SLAV ACHIEVEMENT
IN
ADVANCED SCIENCE

BY

DR BRANISLAV PETRONIEVICS

PROFESSOR AT THE UNIVERSITY OF BELGRADE

LONDON

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INTRODUCTORY

AFTER the ancient Greeks had founded science and philosophy, the modern nations have since the Renaissance assumed their heritage and continued their labours. Italy was the first among these new great powers of the mind; from the seventeenth century, France and England succeeded Italy; in the eighteenth, Germany was added to them; and in the latter half of the nineteenth, Russia came in as the last.

This last statement is sure to provoke doubts in many of my readers, many of whom have perhaps never heard Russia spoken of as a country whence have sprung men of science comparable to the greatest among the peoples of Western Europe. But this is not all. Not only Russia can boast of such men of science, but some are to be found even among the other Slav nations. The Pole Copernicus and the Jugoslav (Serbo-Croat) Boshcovic are the most illustrious instances.

It is true that, taken all in all, the contribution of the Slav world is not so great in the domain of science, and even less in that of philosophy, than that of the English, French, or Germans. But the Slav world is as yet a world of the future; and what I propose to show in this pamphlet is rather the capacity of the Slav mind for scientific and philosophic achievement of the highest importance.

But how are we to discriminate between these achievements of the highest importance and scientific work of an inferior order? What is certain is, that the difference exists, and that it is admitted by almost everybody. Galilei among the Italians,

Newton and Darwin among the English, Lavoisier, Carnot, and Descartes among the French, Leibniz, Gauss, and R. Mayer among the Germans, are names of men to whom no one would attribute merely the importance of ordinary scientific men. Why? Not only because they are the founders of new and well-founded scientific theories, and even of entire sciences, but also, and chiefly, because the theories and sciences founded by them possess an importance which is greatly superior to that of the scientific theories of ordinary scientific men. We find this importance in the *philosophic value* of their theories, in the possibility of deducing from each of these theories immediate results, which touch upon the gravest problems of the human mind, upon the problems which, taken in their entirety, constitute the riddle of the universe. The men of science themselves, the founders of theories of this type, are only rarely conscious of this superscientific value of their work, and in the majority of cases it is better that this should be so. Because science and philosophy are very distinct from each other, the latter being the supreme synthesis of the facts of experience, which the synthesis of science—however generalised—can never include otherwise than partially.

One might, it is true, consider also the extent and the practical value of scientific labour as the criterion; but it is obvious at once that this criterion, because of its manifest relativeness, cannot be considered sufficient to distinguish between a Newton and a Wundt.

The philosophical importance of their scientific theories being the ultimate criterion applied to discriminate between men of science of the highest order and others less important, we will apply this severe criterion to Slav science, and ask whether the Slav world has produced men of science so great that they can compare with Newton, Archimedes, etc.

The answer to this question is decidedly in the affirmative. But the names of the greatest Slav men of science are either well known, without its being known that they belong to the Slav world, or they are very little known. Of course, they are known to the limited circles of their respective sciences, but they are almost unknown to the great civilised public, and even to the men

of science outside their own branch, and this is the case not only in the countries of our enemies, but also in those of our allies.

Among the numerous scientific men of the Slav world the four following are beyond all doubt men of science of the first order, in the sense I have indicated, viz.: the Pole Nicolas Copernicus, the Russians Dimitrije Mendeljew and Nikolay Lobatchevski, and the Serbo-Croat Rogerus (Rudjer) Boshcovic. A brief sketch of the life and work of each of these four, given in the chronological order of their appearance, will surely not be without interest.

I

NICOLAS COPERNICUS

De revolutionibus orbium cœlestium, libri vi., Norimbergæ, 1543.

COPERNICUS was born on February 19, 1473, in the city of Thorn, which was then in Poland. His father, Niklas Kopernigk, was a native of Cracow, who settled in Thorn in 1460, where he married Barbara Watzelrode, the mother of Copernicus. Copernicus lost his father while he was yet a child, and his uncle Lucas Watzelrode, Bishop of Ermeland, became a second father to him. In 1491 he was sent to the University of Cracow to study medicine. But, even while preparing to take his medical degree, he took up the study of philosophy and mathematics, the latter under the celebrated Alb. Brudzewski. After taking his degree he spent a short time in Thorn, and then travelled to Italy, where he visited Padua, Bologna, and finally Rome. During his sojourn in Italy he made astronomical observations, and it was there that he definitely found his vocation as an astronomer. He remained in Rome for seven years, as professor of mathematics at the University of Rome. On his return to his native country, Copernicus entered the Church and became a canon in Frauenburg. There he found a peaceful life, dividing his time between his medical profession, which he practised for the benefit of the poor, and his astronomical studies. The slow but fertile result of these studies was his immortal work, *De*

revolutionibus orbium cœlestium, which was published in the very year of his death, in 1543, by his pupil, J. Ræticus.

At the time of his death he was known only to a small circle of scientific men of his day, who knew of his doctrine, indirectly, even before the appearance of his book, Copernicus having communicated his discovery to his intimate friends. Both Poland and Europe were, as Flammarion remarks, at the time too distracted by wars and the religious conflicts of the Reformation to take notice of the man who was to play one of the greatest parts in the intellectual development of mankind. His native country of Poland remembered him gratefully only after the lapse of several centuries, by dedicating a monument to him in Warsaw, the magnificent statue by Thorwaldsen, which was unveiled on May 5, 1829.

Copernicus' character has been admirably described by Bertrand: "For us Copernicus is all contained in his book. His private life is little known. What is known, gives the impression of a firm, but prudent man, with an absolutely upright character, altogether devoted to his speculations, and, as if wrapped up in himself, he loved peace, solitude and silence. Simple and sincere in his piety, he could never understand how truth could endanger faith, and he always reserved the right for himself to seek for it, and to believe it. No passion troubled his life . . .; a foe of unprofitable discussions, he sought neither praise, nor the noise of fame; independent without pride, content with his fate and content with himself, he was great without glitter. . . ."

The great reform in our knowledge of the universe brought about by Copernicus is known to all the world. He reversed the theory of the immobility of the earth, which is so obvious to the evidence of the senses and had been elaborated and placed on a scientific basis by the great scientific men of the Old World, Aristotle and Ptolemy, and counted among its adherents even greater scholars, such as Hipparchus and Archimedes. Finally, it was the doctrine of the Church, the supreme spiritual power of that age.

In rejecting this theory, and substituting for it the theory of the earth's daily movement about its own axis, Copernicus,

as he himself acknowledged, was following in the footsteps of the ancient Pythagoreans. His second principal theory, however, that of the earth's movement around the sun together with all the other planets, was almost entirely evolved by himself.

But with these two theories, which were altogether new to his age, Copernicus combined in his system many of the old ideas of the Ptolemaic system. For him, as for Aristotle and Ptolemy, the cosmos was finite in space, terminating in the immovable sphere of the fixed stars, which receive their light from the sun, which he proclaimed the immovable centre of the entire cosmos. He also retained to a certain extent the epicyclic and excentric circles, that great encumbrance of the older system.

But in spite of these imperfections, which have since been eliminated during the subsequent evolution of modern astronomy (Galilei, Kepler, Newton), the new world system remains the creation of Copernicus. "Kepler and Newton," says Bertrand, "have penetrated far deeper into the mysteries of the movements of the heavenly bodies; but it is Copernicus who gave them the key, and even to-day, after their immortal labours, the true world system is called the Copernican system."

No one has expressed the greatness of the revolution inaugurated by Copernicus in more eloquent fashion than Bailly: "We are to forget the movement we see, and to believe in one which we do not feel. It is one man alone who dares to propose it, and all this in order to substitute a certain probability of the mind, felt by a small number of philosophers, for that of the senses, by which the multitudes are carried away. This is not all: he had to destroy an accepted system, approved by three-fourths of the world, and overthrow the throne of Ptolemy, who had received the homage of fourteen centuries."

Poggendorf says: "Copernicus is and remains a bright luminary in the firmament of science."

The originality of his theory, in spite of his Greek predecessors, has been thus summarised by Delambre: "Finally, if I admit, in spite of the universal silence of all their writers, and against my inmost conviction, that the ancients possessed these ideas, it is at least incontestable that not a vestige of them

was preserved. Copernicus was obliged to imagine them anew. His system is his very own: for us this system is not that of Philolaus, nor of Aristarchus, whose writings have not come down to us; it is that of Copernicus, who deserves to have his name attached to it, by the pains he has taken to explain all its parts, and to make it account for all the phenomena we observe."

And Herder, who considers that Copernicus has done more for philosophy with his system than all the Greek schools with their dialectics, expresses himself as follows in his *Philosophy of the History of Mankind*: "It is in the Heavens that our philosophy of the history of the human race must begin, if it is in any way to be worthy of the name. . . . Invisible, eternal bonds link the earth with the sun, the centre whence it derives light, heat, life, and fruitfulness. Without the sun, we cannot conceive our planetary system, any more than one can imagine a circumference without a centre. . . . Nothing offers so sublime an aspect as the spectacle of this great world structure; and never perhaps has human reason taken a bolder and happier stride, than when Copernicus, Kepler, Newton, Huyghens, and Kant discovered and established the simple, eternal, and perfect laws of the formation and motion of the planets."

The intellectual revolution effected by Copernicus is psychologically the greatest possible; he had to oppose his scientific opinion not only to another scientific opinion, but to a scientific opinion which was at the same time the opinion of all the world. No other intellectual revolution, save that of Descartes—who for the first time affirmed the subjectivity of the sensations, contrary to the conviction of everybody,—is comparable to that of Copernicus. Therefore it is not strange that his achievement is looked upon as an intellectual revolution in the highest degree, and that all other similar revolutions are measured by the standard of that of Copernicus. Thus, Du Bois-Reymond, speaking of the great intellectual revolution effected by Darwin's theory of Evolution, could not express his lofty admiration of Darwin otherwise than by saying: "For me Darwin is the Copernicus of the organic world." Finally, Copernicus' discovery is philosophically of capital importance. How can mankind

hope to attain the high goal towards which all its efforts must in a last analysis be directed—the goal of solving the great Riddle of the Universe—without first knowing the physical structure of the cosmos? And it is Copernicus who has laid the correct foundation of this preliminary knowledge, which is indispensable to all higher speculation of the human mind.

The Slav spirit has produced in Copernicus one of humanity's highest contributions to science, and in him the Slav race has shown an intellectual capacity equal to that of any other race in the world.

II

ROGERUS JOSEF BOSHCOVIC

De viribus vivis, 1745.
De materiæ divisibilitate et principiis corporum dissertatio, 1748.
De continuitatis lege et eius consectariis pertinentibus ad prima materiæ elementa eorumque vires dissertatio, 1754.
Theoria philosophiæ naturalis redacta ad unicam legem virium in natura existentium, 1758 (other editions 1759, 1763, 1764, 1765).
Elementorum universæ Matheseos, t. i., 1752.
Trigonometria plana et spherica, 1761.
Opera pertinentia ad Opticam et Astronomiam, 5 vols., 1785.
Stay B. Philosophiæ recentioris versibus traditæ libri X., cum adnotationibus et supplementis R. J. B., 1755.

ROGERUS JOSEF BOSHCOVIC (Rudjer Josif Boshcovic in Serbo-Croatian) was born in Ragusa (Dubrovnik) in South Dalmatia on September 18, 1711. Boshcovic's father, Nikola, was a native of Hercegovina, and an Orthodox Serb, who became a Catholic on settling in Ragusa. His mother belonged to the Italian family of Betere, which had been settled in Ragusa for nearly a century.[1]

Having completed his primary and higher school education at the Jesuit Grammar School of his native city, he entered the Society of Jesus in 1725, and was sent to Rome to continue his studies. There in the Collegium Romanum he studied philosophy and physico-mathematics until 1733; after this he

[1] How little is known of our Boshcovic's nationality may be gathered from the biographical note devoted to him in the *Encyclopædia Britannica*, 11th ed., 1910, which says: "Boskovich, Rogerus Josef, Italian mathematician and natural philosopher. . . ."

spent five years as a teacher of languages and poetics at various schools, but subsequently became professor of mathematics at the Collegium Romanum itself. His literary activity began in 1736 with his scientific treatise in verse, *De Solis et Lunæ Defectibus*, and almost every year after this he published scientific treatises on various mathematical, physical, and astronomical subjects. In 1744 he became a priest; in 1756 we find him sent on a mission of arbitration to Vienna.

Boshcovic soon achieved a considerable reputation in the scientific world by his researches, and in 1759 he was already fellow of several scientific societies, such as the Royal Society in London, the Academy of Science in Petrograd, etc. That very year he left Rome and spent several years in travelling from one town to another. In 1760 we find him in Paris, but as a Jesuit he did not feel at home in this free-thinking and anti-clerical city. That same year he proceeded to London, where he was most cordially received. In 1761 he was sent by the Royal Society to Constantinople to observe the transit of Venus from there. From Constantinople he returned to Rome in 1763, traversing Bulgaria, Roumania, and Poland on his journey. The years from 1764 to 1773 he spent in Italy as professor at the University of Pavia, and director of the Observatory in Milan. The order of the Jesuits having been suppressed by a papal decree in 1773, Boshcovic, now a free agent, went that very year to settle in Paris, where he became naturalised and obtained a Government appointment. He remained in Paris until 1782. In 1782 he returned to Italy, where he remained until his death on February 13, 1787.

In 1745 Boshcovic published his first philosophical treatise, *De viribus vivis*, in which he for the first time put forward his new theory of matter. In 1754 he published a second, more detailed treatise on the same subject under the title *De Continuitatis Lege et Consectariis Pertinentibus ad prima Materiæ Elementa eorumque Vires*. In 1755 and 1757 he published further treatises on the same subject; and finally, in 1758, his chief work appeared, his *Theoria Philosophiæ Naturalis redacta ad unicam Legem Virium in Natura existentium*. This work passed through several editions, in 1759, 1763, 1764, and 1765.

We will not speak here of the extensive labours of Father Boshcovic in the domains of physics, astronomy, and mathematics. His importance is far greater in the domain of natural philosophy, where he occupies a foremost place by his original theory of Matter. We will confine ourselves to a brief exposition of the principal points of this immortal theory.

There are three principal points in Boshcovic's atomic theory :

1. The ultimate elements of matter, the atoms, are real indivisible points ;

2. The atoms are centres of force ; and

3. Force varies both qualitatively and quantitatively in proportion to distance.

The first two of these three points Boshcovic deduces from the same fact of experience, the contact of bodies. This deduction is based on the following natural laws, which Boshcovic regards as proved, viz. : the Law of Continuity and the Law of the Impenetrability of Bodies.

The Law of Continuity, proclaimed for the first time in its completeness by Leibniz, says that a given quantity, passing from a given value to another, must pass through all the intermediate values. According to Boshcovic, geometrical space, time, and motion obey this law.

The second law, recognised almost as an axiom in physical science, says that two bodies cannot simultaneously occupy the same point in space.

Let us now assume two inelastic bodies, A and B, travelling in the same direction, A with a velocity of 12, and B with a velocity of 6 per second. After a certain time the first body will come in contact with the second, and after their collision they will continue their course with an equal mean velocity of 9 per second, the first having lost as much of its initial velocity as the latter will have gained. When did this equalisation of their velocities take place? It is usually supposed to take place at the moment of contact, but Boshcovic asserts that this supposition is contradictory and impossible.

His line of argument is, simplified, as follows (and the same argument applies also to elastic bodies) :—

One must assume that the equalisation of the two velocities

during contact takes place either in a single indivisible instant of time, or during a very short space of time.

In the first case (*cf.* fig. 1) the first body, A, must reduce its velocity from 12 to 9, and the second, B, increase its velocity from 6 to 9, abruptly and without their velocities passing through the intermediate stages 8, 7, etc. The Law of Continuity would be therefore violated. In the second case (fig. 2), the front of A would have to penetrate the rear of B, which would be against the Law of Impenetrability. It is therefore impossible to suppose that the equalisation of velocities takes place during the actual contact of the two bodies.

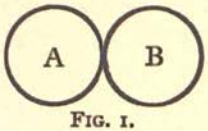

FIG. 1.

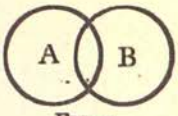

FIG. 2.

Granted this impossibility, it is necessary to suppose that it takes place *before* the two bodies come into contact. And in order that contact may never become possible, it is further necessary to assume a force acting at a distance *between* the two bodies and preventing them from approaching each other so closely as to touch each other. And this force must obviously be repulsive, and, by the same argument, it is bound to increase with the reduction of the distance between the bodies, until it becomes infinitely great when the distance becomes infinitely small.

Such being the nature of the repulsive force acting between the bodies, two important propositions result from this, viz. :

1. The ultimate elements of matter must be *simple* points. Because if we assume them to be composite (like the corpuscules of Descartes and Newton), their component parts could not remain coherent, as the repulsive force acting between them would separate them one from the other.

2. The simple atoms of matter must be conceived as being kept separate in space by the repulsive forces which actually dwell in themselves. They are therefore the centres of these forces.

But experience shows us that the forces acting between bodies are not only repulsive forces. There are also forces of attraction, such as the cohesion between the molecules of bodies and the Newtonian gravitation acting between visible bodies.

Boshcovic is of opinion that it is not necessary to conceive these forces as being qualitatively irreducible one to the other; he conceives them all as different forms of one and the same force, and assumes that this one force varies, not only quantitatively (as Newton assumed), but also qualitatively, according to distance. He assumes that in minimal distances between atoms it is at first repulsive; that it changes its nature when the distance between the atoms reaches a certain definite limit; that it again changes its nature several times as the distance increases; and that, finally, for visible distances it becomes Newton's force of attraction. Boshcovic represents this general law of force by a special curve, known as Boshcovic's curve.

The impossibility of a further qualitative change of the force is enforced by Boshcovic by his theory of finiteness of discrete magnitude. Whatever is discrete cannot be infinite, the infinite number not being possible. The atoms being simple points, separated by intervals of space, their number can only be finite. And existent space itself, being conditioned by the forces emanating from the atoms, must likewise be finite. Regarded as an abstract possibility, as it is in geometry, space is infinite, *i.e.* it can be produced indefinitely; but existent space can only be finite.

Boshcovic puts forward his atomic theory as the synthesis of Newton's and Leibniz's theories of matter. His theory coincides with Newton's in the idea of force acting at a distance, and with Leibniz's in the idea of simple atoms. But the deduction of this synthesis is Boshcovic's very own work, carried out in a manner absolutely original, ingenious, and profound.

The scientific value of Boshcovic's theory is twofold. It is, first of all, of considerable historical value. One of the most eminent among modern historians of philosophy declared his principal book for "the principal work of the philosophy of nature in his epoch" (*cf.* E. Cassirer, *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*, vol. ii., 2nd ed., 1911, p. 506). Its contemporary scientific value is proved by the discussion of his theory by modern physicists such as Sir William Thomson (Lord Kelvin) and J. J. Thomson.

In his book *Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light* (London, 1904) Lord Kelvin several times alludes to Boshcovic's theory. On p. 285 he says: "This mutual action (called force) is different at different distances, so as to fulfil some definite law. If the particles were hard elastic globes, acting upon one another only by contact, the law of force would be zero force and infinite repulsion. This hypothesis, with its hard and fast demarcation between no force and infinite force, seems to require mitigation. Boscovich's theory supplies clearly the needed mitigation." And on p. 556, speaking of the explanation of chemical phenomena by the hypothesis of electrons, Kelvin says: "and as we are assuming the electrons to be all alike, we must fall back on Father Boskovich, and require him to explain the difference of quality of different chemical substances, by different laws of force between the different atoms."

Finally, on p. 675 he says: "The accompanying diagram, fig. 6, copied from fig. 1 of Boscovich's great book" . . . (*cf.* also pp. 653, 123, 125, 131, and 668).

J. J. Thomson, in his book *The Corpuscular Theory of Matter* (London, 1907, p. 160), applies to Boshcovic's theory in his theory of ions.

But the philosophic value of Boshcovic's theory is even far greater. The importance of a philosophical theory cannot be judged in the same way as that of a scientific theory. A scientific theory must be verified by experience in order to be considered important; such verification not being possible in the case of philosophical theories, and the riddle of the universe being as yet unsolved, all we can demand of philosophical theories is that they should be consistently developed and that they should represent typical possibilities of explanation. Now, Boshcovic's theory fulfils these conditions as perfectly as any other of the great philosophic theories, such as Leibniz's monadic theory, Spinoza's theory of substance, Hegel's theory of the evolution of concepts, Schopenhauer's pessimistic theory, etc. The philosophic importance of Boshcovic's theory has been fully recognised by G. Th. Fechner, who, in the second edition of his book *Ueber die physikalische und philosophische Atomenlehre* (Leipzig, 1864), quoted long extracts from Boshcovic's principal work, and declared

him to be the first who had clearly conceived the idea of simple dynamic atoms.

By this theory Boshcovic placed himself among the boldest minds humanity has produced. As he belongs to the Jugoslav branch of the great Slav family, he is a proof that the Slav race in all its branches exhibits those qualities of the mind which are needful for the attainment of the highest degree of achievement in science.

III

NIKOLAY IVANOVITCH LOBATCHEVSKI

O načalach geometrii ("On the Foundations of Geometry"), in *Kasansky Vestnik*, 1829–30.

Voobražemaja geometrija ("Imaginary Geometry"), Kasan, 1835.

Novija načala geometrii s polnoj teoriej paralelnih ("New Foundations of Geometry, with a Complete Theory of Parallels"), Kasan, 1835–38.

Geometrische Untersuchungen zur Theorie der Parallellinien, Berlin, 1840.

Pangéométrie, ou précis de géométrie fondée sur une théorie générale et rigoureuse des Parallèles, Kasan, 1856.

Polnoe sobranie sočinjenii po geometrii N. I. Lobatchewskago ("Complete Collection of the Geometrical Works of N. I. Lobatschewsky"), vols. i–ii., Kasan, 1883.

NIKOLAY IVANOVITCH LOBATCHEVSKI was born in Nijni Novgorod on October 22, 1793. His father, an architect, died when Nikolay was four years old. After his father's death, Lobatchevski's mother settled in Kazan, where Nikolay entered the High School in 1802; in 1807 he matriculated at the University of Kazan as student of mathematics. His professor in this science was Bartels, a German, and former student under Gauss in Göttingen. As a student, Lobatchevski was rather unruly, and, on taking his degree in 1811, he was requested to promise to amend his ways in future. In 1812 he was appointed assistant lecturer at the University. In 1816 he was appointed extraordinary professor. He had to lecture on mathematics, astronomy, and physics, and only later on returned to mathematics alone. In 1827 he was appointed Rector of the University of Kazan, which post he held for nineteen years, until 1846. From 1847 to 1855 he acted as Deputy-Curator of the University. He died on February 12, 1856.

In 1826 Lobatchevski read his first paper on non-Euclidean geometry, entitled "A Succinct Exposition of the Foundations of Geometry," before the physico-mathematical faculty of the University of Kazan. This paper was never published. Not before 1829–30 did he publish—in the "Kazan Messenger"—his first treatise under the title of "On the Foundations of Geometry," in which he propounded his new teaching in propositions without detailed proof. But in 1835–38 he published his second treatise, "New Foundations of Geometry with a Complete Theory of Parallels," in *Scientific Proceedings of the University of Kazan.* This treatise represents a systematic and almost complete exposition of the new geometry. In 1840 he published a pamphlet in German under the title *Geometrische Untersuchungen zur Theorie der Parallellinien*, which is now recognised as the classic introduction to non-Euclidean geometry.[1] Finally, in 1856, his last book on non-Euclidean geometry, entitled *Pangéométrie*, was published in French.

During his lifetime Lobatchevski remained almost entirely unknown. His geometry remained wholly without recognition in his native country, and abroad it was only the great mathematician Gauss who honoured him by expressing approval of the new doctrine to him in a letter, and by furthering his election as a corresponding member of the Scientific Society of Göttingen. But when, after the death of Gauss, his correspondence with his friend Schumacher was published, the amazed mathematical world heard, for the first time, the name of the great Russian mathematician. After the labours of Riemann, Belfrani, Helmholtz, etc., the new theory was finally recognised, and when, in 1893, the first centenary of Lobatchevski's birth came to be celebrated, it was possible by international subscription to dedicate a double monument to his memory—a statue and a prize. The statue was unveiled in 1896, and the Lobatchevski prize was awarded for the first time in 1897.[2]

[1] This paper was translated into French by Hoüel in 1866, into English by G. B. Halsted, and published by the Open Court Publishing Company (London, new ed. 1914), etc. I myself have translated it into Serbian in 1914, and furnished it with a detailed commentary.

[2] It should be mentioned here that non-Euclidean geometry was discovered independently, and almost at the same time that Lobatchevski discovered it,

Lobatchevski's great discovery is the non-Euclidean geometry. In his celebrated work, *Elements of Geometry*, Euclid, the great Greek geometer of the Alexandrine period, included the following proposition among those he could not prove (the postulates):

Through a point lying outside a given straight line, one, and only one, straight line can be drawn parallel to the given straight line.[1]

Attempts to prove the proposition—known as the fifth postulate, or the eleventh axiom of Euclid—have been made from the days of Euclid (two thousand years ago) down to Lobatchevski, and all these countless attempts have remained vain and fruitless. Why they have been vain, and why they were bound to remain so, nobody before Lobatchevski ever knew. Having himself made several attempts to prove the proposition, Lobatchevski was the first who had the intellectual courage to put the following question to himself: Is it not possible that this proposition is unprovable, because it is *not* the sole possible? Having put the question thus, Lobatchevski answered it positively by showing, that it is possible to evolve an entire geometry by starting from a postulate which is in itself the negation of Euclid's postulate, and that there is nothing contradictory in the propositions of such a system. Non-Euclidean geometry is therefore logically possible.

In order to explain this new postulate of Lobatchevski and the difference between it and Euclid's postulate, we will turn to fig. 3 (see p. 20).

In this figure we have the point M lying outside the straight line AB, and MO ⊥ AB. According to Euclid's postulate the only parallel to AB, that can be drawn through M, is the line CD; all other lines passing through M must intersect the line AB at one point. But we can also suppose that the line CD, which forms a right angle with the line MO, is *not* the only line which does not intersect the line AB, that, *e.g.*, the line MD'''

by the Hungarian mathematician, J. Bolayi. The latter published his discovery in the *Appendix scientiam spatii absolute veram exhibens*, 1832. I have devoted a comparative study to the two inventors of non-Euclidean geometry as a part of a treatise "On Simultaneous Discoverers," which will shortly appear.

[1] Euclid's actual postulate is not identical with this proposition, but it is equivalent to it.

does not intersect it either. In passing from the lines which intersect the line AB—such as MB′—to the lines which do not intersect it—such as MD‴—it is obvious that we must pass through a line—MD′—which marks the limit between the intersecting and non-intersecting lines, and that all the lines between this line and the perpendicular MO, starting from the point M, must intersect the line AB. This boundary line MD′ will therefore be parallel to AB.

On the other hand, we obviously find the same conditions on the other side of the perpendicular MO, *i.e.* one straight line—MC′—which does not intersect the line AB, and which

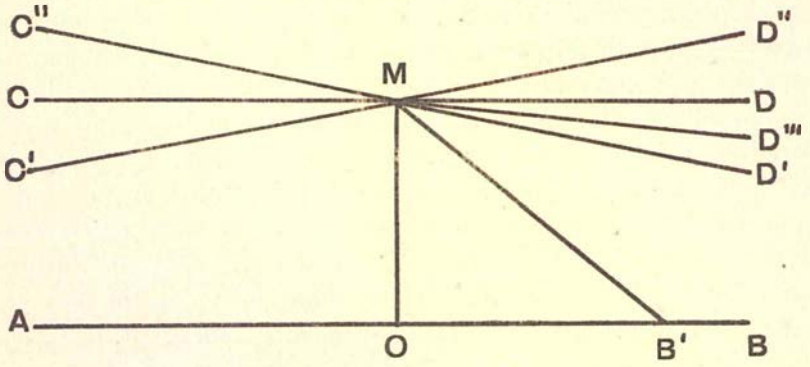

FIG. 3.

is the limit of all straight lines drawn from the point M within the angle C′MO. If we produce the two lines MD′ and MC′ beyond M, MC′ becoming C′D″ and MD′ becoming C″D′, we shall have three species of straight lines passing through the point M, viz. :—

1. An infinite number of non-intersecting lines lying between C′D″ and C″D′;

2. An infinite number of intersecting lines on both sides of the perpendicular MO, lying between the lines MC′ and MD′, and which, when produced, will lie between MC″ and MD″; and

3. The two parallels C′D″ and C″D′.

Lobatchevski then demonstrates that the non-intersecting lines under (1) diverge indefinitely from the line AB, if they are produced in both directions. They are, therefore, *non-inter-*

secting diverging lines. Contrarily, the two parallels C'D'' and C''D' converge indefinitely towards the line AB, if they are produced on that side of the perpendicular MO, where they represent the limit between the intersecting and non-intersecting lines; they are therefore, in one given direction, ***non-intersecting converging*** lines. Being by their definition parallels to the line AB, it is necessary in non-Euclidean geometry to distinguish the ***sense of parallelism*** in these parallels. The parallelism of Lobatchevski's two parallels is such that they stand in the relation of ***asymptotic lines*** to the line AB.

Among the other theorems of non-Euclidean geometry demonstrated by Lobatchevski, the following are the most important:—

1. The sum of the three angles of a rectilinear triangle is less than two right angles (<2 R).

2. There is in non-Euclidean space a surface—the orisphere or boundary surface—in which Euclidean geometry is valid.

3. There is in the non-Euclidean plan a distance x between the two arcs s and s' of two boundary lines for which $\frac{s}{s'}=e^x$, where $e=2{\cdot}718\ldots$, the base of natural logarithms; this distance is the absolute unit of length in the non-Euclidean plan.

4. There are no similar figures in the non-Euclidean plan.

5. The equidistant line to a straight line (CD in fig. 3) is in the non-Euclidean plan a curved line.

6. The area of a rectilinear triangle cannot, in the non-Euclidean plan, exceed a certain fixed value.

In no part of his writings does Lobatchevski expressly say that the non-Euclidean plan is a curved surface, but it is quite certain that he regarded it as such. The researches of subsequent geometricians have shown that Lobatchevski's surface is a curved surface with a constant negative curvature, while the Euclidean plan is a surface whose curvature is zero, and the spherical plan—a new non-Euclidean plan introduced by Riemann—a surface with a constant positive curvature. Lobatchevski's and Euclid's plans have the following two properties in common, which are lacking to Riemann's spherical plan:—

1. Two points always determine *one* straight line, the line of the shortest distance between the two points; and

2. A straight line can be produced indefinitely in both directions.

Lobatchevski's plan is therefore a surface as completely homogeneous and infinite as Euclid's. But, whereas Riemann's plan can be constructed in Euclidean space (it is the surface area of the sphere), Lobatchevski's plan cannot be constructed in Euclidean space.

The scientific importance of Lobatchevski's theory is twofold. First of all, it has given rise to a geometric theory of purely mathematical importance. As Lobatchevski himself foresaw, his geometrical theory has proved most fruitful in mathematical analysis. (Some of the work of the great French mathematician Poincaré has its source in it.) Next, the question of the geometric structure of existent space became imminent. Lobatchevski himself took up this problem, turning to astronomical distances in order to decide the question, whether the sum of the angles in a triangle in our space is exactly equal to two right angles or less. While admitting the possibility that the Euclidean hypothesis ceases to apply to astronomical distances exceeding the dimensions of our visible cosmos, yet he did not consider the supposition probable that magnitudes so "disparate as angles and lines could be dependent one upon the other." For him, then, the Euclidean structure of existent space was more probable than the non-Euclidean structure.

But his numerous successors are not of his opinion. Since the recognition of non-Euclidean geometry, the question of the geometric structure of actual space has been so much discussed by mathematicians, physicists, and philosophers, that a very considerable literature has grown up on the subject.[1]

The philosophic importance of Lobatchevski's discovery is likewise twofold. By it the field of geometry has been greatly

[1] An almost complete bibliography of this literature up to 1911, comprising about 4000 titles, has been published by D. M. Y. Sommerville (*Bibliography of non-Euclidean Geometry, including the Theory of Parallels, the Foundations of Geometry, and Space of n Dimensions*, London, 1911).

enlarged, and a number of new geometrics, which the inventor of the first among them could not foresee, have resulted from it. As the result of these new geometrics arose the problem of their logical connection from their first principles onward, a problem of which the great German mathematician D. Hilbert has suggested a rather provisional solution.

But more important than this purely logical question is the question of the geometric structure of existent space, as already referred to. Besides its purely scientific importance, this question is of capital importance for philosophy. How can philosophy hope to resolve the great riddle of the universe without having previously established the true geometrical nature of existent space? And the non-Euclidean geometry, because of the absolute homogeneity of Lobatchevski's typical space, is the first hypothesis to be examined in this respect besides that of Euclid.

Among the opinions on the importance of Lobatchevski's discovery, I will quote some of the best known. Gauss, in a letter written in 1846 to his friend Schumacher, said in reference to Lobatchevski's paper, *Geometrische Untersuchungen zur Theorie der Parallellinien*: "You know that I have held the same conviction for the last fifty-four years (since 1792). . . . Thus I did not find anything materially new to me in Lobatchevski's work, but the development is made in a way different from that which I have taken myself, and in a masterly manner by Lobatchevski in the true geometrical spirit. I feel in duty bound to call your attention to the book, which is sure to afford you exquisite pleasure." Comparing Lobatchevski with Bolayi, the second inventor of the non-Euclidean geometry, Fr. Engel, Lobatchevski's German translator and commentator, says on his principal work, *New Foundations of Geometry*: "One cannot but describe the *New Foundations* as a truly masterly achievement, for, although one may not deny that they have their shortcomings, yet it would be equally wrong to attach special weight to these shortcomings." And the English mathematician Clifford has compared Lobatchevski's geometrical revolution with the astronomical revolution of Copernicus: "What Vesalius was to Galen, what Copernicus was to Ptolemy, that was Lobatchevski to Euclid."

That the Slav race could produce so bold and so original a mind, which, without precursors, was the first to have the intellectual courage to call in question one of the cardinal points of Euclid's immortal edifice, is surely an obvious proof of the high intellectual capacity of the Slav race.

IV

DIMITRIJE IVANOVITCH MENDELJEW

"The Relations between the Properties of the Elements and their Atomic Weights," in *Journal of the Russian Chemical Society*, Petrograd, 1869.

"The Natural System of Chemical Elements," in *Journal of the Russian Chemical Society*, Petrograd, 1871.

"Die periodische Gesetzmässigkeit der chemischen Elemente," in *Annalen der Chemie und Pharmacie*, viii., Supplementband, 1872.

"The Periodic Law of the Chemical Elements," Faraday Lecture, in *Transactions of Chemical Society*, vol. lv., London, 1889.

Osnowi Chimii, 1st ed., Petrograd, 1869–71; 8th ed., 1908 (*Principles of Chemistry*, English translation, 3rd ed., London, 1905).

D. I. MENDELJEW was born on January 27, 1834, at Tobolsk in Siberia, where his father, a great Russian, was headmaster of the local gymnasium. His mother was a native of Tobolsk, where her people had been settled for a century. Mendeljew lost his father at an early age, and from that time the undivided care of his clever mother was devoted to his education. It was she who, in 1850, took him to Petrograd, where he was entered in the physico-mathematical faculty of the Pædagogic Institute. From 1853 to 1856 Mendeljew went to the Crimea to restore his health, and, having regained it, he became professor at the Odessa High School. From 1856 to 1859 he lived in Petrograd, where he wrote several chemical monographs. In 1859 he was sent abroad by the Government to complete his chemical studies. Mendeljew studied first under Renaud in Paris, and subsequently in Heidelberg. Upon his return to Petrograd, he became lecturer at the University of Petrograd after taking his doctor's degree; in 1866 he was appointed professor in ordinary of chemistry at the same University.

On March 6, 1869, Mendeljew communicated to the Russian Chemical Society his first treatise on the Periodic Law of chemical

elements ("Essay on a System of Elements"), which was published that same year in the journal of this society. This first paper was followed by a second, written in Russian, in 1870, and published in the same journal. At the same time the Periodic Law was proclaimed and made the basis of inorganic chemistry by Mendeljew in his celebrated book, *Principles of Chemistry* (1st ed., 1869–70, 8th ed., 1908), which has been translated into all the principal languages of Europe. Finally, in 1872, Mendeljew published his elemental theory in its final form in a masterly thesis written in German for *Loebig's Annalen der Chemie und Pharmacie*. In 1890 Mendeljew resigned his professorship at the University because of a difference with the Minister of Public Instruction, but in 1893 Witte appointed him director of the Institute of Weights and Measures, where he remained until his death on January 20, 1907. Strange to say, he was never made Member of the Academy of Science of Petrograd, even when his reputation had become world-wide. In 1882 the Royal Society of London conferred the Davy Gold Medal upon him (simultaneously with Lothar Meyer)[1] for his discovery, and in 1889 the Faraday Medal; in 1890 he was mad eFellow of the Royal Society.

Mendeljew's great discovery is the Periodic Law of chemical elements. This law can be formulated as follows: *The properties of simple substances are the periodic functions of their atomic weights.* If we arrange the elements in a series according to the magnitude of their atomic weight, we shall find that in this series there is always after a certain number of elements an element with properties identical with the properties of a previous element in the series. Let us, for instance, take the first fourteen elements of the whole series (leaving out hydrogen, H = 1):—

Li	Be	B	C	N	O	F
7	9	11	12	14	16	19
Na	Mg	Al	Si	P	S	Cl
23	24	27	28	31	32	35·5

[1] Lothar Meyer made the same discovery as Mendeljew, and almost at the same time. In my study already referred to on "Simultaneous Discoverers," I have also devoted a special chapter to the two discoverers of the Periodic Law of chemical elements.

and we shall see that in this series commencing with Li, with an atomic weight of 7, we have first the elements Be, B, etc., whose properties differ greatly from those of lithium, and differ more in proportion to the increase of their atomic weight, so that Li and F are elements of almost completely opposite properties. But in passing from F to Na, we come to an element whose properties are almost the same as those of Li, and, proceeding from Na, the elements Mg, Al, etc., resemble Be, B, etc., in exactly the same way as Na resembles Li. And this periodicity of properties of the elements runs through the entire series of the known elements.

The sub-series Li, Be, B, C, N, O, F, containing dissimilar elements, is called by Mendeljew *one period*, and the whole number of similar elements, such as Li, Na, etc., or Be[a], Mg, etc., he calls a *group*, or *natural family* of elements. Mendeljew has shown that the entire series of elements can be arranged in two different ways into periods and families. These two different ways represent two different periodic systems. In the former system, which is the better known, and to be found everywhere in all chemical text-books, the elements are arranged in twelve periods and eight groups. This is the *System of Small Periods* (*cf.* table).

Periods.	Group I. ... R^2O	Group II. ... RO	Group III. ... R^2O^3	Group IV. RH^4 RO^2	Group V. RH^3 R^2O^5	Group VI. RH^2 RO^3	Group VII. RH R^2O^7	Group VIII. ... RO^4
1	H=1							
2	Li=7	Be=9·4	B=11	C=12	N=14	O=16	F=19	
3	Na=23	Mg=24	Al=27·3	Si=28	P=31	S=32	Cl=35·5	Fe=56, Co=59,
4	K=39	Ca=40	...=44	Ti=48	V=51	Cr=52	Mn=55	Ni=59, Cu=63
5	(Cu=63)	Zn=65	...=68	...=72	As=75	Se=78	Br=80	Ru=104, Rh=104,
6	Rb=85	Sr=87	? Yt=88	Zr=90	Nb=94	Mo=96	...=100	Pd=106, Ag=108
7	(Ag=108)	Cd=112	In=113	Sn=118	Sb=122	Te=125	J=127	... ...
8	Cs=133	Ba=137	? Di=138	? Ce=140	...	...	...	
9	(...)	...	...	...	...	...	...	Os=195, Ir=197,
10	...	...	? Er=178	? La=180	Ta=182	W=184	...	Pt=198, Au=199
11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	...	...	... ...
12	...	...	...	Th=231	...	U=240	...	

The latter system consists of three small and five large periods, and fifteen groups, or sub-groups. This is the *System of Great Periods*. It is very interesting to note that there are elemental properties which vary in accordance with the system of small periods, while there are others which vary in accordance with the system of great periods. From which it follows that only the two systems together fully express the Periodic Law of the elements.

One of the chief chemical properties which varies according to the small periods is their valency with regard to oxygen. In the two small periods already referred to, which in both systems represent the second and third small periods [the first period containing only one known element, viz. hydrogen (H)], we note the following variation of valency :—

Li_2O	BeO	B_2O_3	CO_2	N_2O_5	...	F_2O_7
Na_2O	MgO	Al_2O_3	SiO_2	P_2O_5	SO_3	Cl_2O_7

Here each oxide represents the highest degree of oxydisation possible of a given element. We therefore see that the maximum valency of the elements increases successively from 1 to 7 in each of the small periods.

One of the chief properties which vary in accordance with the system of great periods is the atomic volume (this being the quotient of the atomic weight and the specific weight of an element)—the elements Li, Na, K, Rb, and Cs, which are the first members of two small periods and three great ones, possessing the greatest atomic volumes.

That the Periodic Law does not represent an approximate regularity, but an exact natural law, Mendeljew has shown by drawing bold conclusions from it, which subsequently met with striking confirmation. These conclusions were of a twofold order: they, *firstly*, refer to the correction of the atomic weight of little-known elements, and, *secondly*, they refer to the determination of the chemical and physical properties of unknown elements.

As to the former, Mendeljew had proposed corrections of the atomic weights of the following elements, and his corrections were subsequently accepted, *e.g.* :—

1. For Indium he suggested 114 as the atomic weight instead of 38 and 76.
2. For Uranium he suggested 240 instead of 120.
3. For Cerium he suggested 140 instead of 92.
4. For Yttrium he suggested 88 instead of 60.
5. For Beryllium he suggested 9 instead of 14.

But his predictions of the properties of elements, as yet unknown at the time, are of far greater intellectual value. In particular, he gave an altogether detailed description of three elements, to which he gave the names of Ekaboron, Eka-aluminium, and Ekasilicon, and which were subsequently discovered still during his lifetime by Lecoq de Boisbaudran (1875), Nilson (1879), and Cl. Winkler (1886). The element discovered by the French chemist L. de Boisbaudran, and called by him Gallium, was identical with Mendeljew's Eka-aluminium; the element discovered by the Swedish chemist Nilson, and called by him Scandium, was immediately identified with Mendeljew's Ekaboron by the French chemist Clève; finally, the element discovered by the German chemist Winkler, and called by him Germanium, was identical with Mendeljew's Ekasilicon. The properties of all these three elements are almost exactly like those predicted by Mendeljew. The degree of coincidence between the prediction and its confirmation can be readily seen from the following comparative tables :—

I

Ekaboron (Eb). (Mendeljew.)	Scandium (Sc). (Nilson-Clève.)
Atomic weight, 45.	Atomic weight, 45.
Oxide, Eb_2O_3.	Oxide, Sc_2O_3.
Specific weight of oxide, 3·5.	Specific weight of oxide, 3·8.
Eb_2O_3, a more active base than Al_2O_3.	Sc_2O_3, a more active base than Al_2O_3.
Chloride, $EbCl_3$.	Chloride, $ScCl_3$.
The salts of Eb will be colourless, etc.	The salts of Sc are colourless, etc.

II

EKA-ALUMINIUM (Ea). (Mendeljew.)	GALLIUM (Ga). (Lecoq de Boisbaudran.)
Atomic weight, 68.	Atomic weight, 70.
Specific weight, 5·9.	Specific weight, 5·95.
Ea will be a more volatile metal than Al.	Ga is a more volatile metal than Al.
Ea will probably be discovered by spectrum analysis.	Ga was discovered by spectrum analysis.
Oxide, Ea_2O_3.	Oxide, Ga_2O_3.
Chloride, $EaCl_3$,	Chloride, $GaCl_3$,
etc.	etc.

III

EKASILICON (Es). (Mendeljew.)	GERMANIUM (Ge). (Winkler.)
Atomic weight, 72.	Atomic weight, 72·32.
Specific weight, 5·5.	Specific weight, 5·46.
Es will be a metal.	Ge is a metal.
Oxide, EsO_2.	Oxide, GeO_2.
EsO_2 will be a powder.	GeO_2 is a powder.
Chloride, $EsCl_4$.	Chloride, $GeCl_4$.
$EsCl_4$ will be a liquid.	$GeCl_4$ is a liquid.
The boiling-point of this liquid will be under 100°.	The boiling-point of $GeChl_4$ is 86°.
The density of $EsCl_4$ is 1·9.	Density of $GeCl_4$ is 1·88.
Fluoride, EsF_4.	Fluoride, $GeF_4 . 3H_2O$.
Metallorganic compound, $EsAe_4$.	Metallorganic compound, $Ge(C_2H_5)_4$
Specific weight of $EsAe_4 = 0·96$.	Specific weight of $Ge(C_2H_5)_4 = 0·97$.
Boiling-point of $EsAe_4 = 160°$.	Boiling-point of $Ge(C_2H_5)_4 = 160°$.

As we see, the coincidence between the properties of the predicted Ekasilicon and the discovered Germanium is almost complete. But perhaps still more astonishing is a very little-known fact concerning Gallium, the first of these elements to be discovered. No sooner had the discovery of Gallium been announced in the *Comptes Rendus de l'Academie des Sciences*, than Mendeljew sent a note to the Academy expressing his conviction that the newly discovered element ought to be

identical with his Eka-aluminium. At the same time—and this is the important point—he expressed his doubts concerning the correctness of the specific weight, 4·7, attributed to Gallium by its discoverer, and suggested the probability of some impurity in the metal that had been used. A subsequent experiment to determine the weight of Gallium, carried out by Boisbaudran with a purer metal, completely confirmed Mendeljew's estimate of the specific weight, as deduced by him from the Periodic Law. We are here confronted by the strange fact that theory had gauged the weight of a substance, never so far beheld, more correctly than he who was the first to weigh the actual substance. In this case Mendeljew's genius proved itself truly wonderful.

The discovery of these three elements by Mendeljew has been compared to Leverrier's discovery of Neptune. But, however wonderful that discovery was, Mendeljew's was greater. Leverrier's discovery was the result of applying the already known principles of celestial mechanics, whereas Mendeljew had first himself to discover the principles from which he could logically deduce the properties of the three elements. Leverrier's intellectual courage was great, but Mendeljew's was extraordinary. If Mendeljew had lived in more superstitious times he would have been declared a wizard able to see invisible things; fortunately, in our more enlightened age we need only regard him as one of the most greatly daring men of genius whom humanity has ever produced.

But the discovery of the three elements was only one result among many of the establishment of the Periodic Law, which has since become recognised as an incontestable scientific truth. And the definite value of this law was sealed when several gaseous elements discovered in the atmosphere by Sir William Ramsay were incorporated by himself in the periodic system of elements.

The scientific value of Mendeljew's great discovery has been admirably described by the English chemist W. A. Tilden, in his "Mendeleeff Memorial Lecture" (cf. *Transactions of the Chemical Society*, vol. xcv., London, 1909, p. 2105): "At the beginning of the nineteenth century Dalton gave to chemistry the Atomic Theory, of which it is not too much to say that it

provided the scaffold by the aid of which the entire fabric of modern theoretical chemistry has been built up. Sixty years later this conception, developed and adorned by the labours of an army of earnest workers, has been shown to us in a brilliant new light thrown over the whole theory by Mendeleeff. The views of Boyle, of Lavoisier, and of Dalton have been corrected by experience and broadened by extended knowledge, but their names are immortal. In like manner, there is no reason to doubt that the essential features of the Periodic scheme will be clearly distinguished through all time, and in association with it the name of Mendeleeff will be for ever preserved among the fathers and founders of chemistry."

Others have declared Mendeljew's discovery to be the greatest made in inorganic chemistry since Lavoisier.

But the philosophic value of Mendeljew's Periodic Law is perhaps even greater. It shows clearly that the simple substances of our chemistry cannot be simple in themselves, that they must be considered as compounds of a very small number of the primordial elements. In a word, the Periodic Law authorises us to proclaim the unity of matter, a great truth, serving as foundation for the loftiest speculations of the human mind. Mendeljew himself, by the way, was not disposed to recognise the capital philosophical value of his theory. He was, and desired to remain, solely a man of science, in spite of the boldness of his genius.

. . . . . .

It goes without saying that Slav achievement in science is not restricted to the four great names with which we have occupied ourselves in the preceding pages. A whole series of scientific men of the second and third rank have added important contributions to humanity's general store of knowledge. But, in our opinion, not one of these can be ranked with Copernicus, Boshcovic, Lobatchevski, or Mendeljew. Certainly, a man of science like the Russian Metchnikov, who recently died as Vice-Director of the Pasteur Institute in Paris, has made a discovery of capital importance in his theory of phagocytes, and certainly this theory will remain, as a Frenchman has put it, "on the intangible

heights wherein in indelible characters are inscribed the great discoveries of humanity"; but, in spite of its importance for biology and pathology in general, we cannot but see in it a special scientific theory, possessing no immediate philosophical value. In the same way the masterly labours of the Russian palæontologist W. Kovalevski (1842–83), which gained for him the name of "the second founder of palæontology," are also an important contribution to science in general, but their philosophical importance does not appear to us to be very great. We must not omit to mention here the name of the celebrated Russian mathematician Sonja Kovalewskaja (1850–91), the great Czech reformers of medicine Skoda and Rokitansky, the Russian philosopher Vladimir Solovjev, who was even more a brilliant writer and prophet than a philosopher, the celebrated Polish physicist Marie Sklodowska (Madame Curie), etc. etc.

(*Translated from the French Manuscript.*)

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