Addenda and Corrigenda

January 2008

for *Cut-Elimination in Categories*, by Kosta Došen,

Trends in Logic vol. 6, Kluwer, Dordrecht, 1999

p. viii, line 12 bottom, add "§ 5.11. The links of adjunctions and the links of comonads"

p. 6, line 6, replace "definable" by "explicitly definable"

p. 28, line 2, replace "about them" by "around"

p. 50, line 2 bottom, replace "$\tau^{G(t)}$" by "$\tau^{G(\tau)}$"

p. 60, line 13 bottom, p. 78, line 4 bottom, p. 101, line 8, p. 178, line 11, replace "§ 1.7" by "§ 1.8.1"

p. 118, line 8, delete "topmost"

p. 118, lines 9-10, delete parenthetical text

p. 136, replace penultimate paragraph by:

Note that for $h : C_1 \to C_2$ every F or G in C_1 or C_2 is linked in $\Lambda(h)$ to exactly one other occurrence of F or G in C_1 or C_2.

The links we have just defined are related to the *graphs* of [Eilenberg & Kelly 1966], which one also finds in [Kelly & Mac Lane 1971] (see also [D. & Petric 1997, section 2]). The term "link" is used in knot theory for a collection of knots, and this is, of course, a different notion from our notion of link. In knot theory, a set of our links is a special kind of *tangle* (see [Murasugi 1996, Chapter 9]). Categories of tangles have played recently a prominent role in the theory of quantum groups, in low-dimensional topology and in knot theory (see [Kassel 1995, Chapter 12], [Kauffman & Lins 1994], and references therein).

p.158, line 12, replace "omitted" by "omitted it"

p. 170, lines 8-9, delete parentheses, and add at the end of the paragraph: "(Matters of this section and of the preceding one should be compared with [Pumplün 1970] and [Auderset 1974].)"

p. 193, line 13, replace "does not hold" by "is not satisfied"

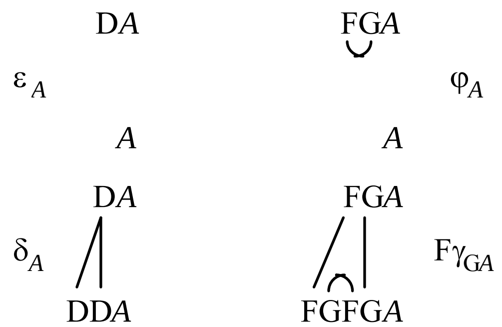
p.194, add new section at the end:

§ 5.11. THE LINKS OF ADJUNCTIONS AND THE LINKS OF COMONADS

We have remarked in § 4.5.1 that the graphs of categories in free adjunctions generated by arrowless graphs are disjoint unions of graphs of categories involved in free adjunctions generated by pairs of graphs $(\mathcal{G}, \mathcal{H})$ where one of $\mathcal{G}$ and $\mathcal{H}$ is arrowless with a single object and the other is the empty graph. Take now the free adjunction $\langle \mathcal{A}, \mathcal{B}, \mathrm{F}, \mathrm{G}, \varphi, \gamma \rangle$ generated by $(\mathcal{G}, \mathcal{H})$ where $\mathcal{G}$ is arrowless with a single object C and $\mathcal{H}$ is empty. The objects of $\mathcal{A}$ are then of the form $(\mathrm{FG})^n C$, where $(\mathrm{FG})^n$, with $n \geq 0$, stands for a possibly empty sequence of n blocks of FG. The objects of $\mathcal{B}$ are the objects of $\mathcal{A}$ with G prefixed.

It follows from Propositions 2 and 3 of § 5.2.2 that $\mathcal{B}$ is isomorphic to $\mathcal{A}_{\mathrm{F}\Gamma}$, the delta category of the comonad of the adjunction. The objects of the subcategory $\mathcal{A}_{\mathrm{F}\Gamma}$ of $\mathcal{A}$ are $(\mathrm{FG})^n C$ with $n \geq 1$, i.e. all the objects of $\mathcal{A}$ except C. It follows easily that the adjunction $\langle \mathcal{A}, \mathcal{B}, \mathrm{F}, \mathrm{G}, \varphi, \gamma \rangle$ is isomorphic to the adjunction $\langle \mathcal{A}, \mathcal{A}_{\mathrm{F}\Gamma}, \mathrm{I}, \mathrm{FG}, \varphi, \mathrm{F}\gamma_{\mathrm{G}} \rangle$, where I is inclusion. This isomorphism is based on the functor F, which maps $\mathcal{B}$ isomorphically onto $\mathcal{A}_{\mathrm{F}\Gamma}$.

Take now the free comonad $\langle \mathcal{A}, \mathrm{D}, \varepsilon, \delta \rangle$ generated by the arrowless graph $\mathcal{G}$ with a single object C. We can show that this comonad is isomorphic to the comonad of the adjunction above by the comonofunctor that maps D to FG, ε to φ and δ to $\mathrm{F}\gamma_{\mathrm{G}}$. This isomorphism (which was considered from a 2-categorial point of view by [Auderset 1974] and [Schanuel & Street 1986]) is demonstrated most easily with the help of the links of § 5.9 and § 4.10.1. It suffices to correlate the links on the left-hand side with those on the right-hand side:



So the links of comonads of § 5.4 can replace the links of $\mathcal{A}$ of our free adjunction. For the links of $\mathcal{B}$ it suffices to note that they have an isomorphic copy in $\mathcal{A}_{\mathrm{F}\Gamma}$, and these links reduce again to the links of comonads. When our free adjunction is generated by an empty $\mathcal{G}$ and an arrowless $\mathcal{H}$ with a single object, we rely analogously on the links of monads, which are obtained by reversing the links of comonads. So the links of adjunctions of § 4.10.1 could be replaced by the links of comonads of § 5.9. We have nevertheless preferred to introduce the former links because the approach through them is more direct, and because of their connection with the graphs of [Eilenberg & Kelly 1966] and with the theory of tangles mentioned in § 4.10.1. By the same token, we could use the links of adjunctions to work with the free comonad.

The maximality of adjunction of § 4.11 can now be deduced from the maximality of comonad of § 5.10. Adding any further equality to the free adjunction makes idempotent the comonad of the adjunction, and this yields the preordering equalities. Our direct proof of the maximality of adjunction in § 4.11 shows however that we can obtain the same result by relying on syntactical methods and cut elimination. (Matters of this section are treated in [D. 2008].)

p. 195, line 14, replace "were" by "where"

p. 205, replace the last paragraph by:

Strong normalization for these reductions is a consequence of the fact that cuts in contracta either disappear or are of strictly smaller degree than cuts in the redexes, and of the fact that the degree of contracta of the last three kinds is strictly smaller than the degree of the corresponding redexes. More precisely, we can take as the complexity measure of an arrow term f a triple $\langle n_1, n_2, n_3\rangle$ where n_1 is the number of cuts in f that are not topmost; next, if $d_1,\ldots, d_k$ are the degrees of all the topmost cuts in f, then $n_2 = 3^{d_1}+\ldots+3^{d_k}$ (if there are no topmost cuts in f, and hence no cuts, then $n_2 = 0$), and n_3 is the degree of f. The triples $\langle n_1, n_2, n_3\rangle$ are lexicographically ordered by an order of type ω^3 with the definition $\langle n_1, n_2, n_3\rangle < \langle m_1, m_2, m_3\rangle$ iff $n_1 < m_1$ or ($n_1 = m_1$ and $n_2 < m_2$) or ($n_1 = m_1$ and $n_2 = m_2$ and $n_3 < m_3$). In the first two reductions, n_1 either decreases, or it is kept constant while n_2 decreases. In all other reductions n_1 is kept constant. It is clear that in the reductions corresponding to (K^{1a} 1), (K^{1a}), (K^{2a} 1) and (K^{2a}) the number n_2 decreases (while n_3 decreases or is kept constant, which is without importance). The number n_2 decreases in a (distr) reduction too, because if $d_1 < d$ and $d_2 < d$, then $3^{d_1}+3^{d_2} < 3^d$ (although d_1+d_2 might be strictly greater than d).

This is inspired by a trick of Gentzen from [1938]; the general fact is that if for every i such that $1 \leq i \leq k$ we have $m_i < m$, then $\Sigma^k_{i=1}(k+1)^{m_i} < (k+1)^m$. The fact that n_3 increases in a (distr) reduction is of no consequence. In the ($K^{1a}K^{2a}$), (K^{1a} distr) and (K^{2a} distr) reductions the number n_2 decreases or is kept constant while n_3 decreases. (We could have used a complexity measure of this kind for proving strong normalization in § 4.6.3 and § 5.8.3, but there we could do with a simpler kind; the complications we have here are due to the (distr) reductions.)

p. 207, replace the penultimate paragraph by:

We can also extend the foregoing to obtain a decision procedure for the commuting problem in the graph $\mathcal{A}^*$ of a free $\times$-category generated by an arbitrary graph. We have first to redefine normal form as having all cuts molecular, instead of being cut-free, besides lacking subterms of the special forms we had before. Right-normal form has all cuts right-molecular. Our collection of reductions now applies to topmost nonmolecular cuts and is enlarged by (cat 2 mol) reductions. To demonstrate strong normalization, the complexity measure of an arrow term f will be $\langle n_1, n_2, n_3, n_4\rangle$ where n_1, n_2 and n_3 are as above and n_4 is obtained as in the parenthetical remark after the introduction of (cat 2 mol) reductions in § 4.6.4. With every right parenthesis in a molecular subterm h of f we associate the number of cuts in h on the right of this parenthesis. Then n_4 is the sum of all these numbers; n_4 is zero for an arrow term in right-normal form. The quartets $\langle n_1, n_2, n_3, n_4\rangle$ are ordered lexicographically. The Church-Rosser property is established as before.

p. 211, line 9 bottom, delete "the end of"

p. 221 and later, add new references:

Auderset, C. [1974] Adjonctions et monades au niveau des 2-catégories, *Cahiers de Topologie et Géométrie Différentielle* vol. 15, pp. 3-20.

Došen, K. [2008] Simplicial endomorphisms, *Communications in Algebra* vol. 36, pp. 2681-2709 (available at: http://arXiv.org/math.GT/0301302).

Kassel, C. [1995] *Quantum Groups*, Springer, Berlin.

Kauffman, L.H., and Lins, S.L. [1994] *Temperley-Lieb Recoupling Theory and Invariants of 3-Manifolds*, Princeton University Press, Princeton.

Murasugi, K. [1996] *Knot Theory and its Applications*, Birkhäuser, Boston.

Pumplün, D. [1970] Eine Bemerkung über Monaden und adjungierte Funktoren, *Mathematische Annalen* vol. 185, pp. 329-337.

Schanuel, S., and Street, R. [1986] The free adjunction, *Cahiers de Topologie et Géométrie Différentielle Catégoriques* vol. 27, pp. 81-83.

p. 228, line 4 bottom, replace “4.14” by “4.1.4”

added in July 2013:

p. 114, line 18, after “*special* adjunction” add “(for references concerning this notion, which is called in various ways, see [Clark & Wisbauer 2011], 3.5 Remarks, and [Herrlich & Hušek 1990], section 4)“

p. 221 and later, add new references:

Clark, J., and Wisbauer, R. [2011] Idempotent monads and $*$-functors, *Journal of Pure and Applied Algebra* vol. 215, pp. 145-153 (available at arXiv)

Herrlich, H., and Hušek, M. [1990] Galois connections categorically, *Journal of Pure and Applied Algebra* vol. 68, pp. 165-180

I am grateful to Tatsuji Kawai for telling me about the second of these references.