Kurepa prime numbers

Žarko Mijajlović

Using computing facilities of the Mathematical Institute of SASA, the author of this article found in November 2025 three large primes related to the Kurepa left factorial hypothesis. The numbers have respectively 352509, 401827 and 415436 decimal digits and are named after prominent Serbian mathematicians Zoran Marković, Miodrag Rašković and Slaviša Prešić.

Đuro Kurepa defined in 1964, [1], the left factorial function as $K_n \equiv !n = \sum_{i=0}^{n-1} i!$, $n \geq 1$, sequence OEIS-A003422. First few terms in this sequence are $1, 2, 4, 10, 34, 154, 874, \ldots$ Kurepa formulated in [2] the conjecture: Assuming $n \geq 2$, the only common divisor of $n!$ and K_n is 2, i.e. $(K_n, n!) = 2$. This conjecture is known as ***Kurepa left factorial hypothesis***, KH. In the same paper, Kurepa proved the equivalence KHp to KH: If p is an odd prime, then p does not divide K_p. KHp is usually used in attempts to solve KH. In spite of many efforts and search for counterexamples, KH is still not resolved, see [4], [5].

A prime p is a Kurepa prime if $p = \frac{1}{2}K_n$ for some n. First few Kurepa primes are 2, 5, 17, 2957, 23117,… Yves Gallot, asked in [3]: Is the number of Kurepa primes finite? The significance of this question is: If there are infinitely many Kurepa primes, then KH is true. Proof of this statement is simple. Namely, if q is an odd prime number that violates KHp, i.e q divides K_q, then it is easy to see that q divides K_n for all $n \geq q$, hence in that case all $\frac{1}{2}K_n$ are composite. So, all Kurepa primes are bounded by K_q, therefore there are only finitely many Kurepa primes. The sequence a_k, OEIS-A100614, is defined as $a_k = n$, where $\frac{1}{2}K_n$ is the k-th Kurepa prime. The previously known terms of the sequence a_k, the last found on 22, 2017, are: 3, 4, 5, 8, 9, 10, 11, 30, 76, 163, 271, 273, 354, 721, 1796, 3733, 4769, 9316, 12221, 41532. The author of this article computed in 2025 three new Kurepa primes with indices 78981, 88997, 91743.

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{121}	a_{13}	a_{14}	a_{15}
3	**4**	**5**	**8**	**9**	**10**	**11**	**30**	**76**	**163**	**271**	**273**	**354**	**721**	**1796**
1	1	2	4	5	6	7	32	110	289	541	546	748	1747	5064

a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
3733	**4769**	**9316**	**12221**	**41532**
11712	15470	32930	44640	173772

a_{21}	a_{22}	a_{23}
78981	**88997**	**91743**
352509	401827	415436

Indices n of Kurepa primes $\frac{1}{2}K_n$ are presented in the second rows of the tables, while in the third row the numbers with corresponding decimal digits are written. The term "gigantic prime" refers to primes with over 10,000 digits. Hence Kurepa primes with indices a_{16} - a_{23} are gigantic.

Y. Gallot computed numbers $a_1 - a_{13}$, 2000y; Robert G. Wilson a_{14}, 2004y; Ray Chandler a_{15}, 2004y; T. D. Noe a_{17}, 2004y; Eric W. Weisstein a_{16}, a_{18} $(2005y)$, a_{19} (2006y); Serge Batalov a_{20}, 2017y.
Ž. Mijajlović found a_{21}, a_{22}, a_{23}, finishing computation on Nov 26, 2025.
Computation is executed on a server with two AMD CPU's, each having 128 cores. Sieving took 20 days, while primality testing took eight months.

The author of this article developed algorithms and wrote parallel programs for concurrent computation. We tested for primality all numbers $\frac{1}{2}K_n$ for $n \leq 197651$. Hence, there are exactly 23 Kurepa primes with indices in the integer interval $[1,197651]$. So, possible search for new Kurepa primes $\frac{1}{2}K_n$ should start with $n = 197652$. Hence, the next Kurepa prime number with index a_{24}, if such prime exists, would have at least 906904 decimal digits.

The computation was divided into two stages. The first one is sieving, a filtering of the sequence $\frac{1}{2}K_n$ for elements of the sequence with a small prime factors. Our method of sieving is based on Euclid algorithm applied on primorials (consecutive products of primes) and certain partitions of the sequence $\frac{1}{2}K_n$. In this way all elements from the sequence having a prime factor $\leq 10^{12}$ are eliminated. The remaining set of numbers that should be further tested had 80053 elements of the original set with 197651 elements, what is less than 41% of the original size. We note that this method of sieving is several magnitudes more efficient than standard Trial division test, or Filter wheel factorization. The drawback of the method is that it eats a lot of memory; at least 4 GB per core of memory is needed. As the program manipulates with very large numbers which size could be several GBs, small number of memory channels, e.g. 2 or 4, could be the second bottleneck. Hence, applying this procedure on personal PCs on very large numbers could be slow and ineffective. Filtering in our case on a server took about 20 days.

The second part of computation was the primality testing of the so obtained filtered sequence with 80053 elements. We used first a Miller–Rabin strong pseudoprime test and then a strong Lucas pseudoprime test is applied. Strictly speaking, obtained Kurepa primes are in fact pseudo primes but with incredibly small probability that they are not primes. Anyhow, one can perform the existing certified primality tests on the newly found primes if he thinks it is worth's it.

Nenad Filipović, system engineer, tested the programs, while technician Dragan Aćimović helped much in running computation smoothly.

In honor to Serbian distinguished mathematicians the new primes (their indices) are named as follows:
a_{21} - Prešić number, after Slaviša Prešić (1933- 2008).
a_{22} - Brča number, after Miodrag Brča Rašković (1951 - 2025),
a_{23} - Zoran number, after Zoran Marković (1948 - 2025),

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