

STUDENT INTERNSHIPS AT THE MATHEMATICAL INSTITUTE SANU
JUNE 2024



Research in combinatorics

Mentor. Luka Milićević

Description. Each candidate will get three open problems in combinatorics. During the first week, every candidate will choose a single problem and work on it for the remaining duration of the project. In order to help the students develop their own mathematical interests further, each of the three problems will come from a different area of combinatorics, such as combinatorial geometry, extremal combinatorics or graph theory.

Development of algorithms for optimization problems

Mentors. Tatjana Davidović, Dragan Urošević, Slobodan Jelić, Raka Jovanović, Dušan Ramljak

Description. This topic belongs to the Operations Research scientific discipline. The work would be focused on the development of mathematical models, exact and heuristic optimization methods for various, academic and practical optimization problems (graph optimization, scheduling, transportation, location, newer variants of the traveling salesman problem, etc.). In addition to the application of various general-purpose exact methods (e.g., CPLEX, Gurobi, LINGO), exact and heuristic algorithms the explore a priori knowledge about the problem under consideration would be developed. Special attention would be paid to metaheuristics, general-purpose approximate methods that allow to overcome the shortcomings of exact methods, i.e., high demands on time and memory resources. The metaheuristics developed by Serbian researchers, Variable Neighborhood Search (VNS) and Bee Colony Optimization (BCO), would be specially promoted. Research would also be directed towards hybridizing metaheuristics with existing and newly developed machine learning techniques. In addition to development and implementation, work would be done on parallelization, theoretical and empirical analysis of metaheuristics. A significant segment of research within the summer internship would be focused on the implementation of optimization methods for real optimization problems that occur in science and industry. Through this work, students are enabled to develop and apply skills and techniques related to programming, optimization, graph theory and machine learning. Interested students will be able to apply data science techniques, such as visualization, data analysis, game theory, reinforcement learning, collaborative learning, and recommender systems.

Prerequisites. Programming skills, C(C++), C#, Java, R, Python. Also, we suggest students to familiarize themselves with existing software for finding optimal parameter values of metaheuristic methods (eg iRace).

Suggested material.

- IBM ILOG CPLEX 12.7 Optimization Studio CPLEX User's Manual, 2017.
https://www.ibm.com/support/knowledgecenter/SSSA5P_12.7.1/ilog.odms.studio.help/pdf/usrcplex.pdf
- LINGO - The Modeling Language and Optimizer, LINDO Systems Inc. 2020.
<https://www.lindo.com/downloads/PDF/LINGO.pdf>
- Gurobi Optimizer Reference Manual, Version 9.0, Gurobi Optimization, LLC, 2020.
https://www.gurobi.com/wp-content/plugins/hd_documentations/documentation/9.0/refman.pdf
- Woeginger, Gerhard J. "Exact algorithms for NP-hard problems: A survey". Combinatorial optimization—eureka, you shrink!. Springer, Berlin, Heidelberg, 2003. 185-207.
- Talbi, El-Ghazali, "Metaheuristics: from design to implementation", John Wiley and Sons, 2009.
- Hansen, Pierre, et al., "Variable neighborhood search: basics and variants", EURO Journal on Computational Optimization 5(3):423-454, 2017.
- Davidović, Tatjana, "Bee Colony Optimization: Recent Developments and Applications", (plenary talk), Proc. Balkan Conference on Operational Research, BALCOR 2015, Constanta, Romania, Sept. 9-12, 2015. Mircea cel Batran Naval Academy Scientific Bulletin, 18(2):225-235, 2015.

Solid Mechanics: Theory and Applications

Mentor. Danilo Karličić

Description. This research belongs to the area of Mechanics and Thermodynamics of Soft and Active Matter. The candidate will do their research in one of the following three subtopics.

Topic 1: Coupled field theory of soft actuators.

Within the framework of non-equilibrium thermodynamics of continua, we will consider several fundamental issues involving coupled fields, with a special emphasis on the theory of thermo-electroelasticity and thermo-electro-viscoelasticity in soft and active materials. Additionally, particular attention will be given to anisotropy, namely how different material properties change depending on direction, which is particularly important for understanding the complex behavior of mechanical materials (dielectric and magnetic elastomers) under various loading conditions.

Topic 2: Non-equilibrium thermodynamics model of ionic hydrogel.

The second topic of the proposed research pertains to the analysis of coupled fields of diffusion and elasticity, where the effects of chemical substance diffusion processes on the mechanical properties of materials will be investigated. This research will enable a deeper understanding of the complex interaction of various physical phenomena in soft hydrogels, such as swelling and drying, which is crucial for the development of new technologies and applications in biomedicine and flexible electronics.

Topic 3: Mechanics of biological growth.

The third topic concerns the fundamentals of growth mechanics in biological and active materials. The main relations are derived based on the finite theory of elasticity with corresponding tensors of isotropic and anisotropic growth. The primary focus will be on analyzing the basic types of elastic instabilities that occur during the growth of biological tissue.

Suggested literature.

- Holzapfel, G. A., “Nonlinear Solid Mechanics. A Continuum Approach for Engineering”, Wiley, Chichester (2000).
- Gurtin, M. E., Fried, E., & Anand, L., “The mechanics and thermodynamics of continua”, Cambridge University Press, (2010).
- Volokh, Konstantin, “Mechanics of soft materials”, Vol. 337. Singapore: Springer, 2016.

- Dorfmann, L., & Ogden, R. W. (2014). “Nonlinear theory of electroelastic and magnetoelastic interactions” (Vol. 1). New York: Springer.
- Goriely, A. (2017). “The mathematics and mechanics of biological growth” (Vol. 45). Springer.
- Li, H., & Silberschmidt, V. (Eds.). (2022). “The Mechanics of Hydrogels: Mechanical Properties, Testing, and Applications”, Woodhead Publishing.

Yang-Baxter equation in matrices and its applications

Mentor. Bogdan Đorđević

Description. Within this research topic, the candidate would explore the Yang-Baxter like matrix equation $AXA = XAX$, where the input matrix A would possess some peculiar properties (like being invertible, g-invertible, nilpotent, unitary, Hermitian, skew-symmetric, permutation matrix or a doubly stochastic matrix). The goal of the research would be developing new solving techniques for this equation, and obtaining new classes of solutions. Some important properties, like connections to braid groups, would be further investigated.

Prerequisites. Linear algebra, matrix analysis, real analysis, complex analysis, numerical methods in linear algebra and matrix analysis.

Suggested literature.

- N. Č. Dinčić and B. D. Djordjević, “On the intrinsic structure of the solution set to the Yang-Baxter-like matrix equation”, *Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat.* 116:73 (2022)
<https://doi.org/10.1007/s13398-022-01214-8>
- N. Č. Dinčić and B. D. Djordjević, “Yang–Baxter-like Matrix Equation: Road Less Taken, Matrix and Operator Equations and Applications”, Springer (2023) 241-346
https://doi.org/10.1007/16618_2023_49
- B. D. Djordjević, “Doubly stochastic and permutation solutions to $AXA = XAX$ when A is a permutation matrix”, *Linear Algebra Appl.* 661 (2023)79–105
<https://doi.org/10.1016/j.laa.2022.12.013>

Application of Bifurcation Analysis of Dynamical Systems in Population Ecology

Mentors. Petar Ćirković, Predrag Đorđević

Description. The formation and analysis of mathematical models are of great importance in population ecology. They enable us to understand the interactions between species, predict changes in population sizes and ecosystems, and develop strategies for sustainable management and protection of biological resources. Bifurcation analysis is a research method used to track changes in the local and global dynamics of nonlinear dynamical systems as parameter values change. During the practice, students will become familiar with the basic types of bifurcations: saddle-node, transcritical, pitchfork, Hopf, and Bogdanov-Takens bifurcations.