maximum. This talk provides a survey of results on the mean subtree order, related invariants, and generalisations, with a focus on recent developments.

Hamilton paths and cycles in vertex-transitive graphs

Klavdija Kutnar

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A path (cycle) containing every vertex in a graph is called a Hamilton path (Hamilton cycle, respectively). A graph is called vertex-transitive if for any pair of vertices u and v there exists an automorphism mapping u to v. In 1969, Lovasz asked whether every finite connected vertex-transitive graph has a Hamilton path. With the exception of the complete graph on two vertices, only four connected vertex-transitive graphs that do not have a Hamilton cycle are known to exist. These four graphs are the Petersen graph, the Coxeter graph and the two graphs obtained from them by replacing each vertex by a triangle. The fact that none of these four graphs is a Cayley graph has led to a folklore conjecture that every Cayley graph has a Hamilton cycle. (A Cayley graph is a graph whose automorphism group admits a regular subgroup.) Both of these two problems are still open. However, a considerable amount of partial results are known.

I will survey some results about the topic with special emphasis given to results obtained using methods initiated by Dragan Marušič.

On a conjecture of Mohar about HL-index

Bo Ning

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Mohar conjectured that every planar subcubic graph G satisfies that $R(G) \leq 1$, where $R(G)$ is the HL-index of G. Recently, Wu and Zhang proved that this conjecture holds for K_4-minor-free subcubic graphs. In this talk, we first give a short survey on HL-index and some related results. In particular, we shall report some result which generalizes Wang and Zhang's result.

This is a joint work with Xu Liu, an undergraduate student of Zhengzhou University, China.

About the Merino-Welsh conjecture

Péter Csikvári

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In 1999 Criel Merino and Dominic Welsh proposed a conjectured inequality between three evaluations of the Tutte polynomial of a graph. Since then the conjecture was extended to matroids. In the last two decades there have been many partial results about this conjecture. In this talk I will show a counter-example to the matroidal version. I will also discuss a tool called permutation Tutte polynomial that provided the intuition behind the counter-example and also enabled us to improve on a result of Bill Jackson.

This is a joint work with Csongor Beke, Gergely Kál Csáji and Sára Pituk.

Values for cooperative games with a prior unions and a communication graph based on combined effects

Rong Zou, Genjiu Xu, Wenzhong Li, and Panfei Sun

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In this paper we focus on restricted cooperation modeled by TU-games with a prior unions and a com- munication graph. We deal with the two structures in a comprehensive way by considering two potential combined effects of them, called integration and limitation. While the former makes it possible for sets of players who are unconnected in the graph but intersect with same unions to cooperate, the latter indicates that cooperation only occurs among players who are connected in the graph and within the same union. The two partial cooperative relationships resulting from the two effects are visualized by the integration graph and the limitation graph respectively. Then, we introduce the integrated (limited) Myerson value by applying the Myerson value to TU-games with the integration (limitation) graph. Finally, two sets of comparable axiomatizations are provided to show the differences and similarities between the two values.

Limit points of spectral radius of graphs

Vilmar Trevisan

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In 1972, A. J. Hoffman introduced the concept of limit points of eigenvalues of graphs. Let $\mathcal{A}$ be the set of all symmetric matrices of all orders, in which every entry is a natural number, and $R = \{\rho : \rho = \rho(A) \text{ for some } A \in \mathcal{A}\}$ where $\rho(A)$ is the largest eigenvalue of A. Hoffman asked which real numbers can be limit points of R, showed that it is sufficient to consider matrices of $\mathcal{A}$ having entries in $\{0, 1\}$ and 0 diagonal, e.g. adjacency matrices of graphs, and determined all limit points of $R \leq \sqrt{2 + \sqrt{5}}$. In 1989, a remarkable result due to J. B. Shearer extended the work of Hoffman. He showed that every real number larger than $\sqrt{2 + \sqrt{5}}$ is a limit point of R.

In this talk we will present the rich history of these findings. Moreover, we will explain our results that extend Hoffman's original question to other matrices.

We recall from the work of V. Nikiforov that for an undirected graph G the matrix $A_\alpha(G) := \alpha D(G) + (1-\alpha)A(G)$, for $0 \leq \alpha \leq 1$. We study an A_α version of Shearer's results. We are able to generalize the work of Shearer obtaining, for every $0 \leq \alpha < 1/2$, intervals where all points are A_α-limit points of spectral radius of graphs.

We also study limit points of (signless) Laplacian spectral radii of graphs. We will explain our findings and show that this seems to be a much harder problem.

Approximation Algorithm for Unrooted Prize-Collecting Forest with Multiple Components

Wei Liang, Zhao Zhang

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In this talk, I will introduce our work on a polynomial-time 2-approximation algorithm for the Unrooted Prize-Collecting Forest with K Components (URPCF_K) problem, which aims to find a forest with exactly K connected components while minimizing the sum of the forest's weight and the penalties incurred by unspanned vertices. In particular, for $K = 1$, the URPCF_1 problem is exactly the prize-collecting Steiner tree (PCST) problem, which has received extensive studies. Unlike the PCST problem, whose unrooted version can be solved by transforming it into an unrooted version by guessing the root, the unrooted PCF_K problem cannot be readily solved using its rooted analogue, because guessing its roots may lead to exponential time complexity for non-constant K. To address this challenge, we propose a rootless growing and rootless pruning algorithm. We also apply this algorithm to improve the approximation ratio for the Prize-Collecting Min-Sensor Sweep Cover problem (PCMinSSC) from 8 to 5.

ABSTRACTS
CONTRIBUTED TALKS

On odd induced subgraphs of a graph

Baoyindureng Wu

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A graph G is called *even* (resp. *odd*) if $d_G(v)$ is even (resp. odd) for every vertex $v \in V(G)$. Gallai proved that the vertex set $V(G)$ can be partitioned into two sets V_1 and V_2 such that $G[V_i]$ is even for each $i \in \{1, 2\}$. This is not the case for the formulation of the 'odd version'. However, Scott showed that every connected graph of an even order has a partition $V_1, \dots, V_k$ of $V(G)$ such that $G[V_i]$ is odd for any $i \in \{1, \dots, k\}$. In this talk, we discuss some new results on this topics.

On the σ-irregularity index

Goran Popivoda

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The σ-irregularity index is a modified version of the well-known Albertson irregularity index. For a graph $G = (V, E)$, this index is calculated as $\sigma(G) = \sum_{uv \in E}(d(u) - d(v))^2$, where $d(u)$ and $d(v)$ represent the degrees of vertices u and v, respectively. In this article, we present a characterization of chemical trees of a specific order that have the maximum σ-irregularity index.

This is joint work with Žana Kovijanić Vukićević, Saša Vujošević, Riste Škrekovski and Darko Dimitrov.

Some results on Gallai-Ramsey type problem

Xueliang Li

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Let G and H be two non-empty graphs and k be a positive integer. The Gallai-Ramsey number $gr_k(G : H)$ is defined as the minimum positive integer N such that for all $n \geq N$, every k-edge-coloring of the complete graph K_n contains either a rainbow subgraph G or a monochromatic subgraph H. In this talk, we shall survey some results on the Gallai-Ramsey numbers we obtained recently. Some unsolved problems are presented for further study.

This is a joint work with Luyi Li, Yapig Mao and Yuan Si.

The absence of monochromatic triangles implies various properly colored spanning trees

Ruonan Li, Ruhui Lu, Xueli Su, Shenggui Zhang

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An edge-colored graph G is called properly colored if every two adjacent edges are assigned distinct colors. A monochromatic triangle is a cycle of length 3 with the edges assigned a same color. Given a tree T_0, let $\mathcal{T}(n, T_0)$ be the collection of n-vertex trees that are subdivisions of T_0. It is conjectured that for each fixed tree T_0 of k edges, there is a function $f(k)$ such that for each integer $n > f(k)$ and each $T \in \mathcal{T}(n, T_0)$, every edge-colored complete graph K_n without containing any monochromatic triangle must contain a properly colored copy of T. The case that T_0 is a star is confirmed. A weaker version of the above conjecture is also obtained. Moreover, to get a nice quantitative estimation of $f(k)$ requires determining the Constraint Ramsey number of a monochromatic triangle and a rainbow k-star, which is of independent interest.

The chromatic number of the associahedron

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The n-associahedron is an $(n\ 2)$-dimensional convex polytope in which each vertex corresponds to a way of correctly inserting opening and closing parentheses in a string of n letters, and the edges correspond to single application of the associativity rule. Its facets correspond to the inserting of a single pair of the parenthesis in the string. The chromatic number related to a colouring of facets of certain classes of the n-associahedron is determined.

The talk is based on joint research with Jelena Ivanović and Zoran Petrić.

Edge coloring of hypergraphs

Zhimin Wang, Xia Zhang

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A k-edge coloring of a hypergraph H is an edge coloring of H with k colors such that any two intersecting edges receive distinct colors. Let $D(H)$ denote the maximum strong degree of H. In 1980s, Berge and Meyniel, independently, conjectured that each loopless linear hypergraph H has a $(D(H)+1)$-edge coloring, which implies the Erdős-Faber-Lovász conjecture. In 2000, Dvořák generalized Shannon's result to hypergraphs without multiple 2-edges and conjectured that every loopless hypergraph H has a $\lfloor 1.5D(H) \rfloor$-edge coloring. In this talk, we confirm the Berge-Meyniel conjecture for two classes of linear hypergraphs, and verify the Dvořák's conjecture for weakly 2-edge-sparse hypergraphs and hypergraphs without intersecting multiple 2-edges. The above results strictly extend several related results due to Bretto, Faisant and Hennecart (Discrete Math. 2020), Alesandroni (Discrete Math. 2021), Dvořák (Eur. J. Comb. 2000), respectively.

The signless Laplacian spectral radius of $2K_3$-free graphs

Yanting Zhang, Ligong Wang

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Let $2K_3$ denote the disjoint union of two triangles. Given a graph H, a graph is said to be H-*free* if it does not contain H as a subgraph. In spectral extremal graph theory, it is interesting to determine the maximum (signless Laplacian) spectral radius of H-free graphs. In this paper, we characterize the unique extremal graph with the maximum signless Laplacian spectral radius among all $2K_3$-free graphs of order $n \geq 44$.

On nut graphs with a given automorphism group

Nino Bašić, Patrick W. Fowler

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A *nut graph* is a simple graph of order 2 or more for which the adjacency matrix has a single zero eigenvalue such that all non-zero kernel eigenvectors have no zero entry. We show by construction that every finite group can be realised as the group of automorphisms of infinitely many nut graphs. Moreover, we show that such nut graphs exist even within the class of regular graphs.

Graph operations and a unified method for kinds of Turán-type problems on paths, cycles and matchings

Jiangdong Ai, Hui Lei, Bo Ning, Yongtang Shi

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We develop a method that provides a unified approach for solving some Turán-type and generalized Turán-type problems, degree power problems, and extremal spectra problems (mainly under spectral radius conditions and signless Laplacian spectral radius conditions) on paths, cycles, and matchings. Our results generalize classical results on cycles and matchings due to Kopylov and Erdős-Gallai, respectively, and provide a positive resolution to an open problem originally proposed by Nikiforov. We improve and extend the spectral extremal results on paths due to Nikiforov, and due to Nikiforov and Yuan. We also offer a comprehensive solution to a connected version of a problem on the degree power sum of a graph containing no P_k, a topic initially studied by Caro and Yuster.

Eigenvalues of Complex Unit Gain Graphs and Gain Regularity

Maurizio Brunetti

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A complex unit gain graph (or $\mathbb{T}$-gain graph) $\Gamma = (G, \gamma)$ is a gain graph with gains in $\mathbb{T}$, the multiplicative group of complex units. In this talk we address the following two problems:

1. Which real numbers are eigenvalues of $\mathbb{T}$-gain graphs?

2. Which real numbers are indices (i.e. largest eigenvalues) of $\mathbb{T}$-gain graphs?

In order to face those problems, non-complete extended p-sums, suitably defined joins of $\mathbb{T}$-gain graphs and a notion of a-$\mathbb{T}$ regularity are considered.

On higher-dimensional Hadamard matrices and designs

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Higher-dimensional Hadamard matrices and other types of higher-dimensional designs were studied intensively in the 1980s and 1990s. The books [1] and [5] still provide a fairly complete overview of the topic as there are few recent results. However, many interesting open questions remain. By the product construction of Yang [4], the spectrum of orders v such that proper n-dimensional Hadamard matrices exist is the same for all dimensions $n \geq 2$. By the famous Hadamard conjecture it includes all orders $v \equiv 0 \pmod 4$. For $n \geq 3$ there are "improper" Hadamard matrices that can also exist for $v \equiv 2 \pmod 4$. Provided the Hadamard conjecture is true, Yang's constructions described in [5] cover all even orders v for dimensions $n \geq 4$. For dimension $n = 3$, only examples of order $v = 2 \cdot 3^k$ were known.

In the first part of the talk I will describe a construction of three-dimensional Hadamard matrices of order v such that $v - 1$ is an odd prime power [2]. The construction covers infinitely many orders that were previously open. In the second part I will present some new results on n-dimensional symmetric block designs [3]. A construction based on difference sets is generalized so that the resulting n-dimensional designs may have inequivalent (v, k, λ) block designs as 2-dimensional slices. For parameters (v, k, λ) of Menon type, they can be transformed into proper n-dimensional Hadamard matrices with inequivalent slices. Previous constructions of n-dimensional designs and Hadamard matrices all give examples with equivalent slices.

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An embedding technique in the study of word-representability of graphs

Sumin Huang, Sergey Kitaev, Artem Pyatkin

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Word-representable graphs, which are the same as semi-transitively orientable graphs, generalize several fundamental classes of graphs. In this talk, we propose a novel approach to study word-representability of graphs using a technique of homomorphisms. As a proof of concept, we apply our method to show word-representability of the simplified graph of overlapping permutations that we introduce in this paper. For another application, we obtain results on word-representability of certain subgraphs of simplified de Bruijn graphs that were introduced recently by Petyuk and studied in the context of word-representability.

On generalizations of Dirac-type problems

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There are various ways to generalize Dirac-type problems, such as the Ramsey-Turán version, the random sparsification version, the algorithm version and so on. In this talk, I shall report some recent results on generalizations of Dirac-type problems.

Results on Bollobás set-pair systems

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Suppose $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$ is a family of pairs of sets, where $A_i, B_i \subseteq [n]$, and $A_i \cap B_i = \emptyset$. Then $\mathcal{P}$ is called a Bollobás system if $A_i \cap B_j \neq \emptyset$ when $i \neq j$, and a skew Bollobás system if $A_i \cap B_j \neq \emptyset$ when $i < j$.

In 1965, to solve a problem on hypergraphs, Bollobás proved that for a Bollobás system $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$, we have $\sum_{i=1}^{m} \binom{|A_i|+|B_i|}{|A_i|}^{-1} \leqslant 1$, which is later called Bollobás Theorem or Bollobás Inequality, and plays a important role in extremal set theory. If we further requests $|A_i| = a, |B_i| = b$ for every i, the inequality above shows that the maximum cardinality of the Bollobás system satisfy $m \leqslant \binom{a+b}{a}$. In 1977, Lovász generalized this (uniform) result to Bollobás system of spaces, and decided its maximum cardinality. In 1982, Frankl proved that both (uniform) statements remain true if the Bollobás system is replaced by skew Bollobás system.

In this talk, we generalize the Bollobás-type Theorem to partitions of sets. Using a polynomial method, we decide the maximum cardinality of skew Bollobás system on it. Also, using exterior product method, we consider the corresponding question of spaces, which is known as singular linear spaces.

For the nonuniform case, we cannot simply weaken the condition of Bollobás system to skew Bollobás system as for the uniform case. So it is natural to ask what can we say for a (nonuniform) skew Bollobás system. In 2023, Hegedüs and Frankl answered the question with the inequality

$$\sum_{i=1}^{m} \binom{|A_i| + |B_i|}{|A_i|}^{-1} \leqslant 1 + n.$$

Using a probabilistic method, we can strengthen their inequality to

$$\sum_{i=1}^{m} \left((1 + |A_i| + |B_i|) \binom{|A_i| + |B_i|}{|A_i|} \right)^{-1} \leqslant 1.$$

And using similar arguments, we generalize this result to partitions of sets on both symmetric and skew cases.

Some progress on minimum spectral radius of a graph

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The graph with the minimum spectral radius among all connected graphs in a given graph family $\mathbb{G}$ is called the minimizer graph with respect to $\mathbb{G}$. Let $\mathbb{G}_{n,\alpha}$ be the family of connected graphs with order n and independence number α. Stevanović in the classical book [D. Stevanović, Spectral Radius of Graphs, Academic Press, Amsterdam, 2015] pointed out that determining the minimizer graph in $\mathbb{G}_{n,\alpha}$ appears to be a tough problem. In this talk, we will present some new progress of the minimizer graph in $\mathbb{G}_{n,\alpha}$. Moreover, we also discuss the minimizer graph in $\mathbb{G}_{n,\gamma}$, where $\mathbb{G}_{n,\gamma}$ is the family of connected graphs with order n and dominating number γ.

This work is joint with Ji-ming Guo, Yarong Hu and Qiongxinag Huang.

On 2-integral Cayley graphs

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In this talk, we introduce the concept of k-integral graphs. A graph Γ is called k-integral if the extension degree of the splitting field of the characteristic polynomial of Γ over rational field $\mathbb{Q}$ is equal to k. We prove that for any positive integers k and Δ, the set of all finite connected graphs with algebraic degree at most k and maximum degree at most Δ is finite. We study 2-integral Cayley graphs over finite groups G with respect to Cayley sets which are a union of conjugacy classes of G. Among other general results, we completely characterize all finite abelian groups having a connected 2-integral Cayley graph with valency $2, 3, 4$ and 5. Furthermore, we classify the finite groups G that all Cayley graphs over G with bounded valency are 2-integral.

On some spectral properties of graphs with self-loops

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Let G be a simple n-vertex graph with the vertex set $V(G)$, and let $S \subseteq V(G)$, where $|S| = \sigma$ and $0 \leq \sigma \leq n$. The graph G_S obtained by attaching a self-loop at each vertex from the set S is a *self-loop graph* of G. Although there are certain results regarding self-loop graphs in the literature, these graphs become again interesting for study by introducing their *energy* [4]. The authors of [4] conjectured that for any set S, such that $1 \leq \sigma \leq n-1$, $\mathcal{E}(G_S) > \mathcal{E}(G)$, where $\mathcal{E}(\mathcal{G})$ is the energy of the (self-loop) graph $\mathcal{G}$. The conjecture was disproved in [5], while its modified version was posed in [1].

In this talk, we discuss some *unary (self-loop) graph operations*, such as the *subdivision graph* $S(G_S)$ of a self-loop graph G_S of a given graph G, and its *line graph* $L(G_S)$, by using the *incidence matrix* of G_S. It is shown that the eigenvalues of the line graph $L(G_S)$ are not less than -2, as well as that the multiplicity of the eigenvalue -2 in the spectrum of $L(G_S)$ is equal to the multiplicity of the same eigenvalue in the spectrum of the line graph $L(G)$ of G. As a consequence of a slightly general statement, it follows that for a graph G of order $n \geq 2$ and a non-empty set S, $\mathcal{E}(L(G)) < \mathcal{E}(L(G_S))$ holds.

Moreover, we explain the *nullity* of a graph $SP(G)$, which is obtained from the subdivision graph $S(G)$ of a graph G by attaching pendant edges at vertices from the set $S \subseteq V(G)$. The nullity of $SP(G)$ is computed by relating such a graph with a certain multiple-loop graph. Due to this result, the nullities and the nullity sets of some bipartite graphs can be characterized.

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On the classification and characterization of graphs with two main eigenvalues

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We are dealing with the classification and structural characterization of finite connected graphs with 2 main eigenvalues. These graphs denoted by $G = G(a, b)$ having integer parameters $a \geq 0$ and b contain exactly two main eigenvalues denoted by $\rho = \rho(G)$ and $\mu = \mu(G)$ .As it is known, these graphs are called 2-walk (a,b) linear graphs. It is worth noting that the classification of 2-walk linear graphs seems to be an unsolved difficult problem.

In the first part of this study it will be shown that based on parameters $a = a(G)$, $b = b(G)$ all 2–walk linear graphs $G(a, b)$ can be classified into four distinct groups. These are i) ordinary graphs with $a > 0$ and $b > 0$ integers, ii) strictly harmonic graphs with $a \geq 2$ and $b = 0$ integers, iii) semiregular graphs with $a = 0$ and $b \geq 2$ integers, and iv) non-ordinary (opposite-type) graphs with $a > 0$ and $b < 0$ integer parameters.

Moreover, a general existence criterion related for 2-walk (a,b) linear graphs is given. It is based on the following lemma and two propositions.

Lemma 1 *Let G(a,b) be a connected 2-walk (a,b) linear graph. Then it can be proved that if $C(a, b) = a^2 + 4b$ then there are no two-walk (a,b) linear graphs for which $C(a, b) = 6$ or $C(a, b) = 7$ hold.*

The existence criterion based on Lemma 1 can be extended as follows:

Proposition 1 *Let ($a \geq 0$, b) be a pair of two integers and consider the function defined by $C(a, b) = a^2 + 4b$. Then there are no 2-walk linear graphs $G(a, b)$ for which $C(a, b) = C(k)$ holds where $C(k) = 4k + 2$ or $C(k) = 4k + 3$, and $k \geq 1$ arbitrary integer.*

Proposition 2 *Let $(a \geq 0, b)$ be a pair of two integers and consider the function defined by $C(a, b) = a^2 + 4b$. Then there exists at least one 2-walk linear graph $G(a, b)$ for which $C(a, b) = C(k)$ holds where $C(k) = 4k$ or $C(k) = 4k + 1$, for $k \geq 1$ arbitrary integer.*

Families of signed graphs with strong (anti-)reciprocal eigenvalue property

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A graph is said to have the strong reciprocal (anti-reciprocal) eigenvalue property if for any adjacency eigenvalue λ of multiplicity t then $1/\lambda$ (resp. $-1/\lambda$) is an eigenvalue of multiplicity t. In this talk we consider the same property for signed graphs and we present some families of signed graphs with both the strong reciprocal and anti-reciprocal eigenvalue property.

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Density of C_3^*-critical signed graphs

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In this talk, we will show that every n-vertex C_3^*-critical signed graph has at least $\frac{3n-1}{2}$ edges, and that this bound is asymptotically tight. A signed-graph analog of Grötzsch theorem then follows: every signed planar or projective-planar graph of girth at least 6 is circular 3-colorable, and for the projective-planar case, this girth condition is best possible.

On strongly regular signed graphs

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We present our current work related to a concept of strong regularity defined for signed graphs, recently defined by Zoran Stanić. We say that a signed graph $\dot{G}$ is *strongly regular* (for short, $\dot{G}$ is a *SRSG*) whenever it is neither homogeneous complete nor edgeless, and there exist $r \in \mathbb{N},\ a, b, c \in \mathbb{Z}$, such that the entries of the square of its adjacency matrix, $A^2_{\dot{G}}$, satisfy

$$a_{ij}^{(2)} = \begin{cases} r & \text{if } \ i = j, \\ a & \text{if } \ i \overset{+}{\sim} j, \\ b & \text{if } \ i \overset{-}{\sim} j, \\ c & \text{if } \ i \nsim j \text{ and } i \neq j, \end{cases}$$

where $\overset{+}{\sim}$ (resp. $\overset{-}{\sim}$) designates that the corresponding vertices are joined by a positive (negative) edge; similarly, $\nsim$ stands between non-adjacent vertices.

We give an overview of some basic properties of such graphs, and establish a classification of all SRSGs according to the relations among their defining parameters. We investigate the relations that exist between SRSGs with a relatively small number of distinct eigenvalues and symmetric 2-class or 3-class association schemes, and also between properties of net regularity, strong regularity and walk regularity of signed graphs. In the end, we present our results regarding determination of all connected net-regular strongly regular signed graphs with vertex degree at most 4.

Constructions of nonregular cospectral signed graphs

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Recently several variations of joins of unsigned graphs have been studied. To extend this work to signed graphs we primarily consider two different types of joins of two signed graphs $\Sigma_1 = (G_1, \sigma_1)$ and $\Sigma_2 = (G_2, \sigma_2)$. These are the neighbours splitting join $\Sigma_1 \underline{\vee} \Sigma_2$ and the non-neighbours splitting join $\Sigma_1 \overline{\wedge} \Sigma_2$. The adjacency and Laplacian characteristic polynomials of the graphs produced by these operations on any two signed graphs are found and so is the normalised Laplacian characteristic polynomial when the two underlying graphs G_1 and G_2 are regular. In the cases where Σ_1 is (r_1, k_1)-co-regular and Σ_2 is (r_2, k_2)-co-regular we also compute the spectra of the joins with respect to their adjacency, Laplacian, and normalised Laplacian matrices. Applying these results enables the construction of nonregular nonisomorphic cospectral pairs of signed graphs.

Bounds on the largest eigenvalue and energy of the connectivity matrix

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The connectivity matrix of a graph is the matrix whose off-diagonal i-j entry is the maximum number of internally vertex disjoint paths between vertices i and j with only zeros on its diagonal. In this talk, we investigate lower and upper estimates for the largest eigenvalue of such matrices. We provide bounds in terms of the underlying graph's average degree and show that they can be utilized to refine previous bounds on the energy of connectivity matrices.

Reinforcement learning for graphs and beyond

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Adam Zsolt Wagner [1] showed how the cross entropy method, a particular reinforcement learning technique, can be used to construct (counter)examples in graph theory. We have recently provided an improved implementation of this method [2] and here we will showcase how it can be used to construct counterexamples for a set of older conjectures on the Laplacian spectral radius of graphs [2], edge-colorings of complete graphs that lead to new lower bounds on Ramsey numbers [3], new examples of graphs with two main eigenvalues and, with a minor adaptation, also the shape of optimal window overhangs for residential homes.

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Metaheuristics for finding threshold graphs with maximum spectral radius

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We consider the problem of characterizing graphs with the maximum spectral radius among the connected graphs with given numbers of vertices and edges. It is well-known that the candidates for extremal graphs are threshold graphs, but only a few partial theoretical results have been obtained so far. Therefore, we approach to this problem from a novel perspective that involves incomplete enumeration of different threshold graphs with a given characteristic. Our methodology defines the considered problem as an optimization task and utilizes metaheuristic to address it. In particular, we implemented Variable Neighborhood Search (VNS), trajectory-based single-solution method, and Bee Colony Optimization (BCO), a population-based metaheuristic from the Swarm Intelligence (SI) class. We tested the developed algorithms on the graphs with a moderate number of nodes and compared their performance.

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Connected graphs of fixed order and size with minimal spectral radius

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We consider connected graphs of fixed number of vertices n and fixed number of edges m that minimize the largest eigenvalue of the adjacency matrix, i.e. the spectral radius. Such graphs are called minimizers. This study has a strong motivation in modelling complex networks as graphs with a small spectral radius since the smaller spectral radius ensures a better protection against viruses. We conjecture that vertex degrees of a minimizer are as equal as possible, i.e., they take at most 2 different values $\lfloor 2m/n \rfloor$ or $\lceil 2m/n \rceil$. Using a total enumeration the conjecture is fully confirmed for graphs with at most 10 vertices. We give theoretical results that support the conjecture for certain particular classes of graphs. For larger n, due to the enormous number of graphs that should be considered, we designed the Variable Neighborhood Search (VNS) metaheuristic following the Less Is More approach (LIMA). For instances with up to 100 vertices, with various values of n and m, the solutions obtained by the VNS always had the desired structure.

On computational searches for borderenergetic graphs

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A simple connected graph is said to be a non-complete borderenergetic graph if it is not a complete graph and its energy is equal to the energy of the complete graph on the same number of vertices. We expand upon findings of Li, Wei and Zhu [MATCH Commun. Math. Comput. Chem. 77 (2017) 25–36] concerning the counts of borderenergetic chemical graphs, by investigating such graphs with up to 19 vertices. The exploration reveals a single new non-complete borderenergetic chemical graph on 15 vertices. Additionally, we improve Furtula's results [Iranian J. Math. Chem. 8 (2017) 339–344] related to the count of borderenergetic graphs with up to 12 vertices, unveiling several new borderenergetic graphs on 12 vertices.

Uniformly 3-connected graphs

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Let k be a positive integer. A graph is said to be uniformly k-connected if between any pair of vertices the maximum number of independent paths is exactly k. Dawes [R.W. Dawes, Minimally 3-connected graphs, J. Combin. Theory Ser. B 40 (1986) 159-168] showed that all minimally 3-connected graphs can be constructed from K_4 such that every graph in each intermediate step is also minimally 3-connected. In this paper, we generalize Dawes' result to uniformly 3-connected graphs. We give a constructive characterization of the class of uniformly 3-connected graphs which differs from the characterization provided by Göring et al. [F. Göring, T. Hofmann, M. Streicher, Uniformly connected graphs, J. Graph Theory 101 (2022) 161-340]. Eventually, we obtain a tight bound on the number of edges in uniformly 3-connected graphs.

Minimum degree of minimal k-factor-critical graphs

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A factor-critical graph is a graph in which the removal of any vertex results in a graph with a perfect matching. A graph G with at least one edge is called *bicritical* if, after the removal of any pair of distinct vertices of G, the resulting graph has a perfect matching. O. Favaron and Q. Yu independently introduced k-factor-critical graphs as a common generalization of factor-critical and bicritical graphs. A graph G of order n is said to be k-factor-critical for an integer $1 \leq k < n$, if the removal of any k vertices results in a graph with a perfect matching. A k-factor-critical graph is minimal if the deletion of every edge results in a graph that is not k-factor-critical. In 1998, O. Favaron and M. Shi conjectured that every minimal k-factor-critical graph has minimum degree $k+1$ and confirmed it for $k = 1, n-2, n-4$ and $n-6$. Afterwards in 2007 Z. Zhang et al. re-proposed this conjecture, which remains open to now in general case. This talk will present some recent progresses on this topic: J. Guo and H. Zhang have confirmed this conjecture for $k = 2, n-8, n-10$ by using a new method. As joint works with Dr. F. Lu and Q. Li, very recently this conjecture has also be confirmed for claw-free graphs and planar graphs. Moreover, we derive that every 3-connected minimal bicritical claw-free graph G has at least $\frac{1}{4}|V(G)|$ cubic vertices, yielding further evidence for S. Norine and R. Thomas' conjecture on the number of cubic vertices of minimal bricks.

This is a joint work with Dr. Jing Guo.

A Ramsey-Turán theory for tilings in graphs

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An (s,t)-bipartite-hole in a graph G consists of two disjoint sets of vertices S and T with $|S| = s$ and $|T| = t$ such that $E(S,T) = \emptyset$. We use $\alpha^*(G)$ to denote the largest integer s such that G contains an (s,s)-bipartite-hole. Given any constant $\mu > 0$, there exists some constant $\alpha > 0$ such that the following holds for sufficiently large n and any collection of cycles $\{x_iC_i\}_{3\leq i\leq n}$. Let G be a graph on n vertices such that $\delta(G) \geq \sum_{i=3}^{n} x_i + \mu n$ and $\alpha^*(G) \leq \alpha n$, where x_i is an integer. We showed that there exists an $\{x_iC_i\}_{3\leq i\leq n}$-factor in G.

Evolution of geometric graphs of fullerene spectra

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A *fullerene graph* is a 3-regular, 3-connected, plane graph with only pentagonal and hexagonal faces. Fullerene graphs are mathematical models of fullerene molecules. For a given number of atoms, there are many more theoretically possible isomers than there are actually observed ones. The ongoing research on fullerene graph has been to a large extent motivated by search for topological invariants which could serve as reliable indicators and predictors of stable isomers. Among many invariants examined so far, particularly promising are some based on graph spectra. For a graph F_n on n vertices, its spectrum (i.e., the spectrum of its adjacency matrix) can be considered as a point in n-dimensional space. So, for a given number of atoms n, we compute spectra of all fullerene graphs on n vertices. The resulting n-dimensional point set serves as the vertex set of a geometric graph $G(n,d)$ obtained by connecting all points whose distance does not exceed a given positive parameter d. In this talk, we analyze the properties of such geometric graphs, in particular, the increase of their number of edges and the emergence and evolution of connected clusters with increasing values of d.

On Graovac-Ghorbani index of unicyclic graphs

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Topological indices are numerical invariants associated with the chemical constitution for the purpose of correlation of chemical structures with various physical properties, chemical reactivity or biological activity. They have found important application in predicting the behavior of chemical substances. The Graovac-Ghorbani (ABC_{GG}) index is a topological descriptor that has predictive potential compared to analogous descriptors. It is used to model both the boiling point and melting point of molecules and is applied in the pharmaceutical industry. In the last years, the number of publications on its mathematical properties has increased. We consider the problem of characterization of unicyclic graphs that minimize the ABC_{GG} index. While the case of even girth is quite easy to solve, the odd girth case leads only to the partial results, thus forming a conjecture. As an auxiliary result, we compare the ABC_{GG} indices of paths and cycles with an odd number of vertices.

Metallic cubes and beyond

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We define a new two-parameter family of graphs that further generalizes the Fibonacci cubes and offers an alternative definition of the Pell graphs. Namely, the number of vertices in this family of graphs satisfies recurrence $s_n = a \cdot s_{n-1} + s_{n-2}$, a linear recurrence of order two. We show that many appealing and useful properties of the Fibonacci are preserved. In particular, we present recursive decomposition and decomposition into grids and explore some metric and enumerative properties. We also answer the question of the existence of the Hamiltonian paths and cycles. Furthermore, we further extend this family by considering graphs whose vertices satisfy Horadam's recurrence $s_n = a \cdot s_{n-1} + b \cdot s_{n-2}$.

Polynomial indicator of flat bands

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Flat bands have attracted the attention of many scientists dealing with condensed matter and materials research, including topological materials, photonics and unconventional superconductivity. Several approaches have been proposed for their construction, such as the generalized Lieb's method and refinements within graph theory. Here, we introduce the polynomial indicator method, which represents an efficient algebraic method that predicts the existence of flat bands in a given crystal lattice by showing that their existence is equivalent to a nontrivial GCD of the coefficients of an appropriate polynomial related to the lattice Bloch Hamiltonian. We subsequently prove the validity of the disclosed method and finally provide a few examples that illustrate how it can be used on concrete lattices, such as the kagome and the dice lattice.

On the edge reconstruction of six digraph polynomials

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Let $G = (V, E)$ be a digraph having no loops and no multiple arcs with n vertices and m arcs. Denote the adjacency matrix and the vertex in-degree diagonal matrix of G by A and D. Set

$$f_1(G;x) = \det(xI - A), f_2(G;x) = \det(xI - D + A), f_3(G;x) = \det(xI - D - A),$$

$$f_4(G;x) = \mathrm{per}(xI - A), f_5(G;x) = \mathrm{per}(xI - D + A), f_6(G;x) = \mathrm{per}(xI - D - A),$$

where $\det(X)$ and $\mathrm{per}(X)$ denote the determinant and permanent of a square matrix X, respectively. In this paper, we prove that, for any $1 \leq i \leq 6$,

$$(m - n)f_i(G;x) + x{f_i}'(G;x) = \sum\nolimits_{e \in E} f_i(G - e;x),$$

which implies that if $m \neq n$, then $f_i(G;x)$ can be reconstructed from $\{f_i(G - e;x)|e \in E\}$.

Twist polynomial as a weight system

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This talk contains four parts. In the first part, we give the definitions of ribbon graphs and their partial-dual (genus) polynomials, and some progress in this topic. Then we prove the partial dual polynomial as a function of framed chord diagrams satisfies the four term relation, thus can be viewed as a weight system. In the third part, we focus on set systems and their twist (width) polynomials and introduce the 4-term relation of set systems and prove the twist polynomial satisfies it. In the last part, we disclose the connection between the previous two parts.

The Moore-Penrose inverse of a Laplacian matrix of a signed graph

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We present a combinatorial formula of the Moore-Penrose inverse of the Laplacian matrix L of a signed graph G. This is achieved by finding a combinatorial formula for the Moore-Penrose inverse of an incidence matrix of G. This work generalizes related known results on incidence and Laplacian matrices of an unsigned graph.

ℓ_1-embeddability of hypergraphs

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Embeddings of graphs into hypercubes and other ℓ_1 spaces are well studied. Graphs that are embeddable into ℓ_1 space have several useful properties and characterizations, which were, for example, widely used in mathematical chemistry. In this talk we will present several hypergraphs that are ℓ_1-embeddalbe. More precisely we will consider cube hypergraphs and different types of hypertrees.

Hitting times for the simple random walk on cube-like networks

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Let $\mathbb{Z}_2^n = \{i = i_1 \dots i_n \mid i_k = 0 \text{ or } 1, k = 1, \dots, n\}$ be the Abelian 2-group, any simple connected Cayley graph $Cay(\mathbb{Z}_2^n, H)$ is called a *cube-like network*, which includes the classical hypercube network, the enhanced hypercube network, the augmented hypercube network, etc. In this paper, we study the simple random walk (SRW) on cube-like network $Cay(\mathbb{Z}_2^n, H)$. First, using the irreducible characters of group $\mathbb{Z}_2^n$, a uniform formula for the expected hitting time from vertex $i = i_1 \dots i_n$ to vertex $j = j_1 \dots j_n$ is obtained as follows:

$$E_i T_j(Cay(\mathbb{Z}_2^n, H)) = 2|H| \sum_{k_1 \dots k_n \in H_{ij}} \frac{1}{|H| - \lambda_{k_1 \dots k_n}},$$

where $\lambda_{k_1 \dots k_n}$ denotes the adjacent eigenvalue corresponding to vertex $k_1 \dots k_n$, and the vertex subset $H_{ij} = \{k = k_1 \dots k_n \mid |Index(i \oplus j) \cap Index(k)| \text{ is odd}\}$ with $Index(i_1 \dots i_n) = \{t \mid i_t = 1, t = 1, 2, \dots, n\}$. Note that H_{ij} is not only independent of the generator set H, but the elements of which can be listed easily. Hence, the computation of $E_i T_j$ for SRW on cube-like networks can be reduced to some combinatorial counting in some sense. Then as applications of the above formula, explicit expressions of hitting times for SRW on three families of cube-like networks are obtained, and some combinatorial identities are derived at the same time as by product.

How does a necessary condition effect a graph to have hamiltonian property?

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If a graph G has property A implying G also has the property B, then B is a necessary condition for G to have A. There exists some property for a graph to have it such that is the same even we impose a necessary condition on it, while there exists some property for a graph to have it such that is is different from it if one impose a necessary condition on it. For example, to be 2-connected is necessary condition for a graph to be hamiltonian cycle. Ore condition say that if minimum degree sum of two vertices of a graph G is at least its order, then G is hamiltonian. If we impose a necessary condition that it has a hamiltonian path, then the condition could not be changed; while it would be changed to be that one of degree of a pair of vertices of distance two is at most half of the order of G, which be much improved than those without it. In this talk, we focus on hamiltonian property: Hamiltonian cycle or path, and so on.

Some saturation problems for nowhere-zero flows

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A nowhere-zero k-flow of a graph G is a pair (D, f) where D is an orientation of G and f is a mapping with $f : E(G) \mapsto \{\pm 1, \pm 2, \dots, \pm(k-1)\}$ such that the incoming flows is equal to outgoing flows for each vertex. A bridgelss graph G is called k-flow saturated if G does not admit a nowhere-zero k-flow but for any edge $e \notin E(G)$, $G + e$ admit a nowhere-zero k-flow. The concept of group connectivity is a nonhomogeneous analogue of nowhere-zero flow properties. Similarly, we can define a bridgelss graph G A-connectivity saturated if G is not A-connected but $G + e$ does for any edge $e \notin E(G)$. In this talk, we will discuss some results related to k-flow saturated graphs and A-connectivity saturated graphs.

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