Novembertagung
On The History
And Philosophy
Of Mathematics

Mathematical
Institute
of the Serbian Academy of
Sciences and Arts

34th NOVEMBERTAGUNG
Unifying the Old and the New: A Cross-Section of Contemporary Investigations in History and Philosophy of Mathematics

November 11-13, 2024
Belgrade

Programme and Book of Abstracts

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Book Cover Photo – A page from the first PhD thesis in mathematics, defended by Mladen Berić at the University of Belgrade in 1912
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Preface

In lieu of an introduction: a look at the past and the present of the Novembertagung

The following text is based upon the material gathered by Henrik Kragh Sørensen, Tinne Hoff Kjeldsen, and Tilman Sauer, all of whom took part in the organisation of the past Novembertagungen.

The history of the Novembertagung on the History and Philosophy of Mathematics is multifaceted, interesting, and exciting almost as much as the history of the 20th-century history and philosophy of mathematics. In a sense, the development of the Novembertagung over the last three decades is quite telling not only of an interdisciplinary development within history of mathematics, but also of the interdisciplinary interactions between history and philosophy of mathematics.

The "November meetings" were born out of an initiative of a small group of (mostly German) doctoral students and young researchers working in the history of mathematics who gathered at the Summer school "Zum Verhältnis von Mathematik und Anwendungen im 18. und 19. Jahrundert" at the PfalzAkdemie in Lambrecht, Germany, on May 15th, 1990.[1] Being somewhat dissatisfied with the mainstream approaches to the study of the history of mathematics, and motivated by the perceived need to explore new research vistas, foster informal intellectual exchange, and, most importantly, to connect junior scholars from the small and, at times, disparate academic backgrounds, this pioneering group of enthusiasts gathered for the first time at the University of Wuppertal, from November 1st to November 4th, 1990. It was immediately realised that this was the ideal way to start building a continuously expanding network of contacts and, consequently, to ensure the future of the field. The Wuppertal meeting thus marked the beginning of the now already venerable, thirty-four-year long tradition. Since 1990, the Novembertagung has become an annual event, regularly taking place in co-operation with a different university or a research institute.

Over the years, the Novembertagung has become somewhat of a trademark – a friendly, open-minded and inclusive event organised by early-career researchers for early-career researchers (advanced M.A. and PhD students and post-docs) in the history and philosophy of mathematics and the related fields. It provides a unique platform and stimulating environment to meet and interact with colleagues in related areas, opening doors to potential collaborations and receiving valuable feedback. One of the main goals of the Novembertagungen today is to gather and connect as many young researchers from around the world as possible and to promote the study of the history and philosophy of mathematics. In line with the guidelines of the International Commission on the History

[1]This was one of the first events which saw researchers from East and West Germany meet in person following the fall of the Berlin Wall.

of Mathematics (ICHM), the themes are typically construed broadly to include as many interesting applications as possible.

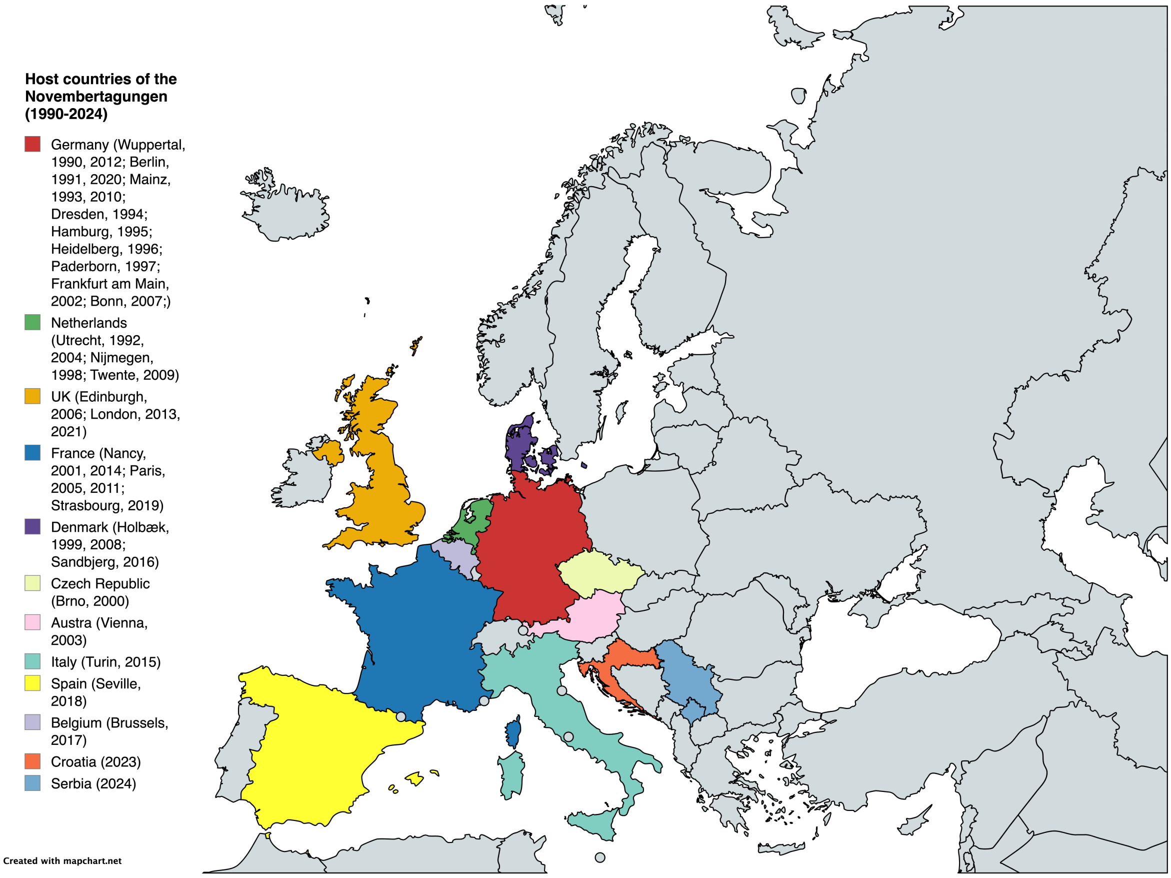
Following Wuppertal and Berlin (1991), the first Novembertagung outside of Germany was organised in Utrecht (1993). Looking at the map of the host countries of the Novembertagungen, Germany holds the record – over the years, twelve conferences were organised under the auspices of German universities. This should not come as a surprise since the Novembertagung was christened as the Novembertagung zur Geschichte der Mathematik, and the official working language of the meetings was German for much of the first decade of the conference's history even though the event quickly started to internationalise by bringing together participants from non-German speaking countries as well, e.g. the Netherlands, Denmark, Czech Republic or France. At the turn of the 21st century, following the expansion of the Novembertagung across the European continent, English became the official language of the conference, and it has remained so until the present day.[2]

In the Introduction to his seminal Proofs and Refutations: The Logic of Mathematical Discovery (published posthumously in 1976), Imre Lakatos (1922–1974), paraphrasing Kant, famously said that "the history of mathematics, lacking the guidance of philosophy, has become blind, while the philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become empty". In writing this, Lakatos intended to emphasize the fact that during the 20th century, at least until the late 1960s, history and philosophy of mathematics were mostly pursued as independent and isolated ventures – predominantly internalist histories tended to write off broader socio-political, institutional, and economic factors, as well as philosophical underpinnings of many important mathematical contributions as irrelevant, whereas various philosophies of mathematics (or, better yet, mathematical philosophies) treated their subject matter, i.e. foundations of mathematics, from an a historical point of view, focusing mainly on technical issues believed to be resolvable by means of mathematical logic.

Thus, in his (in)famous address "History of Mathematics: Why and How" at the Helsinki ICM in 1978, the Bourbakist André Weil even proclaimed that it was hard for him to imagine what history and philosophy of mathematics could have in common. In sharp contrast with such a view, the organisers of the 10th Novembertagung (Holbæk, Denmark, 1999) decided to include philosophy of mathematics explicitly for the first time not only in the name of the conference but also in the conference theme: "How can the Philosophy of Mathematics contribute to the History of Mathematics?". This change was motivated by the gradual broadening of perspectives within both history and philosophy of mathematics, and by an emerging idea characteristic of the beginning of the 21st century, namely, that integrated, practice-oriented historico-philosophical studies of mathematics are the way to go.[3] After 1999, the interactions and interrelations between history and philosophy of mathematics were repeatedly addressed at various meetings, e.g. in Bonn (2007), Sandbjerg (2016) or Rijeka (2023). The 10th meeting also saw the formal introduction of mathematics education in the conference programme.

[2]For the full list of conference venues, along with themes and invited speakers consult the "History" section of the official Novembertagung website: https://novembertagung.wordpress.com/history/.

[3]For more details about this novel approach see, e.g., the 2006 OUP collection The Architecture of Modern Mathematics. Essays in History and Philosophy edited by Jose Ferreirós and Jeremy J. Gray.



The 2024 edition of the Novembertagung aims to revisit these issues under the heading "Unifying the Old and the New: A Cross-Section of Contemporary Investigations and Trends in History and Philosophy of Mathematics". The conference is hosted by the Mathematical Institute of the Serbian Academy of Sciences and Arts, and it will take place in Serbia's capital, Belgrade, from 11–13 November 2024. Following the closure of the Call for Abstracts on September 1st, we witnessed a record turnout of almost seventy high-quality submissions from young scholars affiliated with leading research institutions not only across Europe, but also from Asia (both Middle and Far East) and the Americas, thirty-five of whom will present their work in Belgrade. As far as we are aware, this will be the first ever intercontinental Novembertagung. Keynote lectures will be given by José Ferreirós (Universidad de Sevilla), Vincenzo De Risi (CNRS – Laboratoire SPHERE), and Øystein Linnebo (Universitetet i Oslo).

The Organising Committee of the 34th Novembertagung decided to devote the Belgrade meeting to the following thematic blocks:

- Towards a global perspective in the historiography of ancient mathematics: mathematical manuscripts and treatises in cuneiform, Greek, Chinese, Latin, Arabic, and Sanskrit — new questions and new interpretative strategies;
- Non-Western mathematical cultures: historico-philosophical considerations;
- Interactions between history and philosophy of mathematics in the 19th and 20th centuries;

- Historico-philosophical studies of foundational crises in mathematics (Grundlagenkrisen) and the ensuing reactions to them;

- History and philosophy of "non-standard mathematics" (non-Archimedean mathematics, constructive mathematics, etc.);

- New approaches to traditional issues: the axiomatic method, the status and role of intuition and visual methods in mathematics (diagrammatic reasoning), the historical development of key mathematical notions (e.g., "proof", "algorithm", "rigor" or "exactitude", etc.);

- Historical case studies in contemporary philosophy of mathematics;

- History of logic as a sui generis branch of the history of mathematics: specific problems, case studies, and open questions;

- The institutional history of mathematics (the establishment of mathematical institutions, e.g. societies, associations, institutes, departments or research centres, the founding of mathematical journals, specialised archives and reference libraries or databases, etc.);

- Digitization of mathematical heritage;

- Is there such a thing as "national" mathematical schools?

- History and philosophy of mathematics in the era of digital humanities;

- The challenges of AI in contemporary historiography and philosophy of mathematics;

- History, the present state, and prospects in the didactics of mathematics and mathematics education;

- Mathematics and the arts: around the notion of "mathematical beauty".

As is evident, the flexibility of the Novembertagung allows it to keep answering the various challenges of contemporary scholarship, and to keep adapting to the needs of its participants and attendees, fostering ongoing negotiation around its format and scope. Initially, the meetings were small-scale and low-budget events, however, the Novembertagung has evolved over the years and it is becoming increasingly large-scale which is testified by the larger than expected number of people who expressed their interest in the conference this year. These changes also affected the format of the conference, making parallel sessions a necessity already at the 16th Novembertagung in Paris (2005), with over forty participants. As the event has acquired quite an international reputation, participants have started coming from outside Europe in the last years (e.g. from Argentina, Israel, Mexico, and the USA), and almost a third of this year's submissions came from Asia and the Americas. From an organisational standpoint, expansion and internationalisation made financial planning and fund-raising a pressing matter. Namely, one of the key ideas behind the Novembertagungen is to facilitate exchange between young scholars which is quite often complicated due to financial difficulties and obstacles (i.e. travel and accommodation expenses). This would have been impossible without the help of numerous institutions that sponsored the Novembertagung over the years. The same holds for the 2024 Belgrade meeting which was generously supported by the Ministry of Science, Technological Development and Innovations of the Republic of Serbia, the International Commission on the History of Mathematics (ICHM), the British Society for the History of Mathematics (BSHM), and the RT2183 Histoire des maths et didactique (HiDiM) of the Centre national de la recherche scientifique (CNRS), for which we are most grateful.

Over the years, many of the "founding fathers" and "mothers", i.e. the initial organisers as well as the participants of the Novembertagung became leading experts in contemporary historiography and philosophy of mathematics, returning back to the Novembertagung but now as invited speakers. This shows that the Novembertagung has played a key role in the ensuring the future development of these disciplines. It is our hope and wish for the Novembertagung to continue evolving in the future, remaining both a nursery and a proving ground for talented young scholars partaking in two fascinating and deeply interconnected human enterprises – history and philosophy of mathematics.

Belgrade, November 1st, 2024

Contents

Preface 5
In lieu of an introduction: a look at the past and the present of the Novembertagung 5

List of Abstracts – Talks 11
Invited Lectures 11
List of Abstracts 14

List of Participants 60

List of Abstracts – Talks

Invited Lectures

The Paradigm of Science
Modern Axiomatics and the Discovery of Non-Euclidean Geometry

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The talk discusses the theories of axioms from the early modern age to the discovery of non-Euclidean geometries in the early 19th century. The main question is: Why were non-Euclidean geometries not discovered before 1830? The idea that a system of axioms other than Euclid's produces a different geometric theory is in fact very elementary, and furthermore, non-Euclidean geometry does not require, at least for its fundamental propositions, any part of advanced mathematics (as shown in Saccheri's work, in which the main properties of hyperbolic space are demonstrated with ruler and compass).

The answer to this question is found in the epistemology of axioms of the early modern age. According to Scholastic authors, in fact, the axioms of a theory can be proved from the meaning of their terms. These are, in other words, analytic propositions. This Scholastic theory was still widely endorsed in the 17th and 18th centuries, and was taught by most geometry and textbooks on the theory of science. All major modern scientists and philosophers show that they knew it and adhered to it (e.g., Galileo, Hobbes, Leibniz, Newton, Bernoulli, Euler, etc.). Within this conceptual framework it is impossible to imagine non-Euclidean geometries, because the negation of any axiom of Euclid (conceived as an analytic proposition) would imply a contradiction. Euclidean geometry is therefore necessary.

The emergence of non-Euclidean geometries in the early nineteenth century is thus due to the establishment of a different epistemological paradigm concerning the principles of demonstration. This different paradigm was discussed extensively in Germany during the eighteenth century, both by leading mathematicians (such as Lambert) and leading philosophers (such as Kant). Lambert first proposed an alternative theory of the principles of demonstration, according to which the traditional relationship between definitions and axioms was reversed: according to Lambert, definitions are derived from axioms rather than vice versa. Lambert applied this theory of axioms in his studies on the theory of parallels, and produced an important work discussing the possibility of non-Euclidean geometries.

The emergence of the new epistemological model produced an important conceptual revolution in the conception of mathematics and the axiomatic method. A few decades later, Lobachevsky and Bolyai claimed for the first time that a non-Euclidean geometry is possible. This was, in a sense, the birth certificate of modern mathematics.

From Foundational Issues to Cognitive Studies

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We shall discuss possible ways of analyzing mathematics from a cognitive viewpoint (in a broad sense of the term) as an alternative or a complement to foundational studies. Given that foundational studies cannot fully account for mathematics, an obvious alternative is to consider it as a human activity – to adopt a pragmatist approach. Modern mathematics, despite all its novelties, links back to arithmetic, geometry and mechanics, and they all have strong cognitive roots. Furthermore, in recent decades there has been a lot of activity in cognitive psychology and neuroscience, leading to studies in the field of 'mathematical cognition'. The aim of the talk is to offer some critical remarks on this confluence of interests, and suggest ways in which history and philosophy of mathematics can fruitfully interact with cognitive studies. One of the key points is to argue that semiotic practices –our manipulation of external representations as cognitive tools– are crucial mediators for the emergence of mathematical notions and practices.

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Abstraction and Critical Plural Logic

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Frege's attempt to found arithmetic on a theory of set abstraction foundered because of Russell's paradox. Neo-Fregean have proposed to build instead on consistent forms of abstraction, such as Hume's Principle for cardinality abstraction. However, this proposal faces "the bad company problem", namely, that there are bad forms of abstraction mixed in among the good forms. Dummett wished to solve the bad company problem by requiring that abstraction be predicative in some sense. The dominant way to pursue this idea has been to impose predicativity restrictions on the second-order logic. I review why this strategy has not been a success. I propose an alternative development of Dummett's idea, loosely speaking, that we successively abstract on "available" objects. This alternative is developed by (i) abstracting on pluralities of objects rather than Fregean concepts, and (ii) using the Critical Plural Logic recently developed by Salvatore Florio and myself rather than traditional plural logic. In this way, we obtain a large and natural class of permissible abstractions.

List of Abstracts

An Analysis of Mental Arithmetic Applications in Islamic Mathematical Tradition

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In Islamic tradition, there were at least three primary calculation systems: Hindu calculation (*hisāb al-Hindi*), which relied on Hindu numerals; the sexagesimal system, based on a sixty-based structure; mental calculation (*hisāb al-hawāi*), which focused uniquely on mental processing.
Al-hisāb al-hawā'ī was the second major calculation method in Islamic mathematics. In this system, arithmetic operations are performed mentally without the use of paper or pen, utilizing hands to track intermediate steps. It was called *al-hisāb al-yadd* for using hands, *al-hisāb al-'uqūd* due to the use of finger joints, *al-hisāb al-zihnī* for its rely on mental processing, and *al-hisāb al-hawā'ī* for the impression of being performed 'in the air. Ibrahim al-Uqlidisi (d.980 CE) referred to this system as "Arithmetic of the Byzantines and Arabs" (*hisāb al-Rūm wa'l-'Arab*). Additionally, it was known as *al-hisāb al-maftūh*. Rooted in Greco-Babylonian origins, this system evolved into a new form through the intellectual contributions of Muslim scholars.[1]
Al-hisāb al-hawā'ī aims to develop the ability to provide accurate and quick answers to calculation problems.[2] It is defined as "a science that teaches how to calculate without writing, using only mental power" with unique rules beneficial for merchants and situations where writing is impractical, particularly for those unable to write.[3]
Due to its practicality and capacity to maintain secrecy, *al-hisāb al-hawā'ī* was widely used by scribes and jurists during the Abbasid period.[4] A strong knowledge of this system was essential for efficient calculations in mathematics, jurisprudence and inheritance.[5]
Al-hisāb al-hawā'ī encompasses topics such as multiplication, division, ratios, proportions, and square roots, with multiplication as the fundamental operation. Beginners should master mental multiplication first, followed by fractions and ratios. Mental exercises in the

[1] Saidan, A. S. (1978). *The arithmetic of Al-Uqlīdisī: The story of Hindu-Arabic arithmetic as told in Kitāb al-Fuṣūl fī al-isāb al-Hindī.* Holland: D. Reidel Publishing Company.

[2] These words are taken from the preface of *Irshād al-Hussab*, a work by the Islamic mathematician Ibn Fallūs (d. 1252), who discussed the rules of mental calculation. For details about the work, see *Irshād al-Hussab fi'l-Maftuh min 'Ilm al-Hisāb* (n.d.). (MS 1292, fol.60b). Süleymaniye Library, Hasan Hüsnü Pasha Collection.

[3] Taşköprizâde Ahmed Efendi (d. 968/1561). (n.d.). *Miftāḥ al-Saāda.* (761/1, fol.35a) Manisa Library, Akhisar Zeynelzade Collection.

[4] Ibn Fallūs. (Irshād al-Hussab, fol.60b; Süveysi, M. (1998). Hesap. In *TDV İslâm Ansiklopedisi* (Vol. 17, p.257). İstanbul: TDV Yayınları.

[5] Ibn Fallūs. *Irshād al-Hussab*, fol.62a

sexagesimal scale were recommended to improve proficiency.[6]

Al-hisāb al-hawā'ī was also applied in other branches of mathematics. In applied geometry (*mesāha*), it was used to determine the lengths, areas, and volumes of geometric shapes. In algebra, it was employed to solve first and second-degree equations, while in number theory, it was used for calculations involving number sequences.[7]

In later periods, with the adoption of Hindu arithmetic, the mental calculation system was restructured but not entirely abandoned. The most effective features of the older method were preserved and integrated with Hindu numerals, resulting in a more advanced calculation system for the Islamic world and future generations. This fusion effectively combined the strengths of both mental reckoning and Hindu arithmetic.

Keywords: Mental Calculation, Hisāb al-Hawāi, Islamic Mathematics, Ibn Fallūs

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An Arithmetical Journey: From Şehzade Mosque to Belgrade University

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Throughout medieval history, arithmetic books, among mathematical works, have been most susceptible to cultural influences. They were often written to address practical problems commonly encountered in society. A prime example of this influence can be found in the oriental collection housed at Belgrade University: an arithmetic treatise titled *Risālah fī al-hisāb* (*The Treatise on Arithmetic*). This manuscript alludes to its origins with a reference to Istanbul's Şehzade Mosque on its first folio. Its content links it to the Ottoman period, during which, after the 16th century, a practical version of arithmetic was

[6]Ibn Fallūs. *Irshād al-Hussab*, fol.62b

[7]For examples of the topics, see Akkaya, R. (in press). *İbn Fellûs'un Matematik Külliyâtı.* Türkiye Yazma Eserler Kurumu Başkanlığı Yayınları.

built upon earlier Arabic and Persian works as its mathematical foundations.

What distinguishes this manuscript from typical treatises is its unique structure, which consists exclusively of numbers in the Hindu-Arabic numeral system, various mathematical operations, and concise Arabic titles that identify the name of each operation or problem. Additionally, sporadic parts in Ottoman Turkish provide some indications about the topic or conversion of units. The manuscript has minimal recourse to verbal representation in ordinary language, which was the main form of mathematics at the time and holds historical importance. This characteristic creates ambiguity in understanding it as historical evidence; however, available historical information enables us to deduce its educational and practical purpose and its role in a problem-solving model of arithmetic. It is reasonable to conclude that it was part of administrative education and work, such as accounting, within the Ottoman Empire's state (*diwān*).

The manuscript commences with an introduction to the Hindu-Arabic numeral system in the decimal system. It covers arithmetic operations, including addition, multiplication, subtraction, and duplication, for both categories of numbers: those with fractions and those without. The text also presents multiplication and duplication in the context of aerial calculations (*al-hisāb al-hawāī*), a method favored by administrative agents, allowing them to perform mental calculations using their fingers.

The section on division is more extensive, featuring not only the aerial method but also two other techniques: the chain (zanjīrī) and Frankish (faranghī) methods. The inclusion of the latter demonstrates the awareness of Western arithmetic practices.

The treatise culminates by addressing a critical issue: bankruptcy calculations (Hisāb al-ghuramā). This involves the fair distribution of the remaining assets of a bankrupt individual among their creditors. The section starts with a paragraph in Ottoman Turkish, which outlines the general principle for property division in this situation. Subsequently, it presents scenarios where the asset value and creditors' shares are known, categorizing them into four distinct types of problems and offering their corresponding solutions. This focus on bankruptcy calculations underscores the role of socioeconomic concerns within a broader arithmetic context, within the expansive empire.

The manuscript also incorporates additional minor topics, some of which appear to have been added at a later stage. As a historical resource, this document exemplifies multicultural accomplishments, not only from the Eastern world but also Western influences. Furthermore, it seeks to transcend language barriers, utilizing numbers as a universal language. Our research begins with an overview of the arithmetic tradition established on Arabic and Persian sources during the Ottoman period. Subsequently, we examine the aforementioned manuscript, exploring its content and significance, with a particular focus on its unique emphasis on bankruptcy calculations and its historical implications.

Keywords: Bankruptcy Calculations, Aerial Calculations, Practical Arithmetic, Ottoman Period, History of Arithmetic, History of Mathematics.

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The Treatise on Arithmetic (*Risālah fī al-ḥisāb*), the oriental collection at Belgrade University.

Practical Geometry. A Case from Islamic Era (Connections with Mathematics education and Islamic tiling Art)

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Geometry in the works of mathematicians in the Islamic era included three basic parts: theoretical and practical geometry and geometry in astronomy. Theoretical geometry related to the tradition of Greek mathematicians like Euclid. Calculations of the area of lands, dividing a land between inheritors, opening the road between the lands, mensuration and so on, are examples of practical geometry. There is evidence that artists, carpenters, surveyors, blacksmiths, etc. used practical geometry to create their works in Islamic era. There are references to the subject of practical geometry in science classification books.

The Codex 50944 now kept at the Āstan-i Quds-i Razavī Library (Mashhad, Iran) includes some valuable treatises on Mathematics from the Islamic era. It seems the date of writing Persian manuscript 50944/6 on practical geometry (fols.27v-41v) is around 878 AH/ 1499 AD in Herat. The treatise includes 76 theorems and problems which can be classified into five sections: dividing shapes, carpenter set- square at a semicircle with a fixed compass (constant radius), constructing shapes whose area is equal to the sum of two or more other shapes, constructing shapes in and on other shapes and in a circle, constructing shapes with a compass of certain radius and adding a shape to another shape.

There are some similarities between this treatise and *kitāb al-Nejārat* by Abū'l-Wafa al-Būzjanī, *on constructing regular figures with a circle of constant radius* by Abd al- Rahmān al-ūfī, an anonymous Persian Compendium *On Similar and Complementary Interlocking Figures* and a little treatise from Codex Paris 169 (fols: 113v-115v).[8] Researchers have done research on the above-mentioned treatises, and there is some resources related to them.

Fourteen problems of newly found treatise have geometric proofs, and the rest of the problems have precise drawing methods. Thirty-six problems are not similar to those in previous treatises. In this article, I try to analyze some new problems. For example, I will explain the drawing of geometric shapes that are useful for tiling artists with the special setsquare introduced in this treatise. In addition, I present the connection between its problems with mathematics education and the art of Islamic tiling.

Keywords: Practical Geometry, Setsquare, Islamic era, al-Būzjanī, Art.

[8]Bibliothèque nationale de France, Ms. Persan 169, fols. 180r–199r

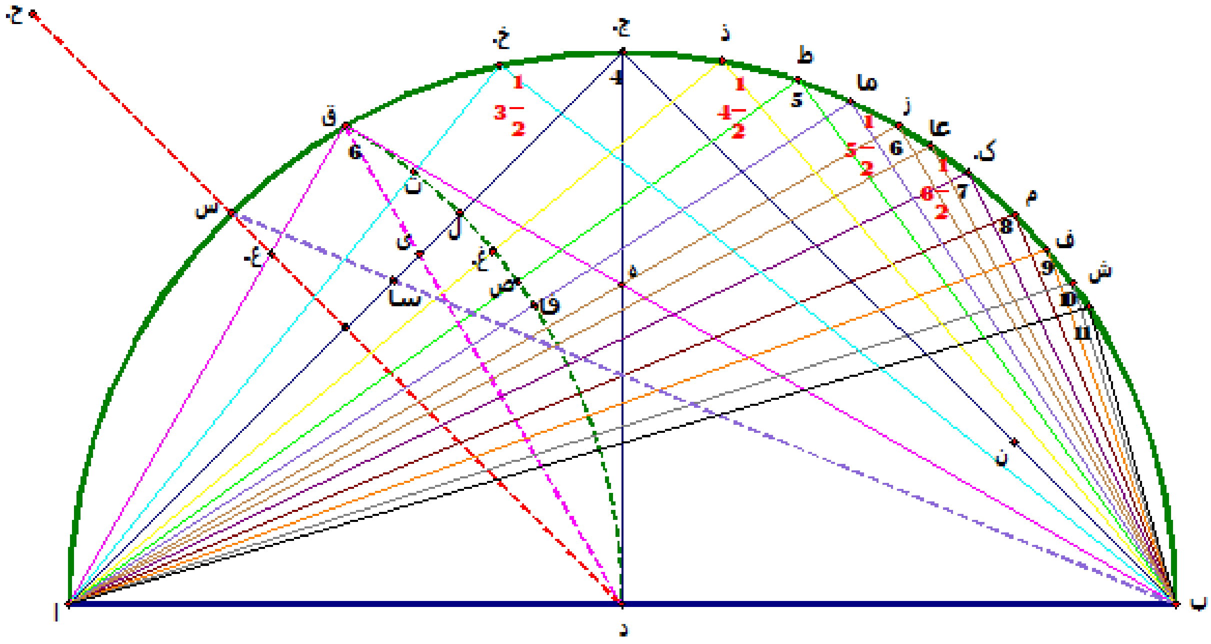

Figure 1: Carpenter setsquare (fol. 31r)

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Does Lebesgue Have a Constructivist Philosophy?

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At the beginning of the twentieth century, mathematics faced what historians of mathematics have commonly called the crisis of foundations. Several strategies have been proposed to resolve this crisis. These have led to different positions on the foundations of mathematics. A group of mathematicians mainly composed of Emile Borel, Henri Lebesgue and René Baire, often appeared as a group claiming a distinct view. Some authors go so far as to consider the idea of a French or Paris school. Certain of these authors even suggest that this French school would defend a kind of “semi-intuitionism”. One of the elements that leads them to identify such a school is the constructivism that these mathematicians would proclaim. In particular, the authors consider that Emile Borel and Henri Lebesgue claimed a specific constructivist project for mathematics. But could we really find such a philosophy

in their works? If so, what kind of constructivism are these mathematicians defending? Are they supporting a unique constructivist project? Are we not rather confronted with several very distinct constructivist undertakings?
This presentation aims to clarify these points. To do this, we will study only the case of Lebesgue, although we will allow ourselves some comparisons with Borel. We will look at Lebesgue's point of view on the question of the existence of mathematical objects. We will clarify what Lebesgue means by "naming" [*nommer*] a mathematical object. This term has frequently been interpreted as the Lebesgue's wish to adopt a constructive or effective attitude in all mathematical reasoning. However, we will show that in reality it is not so obvious. Finally, we will come back to the term "semi-intuitionism" which we believe establishes a historical link between this French school and intuitionism. Here history and philosophy are intertwined: History becomes a means of promoting a certain philosophical conception.

Keywords: Henri Lebesgue, Constructivism; Existence of Mathematical Objects, "semi-intuitionism."

History of Vector Calculus: Scientific Exchanges Between Brazil and France in the First Three Decades of the 20th Century.

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History of Vector Calculus: Scientific Exchanges Between Brazil and France in the First Three Decades of the 20th Century. Delving into the History of Sciences, with a focus on Mathematics and its history, specifically on the History of Mathematics in Brazil, including its notable figures and works that significantly contributed to the development of this science, the present proposal aims to provide historiographical insights regarding the history of vector calculus in Brazil. Previous research data indicate that this subject, as a discipline, was first introduced in the country in 1926 at the Polytechnic School of São Paulo, with engineer-mathematician Theodoro Augusto Ramos (1895-1935) as the course instructor. Ramos was also the author of the first work published in the country (1927) and the first international publication by a Brazilian on the subject (France, 1930). These works have been analyzed and published, along with the academic and professional biography of Ramos, enriched with personal facts previously unknown to the history of Brazilian mathematics. The analysis of Ramos' two works revealed, among other aspects, a strong influence from the tradition of Italian authors, specifically Burali-Forti and Marcolongo, as well as scientific exchanges with French scholars. Furthermore, throughout the character's publications, he mentions his correspondence and the topics discussed with important French figures of the time, such as Borel and Denjoy. Additionally, there is a mention of a significant critique made by the Italian Levi-Civita about the vector calculus work published in France. These facts, along with the questions raised about the scientific exchanges between the countries are the subjects of this research. Contemporary with the works of the Brazilian Ramos, there was a publication in France in 1924 by Albert Châtelet (1883- 1960) and Marie Joseph Kampé de Fériet (1893-1982) on the same subject. In this regard, this work seeks to present a preliminary joint study of these works, opening up the possibility of advancing our understanding of the state of vector calculus in both countries, including their similarities and differences. This leads to a reflection on the scientific relationships established between the two countries during the first three decades of the 20th century. In addition, it seeks to understand the state of vector calculus in France during the period when Ramos was there, such as which works on vector calculus were published at the time. These elements shed light and ignite a promising line of investigation in collaboration with international researchers, enabling the enrichment of the historical-mathematical repertoire regarding the history of vector calculus in Brazil, with previously unknown elements. This is a qualitative research study, and the proposed methodological theoretical framework is based on Ricoeur's reflections on narrative hermeneutics (2007, 2010). As an outcome of this investigation, it is expected to contribute to the expansion of research in the field of history of mathematics, particularly in the history of vector calculus in Brazil and its connections with France. Thus, this endeavor opens avenues for debates, reflection, and internationalization of the subject, paving the way for new studies and collaborations for future cooperative projects.

Keywords: History of Mathematics, History of Vector Calculus, Theodoro Augusto Ramos, Albert Châtelet, Marie Joseph Kampé de Fériet.

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Purity in Mathematics. The Case of Newton's and Bolzano's Work in Analysis

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We address the question of *mathematical purity* in Newton's and Bolzano's work in analysis. From a standard point of view, the differences between the two would be broadly described as follows: whereas Bolzano's approach aims to be *purely analytic*, Newton's formulation of analysis (profoundly involved with physical concepts) is still mathematically impure. Yet, we argue that both approaches share a common criterion of mathematical purity, based on the respect of well-defined *orderings* of mathematical disciplines.

Bolzano places analysis before geometry (Bolzano, 1817: 228) and geometry before mechanics (Bolzano, 1804: 173-174). This justifies his demand for a *purely analytic* proof of his 1817 theorem, that is, a proof independent of the nature of the magnitudes involved. It thus becomes "an intolerable offence against correct method to derive truths of pure (or general) mathematics (i.e. arithmetic, algebra, analysis) from considerations which belong to a merely applied (or special) part, namely geometry" (Bolzano, 1817: 228), or which involve the concepts of time and motion, "as foreign to general mathematics as that of space" (Bolzano, 1817: 229).

On the other hand, Newton's work on analysis from the 1670s onwards is rooted on physical concepts, and assumes that magnitudes are generated through *continuous flux or motion* (e.g., Newton, 1981: 123). Moreover, for Newton, "the genesis of the subject-matter of geometry [...] and the fabrication of its postulates pertain to mechanics" (Newton, 1976: 289). Thus, he places elementary mechanical notions before geometry and analysis, as noted by later authors: John Colson highlights that Newton's method of fluxions is built upon a principle "taken from the Rational Mechanicks" (Newton, 1736: xi); for Lazare Carnot, "it is not acting in opposition to the spirit of Mathematics to define fluxions by velocities" (Carnot, 1832: 94-95).

The above orderings are based on two different ideas: Bolzano's ordering is given by the *generality* with which the truths of a discipline apply, while Newton's ordering focuses on how the objects of study of geometry and analysis are *generated* through mechanical means. However, they both conform to well-defined orderings of mathematical disciplines that determine the methods they accept or reject. It is according to this criterion that the views of both can be regarded as *pure*.

Keywords: Mathematical Purity, Newton, Bolzano, Analysis, Geometry, Mechanics.

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Between Minería and the Fine Hall: Transnational Networks in the Formation of the Mexican Mathematical Community (1939-1968)

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In recent years there has been a proliferation of studies on the history of science and technology during the post and cold war, with a particular emphasis on the mobilization of knowledge in a broad sense, encompassing individuals, objects, and practices. These studies are inserted within the perspective of transnational history, which places emphasis on networks, processes, and the trajectories of actors and materialities that transcend the nation as a category of historical analysis.

The history of mathematics in Mexico during the twentieth century, particularly that which has been written about the period beginning in the 1930s, is informed by a historiography that is framed in the Nation-State as a category of analysis and is imbued with a strong institutional imprint. This historiography has as one of its objectives providing identity to the guild that makes up the Mexican mathematical community. It is shaped by the trajectories of the most emblematic characters of the mathematical community and by the narratives of the emergence of educational, public, and research institutions of mathematical scientific practice. In these narratives constructed within national spaces, the international scenario is presented as part of an "external" context. When transnational connections appear, they are usually interpreted as a sort of "integration" into the hegemonic course of scientific development, reinforcing a diffusionist, linear, and teleological model of the history of science.

The research project I am currently undertaking examines mathematics in Mexico between 1939 and 1968. My approach is informed by a transnational perspective, and one of the objectives is to generate a narrative that deals with phenomena such as World War II, the

Cold War, and the global reconfiguration of the centers of mathematics that took place in the interwar period as processes that had direct effects on the transnational connections and networks of the mathematical community that were established particularly between Mexico and the United States during these years. But this processes also influenced what kind of scientific knowledge and practices were mobilized, why and by what means, who migrated and where, and what resources were instrumentalized to achieve this.

Within this framework, I examine the construction of mathematics practice in Mexico, specifically at the National Autonomous University of Mexico (UNAM). This investigation involves the study of educational practices at the professional level, the lines of research that were established and the transnational connections that were formed.

I explore the historical trajectory of professional mathematics at UNAM to answer which transnational networks contributed to the formation of the mathematical community in Mexico.

To achieve this objective, I examine the migration of individuals and practices between Mexico and the United States within the mathematics field. During the period under study, the United States consolidated its position as the center of mathematics worldwide and two critical figures in the American mathematical community, George Birkhoff and Solomon Lefschetz, established two fundamental nodes in the transnational network that will connect the Mexican mathematical community with the international one, in particular with three institutions: the MIT, Harvard University, and Princeton University, which would become one of the centers of mathematics in the postwar period. Ultimately the goal is to provide a narrative that incorporates global political and social processes into the history of mathematics in Mexico.

Keywords: Transnational History, Mathematics in México, History of Mathematics, Internationalization of Mathematics.

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The Term 'Structure' in Mathematical Discourse From 1889 to 1942. A Bibliometric Study by Using the *Jahrbuch über die Fortschritte der Mathematik*

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This research project explores the historical development of the use of the term 'structure' in mathematical discourse from 1889 to the 1960s, focusing on the *Jahrbuch über die Fortschritte der Mathematik* (JFM), which was published in Germany and was the first internationally extensive review journal in the history of mathematics. It contains information on almost all publications in mathematics and its application areas from the time period from 1868 to 1942. Inspired by Prof. Dr. Ralf Krömer's lecture on *Elie Cartan et l'usage du terme 'structure' dans le discours mathématique*, the project introduces a quantitative investigation using bibliometrics. Key questions include the origin of the term, its distribution across mathematical subdisciplines, and its meaning when used in this context.

The transition from the 19th to the 20th century was a time of change in the methods of mathematics. The so-called structural methods of mathematics began to develop. The development of structural mathematics extends over a long period and has been favoured by various developments in mathematics. It includes contributions from many different mathematicians. Corry (2004) divides the development into the following phases: Modern algebra, as van der Waerden (1930) calls it, the contribution of the Bourbaki group (from 1935 onwards), and finally category theory (mid 1940s/1960s). "The term 'mathematical structure' has unmistakably become one of the central concepts of modern mathematics" (Wußing 1969, p. 9). The rise of structural mathematics is reflected in the JFM and can also be seen on a bibliometric level in the context of scientific publications. In my presentation I would like to present the first results of my bibliometric analysis with the JFM. The data of the JFM is available electronically on the Open Access platform of the*Zentralblatt für Mathematik und ihre Grenzgebiete* (zbMATH). There I searched for the term 'structure' in the title or in the review. Keywords are omitted from this search as they were only added to the database later and are of no use from a historical perspective as they are not included in the original. Furthermore, the different spellings in German (Struktur, Structur), French and English (Structure) and other languages are also taken into account here. The search results were then assigned to the subdisciplines of mathematics to illustrate how the number of subdisciplines in which the term is used is developing and increasing. Using data from the JFM, we can see that the number of submitted papers containing the term 'structure' has increased both in absolute numbers and percentages. On the other hand, the number of mathematical subdisciplines involved has also increased. Overall, a quantitative exploration is carried out in order to work out the role of the concept of structure in the various subfields of mathematics and in a next step to make qualitative statements about mathematical development.

Keywords: Structure, Structural Mathematics, Bibliometrics, Review Journals.

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The Concept of Rigor in the Practice of Analysis in the Mid-19th Century: The Case of Pierre Ossian Bonnet (1819-1892)

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In the standard historiographical account, the nineteenth century is depicted as the period in which rigorous analysis emerged. More recently, it has been emphasized that mathematical rigor is, in itself, a historical concept and, therefore, subject to change. In this presentation, I propose to examine how and why certain methods of doing mathematics, and consequently the accepted criteria of mathematical rigor, were at some point seen as inadequate by certain mathematicians. To do this, I will focus on the contributions to analysis made by the French mathematician Pierre Ossian Bonnet (1819-1892).

Although Bonnet is primarily known for his work in geometry, he devoted several important articles to analysis around 1850 and had, by the end of his career, a significant influence on the principles of analysis in France. By analyzing several texts, particularly his Mémoire sur la théorie générale des series (1849), I will examine the epistemic virtues he emphasizes, his methods, his representativeness, and his originality. I will focus specifically on Bonnet's rejection of certain methods for calculating definite integrals that were inherited from

Cauchy, Laplace, and Poisson. Without introducing a conceptual revolution, he introduced new tools in the service of proof, notably the second mean value theorem, which he used on numerous occasions. I will demonstrate that Bonnet's distinctiveness in his work in analysis is reflected in his focus on real analysis rather than complex analysis, his early questioning of the legitimacy of methods used to solve convergence problems, and his conception of rigor, which was similar to that of German mathematicians, particularly Dirichlet.

Keywords: Pierre Ossian Bonnet, Analysis, Rigor, Second Mean Value Theorem, Definite Integrals.

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Iterating Weyl's Mathematical Process

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Hermann Weyl's *Das Kontinuum* (Weyl 1918) is an influential reconstruction of classical analysis within a predicative setting. In the opening section, Weyl outlines two processes that explain the generation of mathematical objects from a predicativist perspective. These are the *Logical* and the *Mathematical* processes. The latter, in particular, is instrumental to introduce new sets. From this perspective, the idea that the totality of mathematical objects cannot be considered as a *closed* totality emerges naturally. Rather, we should regard it as an *open-ended* totality. Moreover, objects are constructed in stages and the introduction of a new set can refer only to those objects that have been constructed at previous stages. In such a way, *impredicative* definitions are automatically excluded.

The aim of this work is to reconsider Weyl's ideas and provide a modern formalization of them. The result is a version of ramified type theory. It should be observed that this theory differs from the one eventually proposed by Weyl in *Das Kontinuum* (see (Avron 2020)). However, I believe that it provides a useful framework for comparing various insights related to predicativity.

I will present both the classical and the intuitionistic versions of the theory. A crucial issue, indeed, is which logic to adopt as the basis for iterating the Mathematical process. I will show that the extent of what is considered predicative depends on the choice of logic. In *Das Kontinuum*, Weyl opted for classical logic. The notion of *extensional determinateness* – recently analyzed by Crosilla and Linnebo (2023) - plays a fundamental role in his conception. I will discuss this notion and its relationship with predicatively definable sets within the present framework. In *Das Kontinuum*, Weyl also introduces a *restricted* version of the Mathematical process, in which only the sets definable at the first level of the ramified hierarchy are considered. I will show that the system resulting from the restricted process is closely related to the systems proposed by Feferman (1998) for developing predicative analysis à la Weyl.

Palmgren (2018) presents an intuitionistic ramified type theory that is constructively justified through an interpretation in Martin-Löf Type Theory. Additionally, Palmgren shows that a restricted version of Russell's Reducibility Axiom can be constructively justified without compromising the theory's predicative nature. The intuitionistic theory derived from formalizing the iterated Mathematical process proves to be equivalent to Palmgren's theory. I will propose an alternative interpretation of the theory within a version of Martin-Löf Type Theory, that is based on some notions taken from (UF 2013). Finally, I will discuss Palmgren's Functional Reducibility Axiom within this context, highlighting the interplay between classical and constructive predicativity.

The overall aim of this work is to shed light on Weyl's Mathematical Process and to investigate the implications of the choice of logic for a predicative development of mathematics.

Keywords: Predicativity, Type Theory, Hermann Weyl, Potentialism

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How to Conceive of the Algebraic Unknown? Philosophical Tools from Premodern Italy

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The paper puts forth the question 'How to conceive of the algebraic unknown?' and answers it by focusing on the historical and philosophical underpinnings of quantification in premodern European thought, particularly in 14th-century Italy. For the treatment of the unknown in premodern algebra can be seen against the backdrop of 14th-century philosophical reflections on quantity. The central hypothesis posits that the manipulation of the unknown is the result not only of technical practices and needs in mathematics, but also of a many-layered philosophical elaboration stemming from precise ideas and epistemic procedures. I identify these ideas and procedures with the nominalist notion of quantity and with mathematical abstraction.

In the first part of the paper, I will briefly shed light on the premodern Italian context, showing that the educational landscape of 14th-century Italy was characterized by the convergence of universities, studia of mendicant orders, and abacus schools. Such a convergence fostered a unique intellectual milieu. The interaction of university professors, friars, and abacus masters facilitated a cross-pollination of ideas between disciplines. It is within this context, that mathematical abstraction, algebraic practices, and the philosophical treatment of quantity intertwined.

In the second part of the paper, I will show that, in premodern algebraic practices, the equation's starting point and desideratum align with what philosophers (and theologians) of the 14th century recognized as a quantum: to the eyes of 14th-century thinkers – especially those espousing a reductionist ontology – quantity was seen as something endowed with extension, stretchable into three dimensions, and thus a material being – i.e., a quantum.

Finally, I will argue that the conceptualization of the quantum in equations as an unknown required a specific process of mathematical abstraction. Drawing on Aristotelian epistemology, this type of abstraction involves isolating the mathematical properties inherent in material objects from their non-mathematical properties, thus allowing the mathematical properties to be considered as if they exist independently of any material substance. I will further propose that, alongside this Aristotelian epistemological framework, another logical tool emerged in 14th-century logic which could complement abstraction: reduplication. Such logical device facilitated the isolation of the specific aspect under which a predicate is said of a subject by exploiting the qua-operator (i.e., the Latin *inquantum*).

Keywords: Premodern Philosophy, Unknown, Mathematical Abstraction, Quantity.

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Disease Maps and Mortality Lines: The Graphic Method in Dutch Medical Statistics, 1850-1875

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Statistical graphics are visual representations of statistical data, such as charts, diagrams, and quantitative maps. In the second half of the nineteenth century, these visualisations created new ways to give meaning to statistical data (Friendly and Wainer, 2021; Kostelnick, 2016). My research explores how the knowledge and skills required to work with these graphics developed when they were first introduced. Using the Netherlands as a case study, I look at how different groups employed the so-called 'graphic method' in statistics between 1850 and 1900. I investigate what knowledge and skills different actors considered necessary for working with the graphic method, how the visual conventions of this statistical representation evolved, and how graphic literacy circulated from field to field.

In the Netherlands, one of the first fields to adopt visual statistical methods was medical science, especially in the political-scientific programme of the *hygienists* between 1850-1875. This group of doctors advocated for empirical and data-driven public health policies, which would have to be implemented at a national level (Houwaart, 1991; Maas, 2019). Their interest in medical statistics led them to develop and standardise a graphic method to depict mortality, illness, and hygiene. Disease maps, mortality lines, and mortality atlases were the result.

The hygienists primarily developed two forms of graphic representations: quantitative maps and line graphs. They used mortality lines to track and compare seasonal or geographical variations in death rates, while quantitative maps depicted mortality rates across regions, marking areas with higher or lower mortality. Over time, they established conventions such as using specific symbols or shaded areas to represent different mortality rates, which were later codified in resources like a national Mortality Atlas. Professional societies, such as the *Nederlandsche Maatschappij tot Bevordering der Geneeskunst*, played an important role in consolidating and disseminating these practices.

In this talk, I analyse statistical graphics as well as professional debates about the graphic method in medical journals and administrative reports. Through this analysis, I show how the medical community gave shape to the graphic method, for what reasons, and what data practices they developed. My central argument is that this group of hygienist professionals established a *shared* and *consolidated* graphic method, though not necessarily a *universal* or *standard* method.

I argue that this graphic practice emerged in the Netherlands for three main reasons. First, the graphic method aligned well with the theoretical context of Dutch medicine, where it functioned as a bridge between traditional medical knowledge (such as miasma theory) and emerging statistical methods. Second, the decentralised structure of Dutch medical practice allowed doctors to share data informally while retaining their methodological independence, creating a favourable environment for the emergence and development of a new statistical approach. Third, the graphic method provided clarity and a means of standardisation, qualities that resonated with the doctors' goal to professionalise medicine and make it more scientific. Most importantly, they saw the graphic method as a 'positive' scientific method. The final and fourth point is that the hygienists also had a strong political agenda, in which statistical graphics could serve as a powerful rhetorical tool.

Although medical statistics and medical mapping have been carefully studied by historians (Houwaart, 1991; Klep and Kruithof, 2008; Koch, 2017; Tassenaar, 2014; Vandenbroucke, 1991), the role of other visual statistics like charts and diagrams remains underexplored. My research examines the connections between all these graphic forms. Furthermore, I highlight the independent dynamics of the graphic method within medicine, thereby extending upon the work of medical historians who usually consider statistical graphics as one of many ways through which medical theory can be expressed. Ultimately, I hope to contribute a more nuanced understanding of the role that visual representation plays in the history of medicine and statistics.

Keywords: History of Data Visualisation, Statistical Graphics, Medical Statistics, Mortality Atlas, Dutch Medical History, Nineteenth Century.

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Theoretical virtues in Mathematics: There's More Than Beauty

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Theoretical virtues such as simplicity and explanatory power are valued by scientists because they enhance the credibility of scientific theories. Philosophers have long been interested in how these virtues attain their positive epistemic value (Douglas, 2013). Mathematicians also discuss virtues such as simplicity and explanatory power, but they often emphasize mathematical beauty as the primary theoretical virtue that drives their research (Engler, 1990). Although beauty is also valued by scientists (Ivanova, 2017), the ways in which mathematicians rely on beauty appear to be different. As Hardy (1940/2012, §10) put it, there is no permanent place in the world for ugly mathematics.

To explain its alleged epistemic value, mathematicians and philosophers have proposed various definitions of mathematical beauty. However, these definitions often conflict with each other and fail to accurately describe how mathematicians use terms such as "beautiful" or "elegant" (Inglis and Aberdein, 2014). This discrepancy creates a mismatch between the importance mathematicians place on beauty and the apparent failure of philosophical accounts to capture its positive epistemic value.

In this talk, I propose a solution to this mismatch. I argue that mathematicians, like scientists, rely on multiple theoretical virtues when judging proofs, theorems, and conjectures. Not all of these virtues have positive epistemic value: some are truth-conducive, while others may serve merely pragmatic purposes. Theoretical virtues in mathematics are therefore a diverse set, and beauty is only one of them.

To support this argument, I take a two-step approach. First, I'll review prominent defi-

nitions of beauty found in the literature. I'll argue that they fail to explain its epistemic value and fail to generalize. Second, I'll argue that this failure can be explained by taking the components of the definitions at face value: they are all to be found in the diverse set of virtues that mathematicians rely on when judging proofs.

Keywords: Beauty, Theoretical Virtues, Epistemic Value, Mathematical Practice, Comparative Philosophy of Science.

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Alan Turing's Contributions and Influence on Computational Mathematics

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Alan Turing is a leader in theoretical computer science and mathematical logic. The advent of the "Turing machine" in 1936 made Turing one of the pioneers of *Computability Theory.* This also made Turing widely known to computer scientists and mathematical logicians. However, it is worth noting that Turing also made important contributions in other areas of computing. This paper focuses on Turing's contributions in the field of *Computational Mathematics* (*Numerical Analysis*). In 1948, Turing published a paper entitled "Rounding-off Errors in Matrix Processes", which discussed several numerical solutions to linear equations. The numerical solution of linear equations has always been a core topic in *Computational Mathematics.* Based on the original literature, this paper first emphasizes that Turing also made a fundamental contribution to the creation of the discipline of *Computational Mathematics.* Secondly, it further improves the historical route of numerical solutions to linear equations in the 1940s. More importantly, we discuss the real motivation of Turing's proposal of the *LDU Matrix Decomposition* theorem.

Keywords: Alan Turing, Computational Mathematics, System of Linear Equations, Matrix Triangular Decomposition, History of Mathematics in the 20th Century.

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How Are Number Words Used in Early China? On LIU Hui's Understanding of Mathematical Language.

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Ever since Frege (1884 [1988]) argued that number symbols must refer to independent objects, questions about the nature of numbers have taken the shape of questions about reference. Following Frege, it is usually assumed that in mathematics numbers are used as singular terms, as in "the number is four", while in natural language they are used more often as attributive terms, as in "four moons". Even objections against Frege like (Hofweber, 2005) or (Moltmann, 2013) are based on this distinction. Now in ancient Chinese mathematical texts, number words appear most often (though not exclusively) in adjectival position, as in "4 persons" or "7 horses". We might therefore think that they appear as parts of referential noun phrases, for example the expression "4 persons" refers to 4 persons. On this view however some of the referents would be fractions of persons or other impossible entities, a clear sign that we should not take the expressions literally. Chinese mathematics thereby challenges common assumptions about the use of numerical expressions. Martzloff (1997, p. 90) has suggested that for early Chinese mathematicians number words refer to the number symbols on a counting board (and therefore to material objects). Based on careful analysis of a passage from Liu Hui's *Commentary to the Nine Chapters on Mathematical Procedures* I argue that this and similar interpretations run into difficulties as well.

In the next step I argue that theories of reference are not a good tool to analyse ancient Chinese mathematics. Liu Hui's own understanding of mathematical language should be placed in a Neo-Daoist context. I suggest that to him number words can function similar to properties like "tall" or "ugly", which in Daoist philosophy do not belong to any thing intrinsically, but depend on how we judge their relation to other things. This does not imply that the use of number words is subjective, but rather that there are different correct mathematical descriptions for the same situation. Liu Hui sees these different descriptions as an important creative tool to be employed flexibly by the mathematician. I argue that this understanding offers a highly convincing explanation of the mathematical context at hand thereby making it preferable to theories which are based on the mathematics of other cultures. Liu Hui's *Commentary* displays a nuanced and sophisticated understanding of numbers which makes it evident that the use of numbers in adjectival position does not necessarily correspond to an "early state" in the development of arithmetic concepts.

Even more generally, when we apply mathematics to real-life situations number words are only partially determined by counting and measuring while structural relations frequently take precedence over them. Liu Hui's understanding of language can account better for this feature than theories of reference. For him, number words are based on the things around us, but not uniquely determined by them. Even though Liu Hui's interpretation cannot be directly transferred to modern mathematics it has attractive features which point to a viable alternative to current accounts of mathematical language.

Keywords: Chinese Mathematics, Numbers, Numerical Expressions, Quantification and Ontology.

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The Dichotomy of Recreational Mathematics in History

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As many know recreational mathematics is historically grounded, that is, it has a firm body of knowledge and practitioners, with examples of Mathematical Recreations ranging back to the Sumerians. The treatment of this historical material is as versatile as the subject itself. The views range from a redundancy, all of mathematics is recreational, to an antinomy, a formal subject by definition not being ludic.

In this presentation we will take a look at a big picture of the subject, starting today and diving into the past up to the 17th century when the name *Récréations Mathématiques* was first coined as title of printed octavo format anthology. This serves as grounds for discussion on how history of mathematics informed its readers and consequently how mathematicians themselves view what can be considered a multidisciplinary branch of knowledge that joins history, education, popular culture and, of course, our queen and handmaiden, mathematics.

Keywords: Recreational Mathematics.

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A Geometrical Method in Habash al-Hāsib's *Zīj* for Finding Solar Longitude

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During the Islamic Era, diagrammatic reasoning was a key method for solving complex problems, particularly in astronomy. One notable example is analemma, a geometrical tool used to solve problems related to the celestial sphere and Earth. This technique allowed astronomers to transform intricate three-dimensional problems into more manageable two-dimensional representations, thus avoiding the need for complex trigonometric calculations. While the concept of the analemma likely originated in Ancient Greece, it was extensively employed by Islamic astronomers in their works (Brummelen, 2009, p. 66).

One of the rare and complex use of the analemma as described in the MS Istanbul Yeni Cami 784/2 copy of Habash al-Hāsib Marvzi's *Zīj*. Habash al-Hāsib (766 – after 869) was one of the earliest Islamic astronomers to utilize the analemma into his work. This manuscript was written in Arabic and it is one of the most prominent referenced *Zīj* and contains magnificent information (Debarnot, 1987). This Analemma details a graphical procedure for determining the solar longitude (the position of sun along the ecliptic on the celestial sphere) based on the shadow of a gnomon at a specific time and latitude (Debarnot, 1985, pp. 49–50).

The detailed process that is described by Habash al-Hāsib begins with drawing a reference circle centered on the base of the gnomon, determining the cardinal directions and meridian line. Next, the shadow of the gnomon is marked to determine the sun's altitude at the given moment. The next step involves sketching top view of the celestial sphere. Subsequently, the front view of the problem will illustrate, that the equator line is drawn across the referenced circle and marking crucial points to aid in finding the sun's position relative to the celestial equator called which is "declination". Finally, the solar longitude is determined accurately by simulating the ecliptic on the referenced circle and utilizing the derived declination. The whole steps are only graphically. The precision of Habash Hāsib's approach is revealed by mathematical proof.

This exploration emphasizes the influence of ancient Greek knowledge, adapted and refined by Islamic scholars.

Keywords: Habash al-Hāsib.

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Fictionalism and Counterpossibles

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Can we account for the content of mathematical talk without countenancing abstract objects? Platonists say no, nominalists say yes. The platonist accounts for the content of such talk via direct reference to abstract objects. As the nominalist is committed to a different view of the ontological domain – one on which everything that exists is concrete – she cannot take the realist, direct referential approach. She thus incurs the burden of accounting for the content of mathematical statements otherwise. Fictionalism is a prominent anti-realist strategy for meeting the nominalist explanatory burden. Contrary to the platonist's categorical acceptance of the apparent commitments of mathematical statements, the fictionalist accepts such commitments only hypothetically, as a means to capturing the concrete content of mathematical statements. One brand of fictionalism – namely, counterfactualist fictionalism (CF) – captures fictionalism's hypothetical stance via counterfactual conditionals: the existence of abstract objects is entertained counterfactually, and the content of mathematical statements is what holds in such counterfactual scenarios [4, 1]. On this approach, the truth-value of categorical sentences involving apparent commitment to numbers is parasitic on the truth-value of counterfactual statements involving the supposition of the existence of numbers. To bear out his proposal in a principled way, the CF-ist needs a systematic account of the truth-conditions of counterfactuals, one that delivers adequate verdicts of truth/falsity with respect to categorical statements involving abstract objects.

That an account of counterfactuals adequate for CF exists is put into doubt by the orthodox semantics of counterfactuals [6, 3], viz. its vacuous verification of counterfactuals with impossible antecedents (or counterpossibles). If nominalism is read as a thesis that holds of (metaphysical) necessity, the statements the CF-ist hypothetically entertains within the antecedents of counter- factuals (e.g. 'There exist numbers') are metaphysical impossibilities. But then, a combination of nominalism and an orthodox semantics yields the verdict that all fictionalist counterfactuals are vacuously true. This result is unacceptable from the viewpoint of CF.

The trivialising result can be avoided if we accept an alternative semantics, one on which counterpossibles can be non-vacuously true or false [1], [5]. Williamson's recent defence of a vacuous semantics for counterpossibles [7, 8] suggests, however, that this revisionary road is best taken as a last resort.

This presentation considers the prospects of CF in light of the newly sustained case against non-vacuous treatments of counterfactuals. We survey available lines of fictionalist resistance – e.g. endorsing an unorthodox semantics for counterfactuals, modifying the fictionalist strategy – and argue that they are insufficient if they are put forward as ad hoc repairs. This leads us to a powerful desideratum: any defense of an alternative fictionalism must provide a parallel and connected revision of the standard semantics for counterfactuals.

Relying on previous work by Fine [2] and Yablo [9], we suggest that a proposal meeting this desideratum can be formulated via a state-based approach. On our approach, the metaphysical impossibilities entertained by CF are inconsistent, null-states; consequently, the approach con- serves orthodoxy in that counterfactuals whose antecedents refer to such states are vacuously true. Nevertheless, our state-based approach also allows us to extract a proper part of such inconsistent states that involve only concreta, and are thus consistent by nominalist lights. We suggest that the structural properties of such consistent parts of inconsistent states account for the intended predictions of CF. We thus argue that a core of CF can be vindicated despite accepting a vacuist account of counterpossibles.

Keywords: Nominalism, Fictionalism, Counterpossibles, State Semantics.

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An Historical Approach to Abstractionist Mathematical Structuralism

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The literature on mathematical structuralism has focused solely on mathematical objects themselves. Problems pertaining to object definitions have been proliferous in the literature (Keränen, 2001, Shapiro, 2008, Linnebo, 2008, Giovannini and Schiemer, 2021) with little to no reference to the role relations play in the structure. In addition, there is little discussion on historical developments prior to the nineteenth century that is utilised in the abstractionist literature. Resnik (1982) provides a brief treatment for an in re structuralist account, yet the discussion is superficial. There nonetheless remains the question of how a structuralists stance factors into pre-nineteenth century mathematics.

This paper will provide an abstractionist account that looks at objects, relations and their interactions in three historical epochs of mathematical development where abstraction is most evident: ancient Greek mathematics, seventeenth century and the nineteenth century. The aim is to reframe the discussion on mathematical structuralism by focusing on relational abstraction, which is informed by the historical development of mathematics from ancient Greek approach to mathematics, which prioritised object ontology, to the 19th-century axiomatisation that prioritises the role of relations.

Three levels of abstraction are identified:

1. Relational Abstraction: Defining relations extensionally over the abstracted objects. We abstract relations and their properties from physical states of affairs, leading to a system of relations invariant under object permutation. This was the case in ancient Greek geometry where one could see methodological advancements that are informed by discussions surrounding object ontology.

2. Relational Generalization: Removing the determinate nature of abstract objects, focusing solely on their functional roles within a system of abstract relations. The relations become intensionally defined. This is the case with 17th century mathematics, where the loss of object ontology permitted the finding of solutions to previously unsolved problems by ignoring the underlying object ontology. This is also evident by the discussions surrounding the nature of mathematics that followed methodological advancements.

3. Categorical Axiomatization: Defining objects implicitly via categorical (or more

broadly, definitional) axioms based on our needs and the ability to recreate system results. This is the case with 19th century mathematics and the use of concepts in mathematics, as opposed to magnitude and/or quantities. In this period mathematical structuralism as we know it emerged and provided a basis for the field.

These steps, while not exhaustive or completely distinct, represent incremental historical developments in mathematical abstraction. Relational abstraction necessitates an ontology of the objects in a system. The relations in states of affairs will be dependent on the objects to be meaningful. Ancient Greek mathematics exhibited such ontological commitments. The acceptance of mathematical results depended on the ontology of the objects, which restricted the operations (Klein, 1992).

Whereas in the previous step, the nature of the objects in mathematics was known, no such restrictions are present in relational generalisation. The removal of object ontology and the generalisation of the relations brought about the advent of symbolic mathematics in the seventeenth century, the most important advancement of mathematical practice of the time (Mancosu, 1996, Mahoney, 1980).

Given the generalisation of the system and the loss of objects ontology, the third step is finding a suitable basis from which one can recreate the results of the generalised system while simultaenously in a way that ensures reliability. From a structural perspective, this is done via incorporation of intensionally defined relations into categorical axioms that can define the objects. This is the hallmark of nineteenth century mathematics which admits talk of a conceptual approach to the field (Ferreirós and Reck, 2020). In doing so, mathematical entities are no longer abstracted objects, but rather concepts whose properties are derived from the axioms.

Keywords: Philosophy of Mathematics, Mathematical Structuralism, History of Mathematics, Metaphysics of Relations.

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Integrating History of Mathematics and History of Physics: Insights from Hermann Weyl's Unified Field Theory

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Throughout history, mathematics and physics have consistently cross-pollinated each other, with advancements in one field often catalyzing progress in the other. However, the study of the histories of these disciplines has traditionally been treated rather separately, within different scholarly communities. In my talk, I would like to propose an integrated approach to the histories of mathematics and physics, using Hermann Weyl's unified field theory (UFT) as a case study.

Weyl's UFT has been extensively covered in historiographic literature, but often from a rather singular perspective. Some works focus on the history of physics, emphasizing the theme of unification, while treating the mathematical aspects as background information (Vizgin, Sigurdsson). Others approach it from a history of mathematics standpoint, with only cursory references to the historical context of physics (Scholz).

While the existing historiographic work on Weyl's UFT is undoubtedly of high quality, this talk aims to offer an integrated historical perspective informed by both the history of physics and the history of mathematics. This approach promises to yield more than just a synthesis of two viewpoints. Key aspects of Weyl's work, such as the concept of gauge, may take on altered meanings, enabling novel interpretations.

Drawing from this discussion of Weyl's UFT, I would finally like to explore some general benefits of an integrated history of mathematics and physics, as well as potential challenges and obstacles. I believe that by bridging the gap between these disciplines, we can gain a more comprehensive understanding of their intertwined development and uncover new insights that may have been obscured by traditional disciplinary boundaries. If time permits, I will also briefly suggest further historical examples that seem suitable for an integrated treatment, highlighting the broader implications of this approach.

Keywords: Hermann Weyl, History of Mathematics, History of Physics, Unified Field Theory, Gauge Theory, Interdisciplinarity.

Viewpoint Realism in Mathematics

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In this talk, I will articulate a position called 'viewpoint realism', which has its roots in the mathematical and philosophical work of Alexander Grothendieck. According to this position, our grasp of mathematical objects, problems and theories is mediated by a multiplicity of viewpoints, which are partial ways of representing, and therefore of understanding, mathematical data. In some passages of *Récoltes et Semailles* (Grothendieck [1985-1987]), Grothendieck describes the following features of mathematical viewpoints:

- Incompleteness: a viewpoint on X is a partial grasp of X.
- Function: the function (or job) of a viewpoint on X is to generate questions, concepts and statements that unify X.
- Evaluative character: viewpoints on$X$ can be evaluated as better or worse depending on how good or bad they allow for an understanding of X.
- Expressiveness: viewpoints are expressed in language. Different viewpoints require different languages.
- Realism: viewpoints are directed at independently existing mathematical structures.

Although these features do not give us necessary and sufficient conditions for viewpoints, they are a guide to identifying them. In this talk, I will examine two cases that have to do with the history of algebraic geometry:

1. The turn from varieties to schemes had at its core a new, geometric way of understanding rings: for any commutative ring R, there is an affine scheme, called the spectrum of R. If we impose some restrictions on the ring (Noetherian, nilpotent-free, etc.), we get the old varieties, but such restrictions are not necessary for the general theory. The main point here is that some features of rings, their geometry, were not visible before, but are visible now, even if the concept of a ring has not changed at all. This change of viewpoint allowed for a treatment of discrete structures (varieties defined over finite fields) in terms of topological methods and tools, previously reserved for non-discrete or continuous objects (topological spaces)

2. The introduction of the concept of Grothendieck topos can be seen as an algebraic understanding of the old concept of topological space, in the sense that the 'essential' features of the space are captured by its category of sheaves. Unlike a topological space which, in general, does not have a lot of structure, its category of sheaves has a lot of structure that one can take advantage of when doing computations

Viewpoint realism in mathematics has some similarities with perspectival realism in science (Giere 2006, Massimi 2022), in particular, they both emphasize that our grasp of the data is always partial, that theories are essentially incomplete, and that they are, nonetheless, directed at an independently existing reality. The main difference between the two is that viewpoints a la Grothendieck are not models in the sense of philosophy of science. After discussing these similarities and differences with perspectival realism in science, I will turn to consider two arguments against viewpoint realism. The first is the argument from objectivity:

1. If viewpoint realism is true, then our best theories of X depend on viewpoints on X, at least for some X.

2. If mathematics is objective, our best theories of X never depend on viewpoints on X.

3. Mathematics is objective.

Therefore, viewpoint realism is not true.

The second argument is the charge of psychologism:

1. Viewpoint realism is (mainly) a thesis about understanding in mathematics.

2. The concept of understanding applies only to psychological facts and processes. mathematics is about is completely independent from psychological considerations.

Therefore, either viewpoint realism is a form of psychologism, or it's not about mathematics *per se*.

Keywords: Grothendieck, Scheme, Topos, Perspectivism, Realism.

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Two Non-Cantorian Intuitions about Infinite Sizes

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Cantor's notion of cardinality is widely seen as a successful way of extending the familiar notions of counting and size from finite sets to infinite sets. By singling out the existence of a one-to-one correspondence between two sets as the relevant criterion of their equinumerosity, Cantor provided the foundations for a robust mathematical theory of the infinite, and offered a solution to some well-known "paradoxes of the infinite" (Galileo 1958, Russ 2004). However, recent mathematical developments have sparked a renewed interest in the possibility of mathematically viable alternatives to the Cantorian solution. In particular, the recent theory of numerosities of Benci and di Nasso (2003), inspired from the methods of nonstandard analysis, seems to provide solid mathematical grounds to the old Euclidean intuition that the whole should always be greater than any of its proper parts. This raises the question whether, as Gödel thought (Gödel 1947), Cantor's definition is the only possible one (Mancosu 2009).

In this talk based on a recent paper (Massas 2024), I will argue that the debate can benefit from distinguishing between two separate non-Cantorian intuitions. The first one, which I call the Euclidean Constraint, is fully captured by the idea that a set should never have at most as many elements as one of its proper subsets. The second, which I call the Density Intuition, is the intuition that the size of a set of natural numbers is (at least partially) determined by the distribution of its elements along the sequence of natural numbers with their usual ordering. After briefly discussing historical and more recent attempts at developing theories of size that are based on one of these two intuitions, I will argue that the Euclidean Constraint is the only one that can give rise to a viable alternative to the Cantorian notion of cardinality. Time permitting, I will also present what such an alternative may look like.

Keywords: Mathematical Infinite, Part-Whole Principle, Cardinality, Galileo's Paradox.

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From Problems to Objects: the Emergence of the Theory of Quadratic Forms

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Lagrange's *Recherches D'Arithmetique* are usually considered the first systematic study of quadratic forms. However, the notion of quadratic forms that we find in this work is deeply rooted in the resolution of problems of indeterminate analysis, a perspective that differs from the one found in Gauss's *Disquisitiones Arithmeticae*, in which quadratic forms are treated as mathematical objects whose nature and properties will be studied — an essential shift for the development of 19th century mathematics. In this talk we will trace the historical development of the notion of quadratic forms, beginning with their origins in the context of indeterminate analysis and following their gradual transformation into well-defined mathematical objects.

We argue that this conceptual shift was a result of a process that can be followed through three influential works: Lagrange's*Additions* to Euler's *Algebra*, Legendre's *Essai sur la Theorie des Nombres and Gauss's Disquisitiones Arithmeticae* (*D.A.*). Together with the *Recherches D'Arithmetique*, these texts reveal changing views among the authors on the branch of mathematics they address, the nature of the problems they seek to solve, and the tools and methods they devised to the solve them. The theory of quadratic forms presented in the *D.A.* opened the door to a vast number of developments in later mathematics and served as a pivotal moment in the development of number theory (as is thoroughly explored in Goldstein, Schappacher, and Schwermer 2007). However, this theory traces back to Lagrange and Legendre's results, although the theory of quadratic forms found in each of them differs significantly in its conceptual framework. We propose that in order to fully understand the transformation from one into the other, it is necessary to consider not only the shift in perspectives but also a broader methodological development that took place through the production of the authors.

We will point out how the *Recherches* emerged from a set of works by Lagrange written between 1768 and 1770 on indeterminate analysis and numerical equations. These works, brought together and further developed in the *Additions* to Euler's *Algebra*, expanded gradually in scope, motivated by a search for greater generality and the new possibilities introduced by the evolving methods. The ideas that were developed there, together with the increasingly sophisticated tools and methods, led to the questions posed in the*Recherches* — questions that ultimately laid the ground for both the conception of quadratic forms as objects and the formation of a theory dedicated to the study of their properties.

We will then explore how Legendre builds on the questions and results set in the Recherches, while still working within the framework of solving indeterminate equations and viewing these results as a way to derive theorems about prime numbers. It is not until Gauss's *D.A.* that the results obtained by Lagrange and Gauss are reframed within a comprehensive study of the nature of a mathematical object of its own: quadratic forms. By comparing the approaches of these three authors, we aim to illustrate this progression showing how key results concerning the discriminant, reduced forms, and equivalent forms — initially serving as tools in Lagrange's methods— take a different role in Legendre's work and, with Gauss, become fundamental properties of quadratic forms as objects of study. Together, these works capture the development of a theory of quadratic forms from results involving forms that aid in the resolution of equations or exploration of prime number theorems to a study of properties of objects that became central to number theory.

Keywords: Quadratic Forms, History of Number Theory, Lagrange, Legendre, Gauss.

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Hadiyyat al-Muhtadi: A Synthesis of Ottoman and European Mathematical Traditions in 18th-Century Belgrade

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In the 18th century, the Ottoman scientific tradition underwent a significant transformation as it sought to integrate new types of knowledge produced in Europe with its established scientific practices. A notable example of this transformation is the work *Hadiyyat al-Muhtadī* (The Gift of the Converted), written approximately 250 years ago in Belgrade by *Osman b. Abd al-Mannān al-Muhtadī*, who was likely of Hungarian or German origin. This work explores *ilm al-misaha* (the science of measurement), a subfield of *hendese* (geometry at that time) developed within Islamic civilization by combining quantity and magnitude, with practical applications focusing on architecture, land surveying, and artillery. Osman b. Abdulmennân aimed to modernize the science of measurement, establishing it as the primary reference for all measurement-related sciences. To this end, he studied traditional works on geometry and measurement, using contemporary German and French sources, and completed the work over the course of four years.

This study examines the geometric methods applied to measurement science in the author's manuscript (folios 2b-89a), analyzing the types and steps of propositions in both theoretical and practical sections. For this purpose, *Hadiyyat al-Muhtadī* is first compared with two of the most influential classical Ottoman sources in geometry: Kadızâde-i Rumi's *Šarḥ Eškâl al-Te'sīs* (1412) and the measurement section of Bahā' al-Dīn al-'Āmilī's *ulasat al-isab* (17th century). Given the author's reference to German and French sources, the study also considers *Anfangs-Gründe aller Mathematischen Wissenschaften* (1750) by Christian von Wolff and *Nouveau cours de mathématique à l'usage de l'artillerie et du génie* (1725) by Bernard Forest de Belidor as possible influences. This approach also raises questions regarding the origins of the author's style and methods.

The findings reveal that a geometric approach is predominant in the manuscript's presentation. Definitions are provided at the beginning, followed by discussions on the properties, construction, and measurement of shapes and solids, with more than eighty propositions included. The narrative incorporates new types of propositions from European sources, such as lemmas and corollaries. In the measurement sections, the author follows each proposition's steps, using geometric proofs in some cases, while elsewhere applying a sensory proof method (*hissi burhan*). The work references Plato, Archimedes, and Euclid, while also addressing terms and methods used by contemporary geometers and engineers. When comparing the referenced sources, it is evident that the author not only drew inspiration from each source but also interpreted and adapted them, rather than providing a direct translation. This approach aligns with the purpose of his work. The author saw the integration of new knowledge into the Ottoman scientific tradition as essential to continuing it, carefully synthesizing knowledge from various sources into a coherent whole within his own style. Thus, he achieved his goal of updating *ilm al-misaha* by synthesizing classical Ottoman scientific heritage with contemporary European knowledge.

This analysis contributes to a deeper understanding of the history of mathematics in the Ottoman Empire and offers insights into how European developments influenced scientific tradition in the Ottoman Empire.

Keywords: Hadiyya al-Muhtadī (The Gift of the Convert), Geometry, Measurement, Elements, al-Usûl.

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Structural Analogies in Mathematics and the Disagreement Between Turing and Wittgenstein

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The foundational crisis of the 19th century in mathematics led to a renewal of the analogies used to describe mathematics. In particular, structural analogies became popular (e.g., Hilbert's building analogy) and contributed to the shaping of particular images of mathematics and regulating mathematical practice. In fact, many mathematicians and scientists today are influenced by an image of mathematics as like physical structures. For instance, they see contradictions as points of structural vulnerability in pure mathematics that can lead to the collapse of mathematics. In applied mathematics, structural similarities are considered to be at the base of applicability, and contradictions in this context lead to failures in the coordination between mathematics and empirical phenomena. This is a backdrop that heavily influenced Turing's and the later Wittgenstein's opposite views of mathematics, the former endorsing it, and the latter very originally departing from it.

In particular, the disagreements between Wittgenstein and Turing during Wittgenstein's lectures in Cambridge have been used to clarify their views of logic and mathematics (especially Wittgenstein's). The structural analogies employed in their discussion remain underexplored, despite playing a pivotal point in the discussion: they illustrate Turing's arguments for his view of logic and mathematics as structures that in turn mimic empirical structures, and in Wittgenstein's rejection of such a view. In turn, they are key to each one's tolerance (or lack thereof) of contradictions in logic and mathematics, both in pure and applied contexts. Furthermore, both Turing and Wittgenstein attempt to convey and convince each other of their views via these analogies, often to no success, given their limited elaborations on these analogies. I clarify Turing's and Wittgenstein's attitudes towards structural intuitions in logic and mathematics by analyzing structural analogies in the Lectures on the Foundations of Mathematics (Wittgenstein, 1976). This leads to a better understanding of their views and clears up some famous misconceptions about them. One notable example is Chihara's criticism of Wittgenstein's alleged failure to satisfactorily address Turing's objections to Wittgenstein's suggestion of just not deriving anything from a contradiction (Chihara, 1977): an analysis of Wittgenstein's analogies shows that he has a compelling reply to Turing. Furthermore, this enhanced understanding of the later Wittgenstein's philosophy of mathematics allows us to connect it to contemporary debates on the applicability of mathematics.

Keywords: Disagreement in Mathematics, Mathematical Structures, Mathematical Rules, Later Wittgenstein, Turing, Analogies in Mathematics.

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The Academic as Politically Conscious Subject: Towards a Critique of Social Production in History and Philosophy of Mathematics

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As historians and philosophers of mathematics, studying the production and circulation of knowledge makes up our everyday research life. How we thereby produce and circulate knowledge ourselves is, nevertheless, rarely raised as a critical issue nowadays. The goal of our joint presentation is to raise this critical issue with our audience both through the content of our contribution and through its non-standard format.

Through our training as doctoral students in the humanities, we know that the academic industry is not a politically neutral entity: colonial, feminist, and certain historical studies have already long concluded that there can be no disinterested production of academic knowledge. And so, we ask: why is it that this conclusion has also not profoundly affected the way we ourselves organise the academic production processes we are part of?

In other words, as researchers, we are assured of the freedom to choose projects and research questions; we are required, no less, to dismantle the ideologies of the past by availing ourselves of the most advanced tools critical thinking provides. But we are rarely told to stop and consider: What current ideological position are we supporting by following our institutional understanding of 'standard', 'objective', 'scientific' academic production? Where do the resources we are given come from, and what are they paying for?

It is these kinds of questions, we suggest, that academics need to tackle. We find ourselves working in and for academic institutions within a political economic order — institutions that contribute to the fortification of powerful ideological positions within that order. It is precisely this imposition of ideology, we argue, that discourages researchers to reflect upon their own production processes, or indeed, to act upon those reflections as politically conscious subjects with agency to affect the dominant ideology.

Politically conscious academics in Western societies, thus, find themselves entangled in a contradiction. Ostensibly, freedom to say what they wish in their projects, and an obligation to be incisive, innovative, critical. In practice, methods and templates scrubbed clean of political inflection. Academics may say what they wish so long as they say it the way they are told to say it: in the appropriate form, within the walls of jargon, to a community of fellow academics, quietly. Conscious reflection upon how this contradiction affects the institutionalised production process of knowledge can yield some clarity on the structural limitations of our own research projects and, perhaps, unlock suggestions of how to overcome some of those limitations.

In our presentation, we will share how the three of us — doctoral students in the history and philosophy of mathematics working at ETH Zurich — have been confronting the various manifestations of the contradiction in our work. Amongst other examples, we will discuss 1) how we have had to adopt a certain rigid analysis method of mathematical terms to satisfy an academic standard, yet how that method belies the main point of the research; 2) how the imposition of an academic writing style reproduces the precise bias about mathematics that the research itself is trying to dispel; 3) the near impossibility of sharing our research about mathematical practice with a non-elite community for which that research is intended.

Becoming aware of this contradiction that haunts Western academics — being free to talk about politics but unfree to do it in a political manner — must be the first step in tackling that contradiction. It is our hope that our unconventional mode of presentation can encourage more fellow early career scholars to contribute to this effort of reimagining the academic process of knowledge production.

Keywords: Social Production of Knowledge, Institutionalisation of History and Philosophy of Mathematics, Academic Contradiction, Academic Writing, Methodologies, Politics of Science.

The Logic of Mathematical Beauty

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In the proposed talk, I will examine the role of aesthetic judgements in mathematics. There is a long tradition of combining mathematical philosophy and aesthetics, represented by the rationalist idea of objectivity in mathematics, which stretches back from the early modern period to antiquity. My aim is to develop a theory of the semantics of aesthetic expressions concerning mathematics, by showing that the subjective tradition, that become prominent around the middle of the 18th century by Baumgarten and Kant should by no means be understood as a counter-position to the rationalist view, but rather as a special case of it.

Methodologically the analysis focuses on aesthetic expressions attributed to mathematical entities. The conceptual analysis will then be enriched with a metaphysical interpretation in a controlled manner, so that finally a theory of the semantics of aesthetic expressions in mathematics can be developed. The first part contains a historical analysis of rationalistic theories if aesthetics, where mathematics plays a central role. In the second part the results will be transformed into a formal semantics of aesthetics using the technical apparatus of Arthur Prior's hybrid logic.

The root for such an interpretation is set in Prior's late work. Firstly, Prior's philosophical aim of investigating hybrid logic was to model tense logic as a theory of A-series talk, where instants in time are not independently existing objects, but conjunctions of propositions simultaneously being true within a framework of modal logic. But Prior begins to cast doubt on the metaphysical priority of tense and considers a kind of perspectival metaphysics influenced by Leibniz. There he uses examples from aesthetics to explain what he intends. I will explain that this modal way of talking about aesthetics is not accidental. On the contrary: For the present study Prior's hint at Leibniz is crucial, because it provides the analytical instrument to explain that the so called "rationalistic project" implies an aesthetic theory that can be elegantly formalised by a system of modal logic.

In the end we obtain a system designed to represent a formal semantics of aesthetic concepts concerning mathematics, which is indeedly based on the objective tradition of aesthetics containing the philosophical assumptions of the subjective tradition as special cases.

Keywords: Mathematical Beauty, Rationalist Aesthetics, Hybrid Logic, Formalism, Kant, Leibniz, Arthur Prior.

Geometry in a World of Rituals: On the Culture of Abstraction in the Sulvasutras

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One often presumes that ancient mathematics couldn't have been as abstract as its modern counterpart. With specific reference to Baudhayana's ritualistic treatment of geometry, this paper seeks to dispel the myth that apparent under-formalization is the same as under-abstraction. Since modern mathematics gives us ready examples of abstract objects, we are often prone to rate it higher in terms of an achievement in abstraction.

To consider mathematics as one of the manifestations of a culture of abstraction is one way of reframing the investigation in order to avoid falling into such traps. Alberto Toscano's reading of Whitehead(2008) has been insightful in tracing the history of abstraction as a culture vis-a-vis the history of mathematical abstraction. What can one say, in particular, about the nature of abstraction in explicitly pre-scientific domains like myth, magic and ritual? Some answers could be found in anthropology. Wagner (2009) has been one of the first thinkers to boldly employ the study of abstractions in anthropological literature to produce analogies that capture the complex character of mathematical variables.

This paper takes Wagner's cue forward and attempts to show how the abstractions implicit in the pre-mathematical/non-mathematical ritual are harmonious with the philosophy of geometry implied by the actual mathematical content of the proof. Ever since the likes of Seidenberg (1961), there is growing consensus on the supposedly non-Western origin of the Pythagoras theorem. Seidenberg has gone beyond a resolution of the origin debate and underlined the role of ritual in motivating the discovery of a geometrical fact. However, there is little engagement with the manner in which the epistemology of mathematical discovery is tied up intricately with the conceptual nature of the ritual. The formal aspects of the ritual ("the sum of the gods of the two sides of a right-triangle must be the god of the hypotenuse") invoke abstractions that necessitate a specific proof of the theorem over another.

With a deconstruction of the relationship between the distilled mathematical facts and the supposedly non-mathematical world, this paper will lay out a clearer understanding of the nature of the totality that is implicit in the demonstration of certain mathematical facts. (Modern mathematics often takes totalities like infinite space, infinite number line for granted)

Keywords: Abstraction, Formalization, Pythagoras, Geometry, Non-Western Mathematics.

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Imperial Russia and the International Congresses of Mathematicians, 1897 to 1912

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The society and culture in Russia have throughout history developed differently than in Western Europe due to their idiosyncratic religious, political, and cultural traditions. By the mid-1660s, the nascent Western European identity was shaped by the creation of the scientific societies such as *Accademia dei Lincei* in Rome, *Royal Society* in London and *Académie des sciences* in Paris, whose members promoted the study of natural philosophy and questioned the relevance of ancient thinking at a time when new scientific ideas were starting to unfold. This served as an example for the establishment of the *Imperial Academy of Sciences* in the eighteenth-century St Petersburg which signified the beginning of an intellectual era that laid the foundation for the great contributions of Russian scholarship to all branches of science.

The work of N.I. Lobachevsky (1792–1856) on non-Euclidean geometry, the school of mathematics in St Petersburg fathered by P.L. Chebyshev (1821–1894) and his disciples, and the mathematical societies in Moscow and Kharkov attracted considerable attention from the scholarly community in the West. Russian mathematicians contributed to a variety of fields in mathematics, e.g., probability theory, applied mathematics, and partial differential equations, and kept the *Academy* connected with the society at large. They were renowned for their pedagogical heritage and admirable international connections, for instance A.M. Lyapunov (1857–1918) was elected a corresponding member of the *Académie des sciences* and *Lincei*, and Sofia Kovalevskaia (1850–1891) was mentored by eminent professors in Germany surrounded by Karl Weierstrass (1815–1897).

In light of greater opportunities for international connections, the first International Congress of Mathematicians (ICM) took place in 1897 in Zürich, Switzerland, followed by four other meetings before the eruption of the Great War. How did this cooperation in institutional form change the course of development of mathematics in the Russian Empire? And for those delegates that were members of the congresses, was there anything unusual about them being there apart from the scientific motive? Furthermore, how did the expressed goals of the congress, such as internationalism, differ from the actual participation and international undertakings of mathematicians?

My paper explores some of the ways in which Russian mathematicians were involved with the ICMs between 1897 and 1912, drawing upon the conference proceedings and published personal stories. I consider how some Russian scholars were not just motivated by hearing from the main currents of scientific thought about advancements in mathematics. For instance, in addition to turning Kazan University into an important centre of mathematical research and resurrecting the legacy of Lobachevsky, A.V. Vasiliev (1853–1929) was an internationalist who contributed greatly to the promotion of the ICMs through his international world views and correspondences. Opposite this collectivism was Samuel

Dickstein (1851–1939), whose attendance of all five congresses contained an opportunity to demonstrate the individuality of Polish mathematics and, as I find, an element of nationalism. I explore how their involvement had set the dynamic mathematical scene in the Empire, and beyond the frontiers of Russia, as political tensions began to rise in the 1910s.

Keywords: History of Mathematics in the Russian Empire at the Turn of the Twentieth Century, International Congresses of Mathematicians, Communications Between Mathematicians in East and West, Internationalism in Mathematics, Globalisation of Mathematics.

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Mathematical Open-Texture: History, Causes, Benefits

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Waismann's (1947) notion of open texture captures a species of (non-sorietal) semantic- and truth-vagueness; that a concept can both apply and disapply to some given case, within a context of application. Waismann's original conjecture was that all "empirical" concepts displayed some amount of open texture. This is borne-out somewhat by the literature on the notion's application in jurisprudence to our legal and ordinary concepts, and in philosophy of science to many of our best scientific concepts. Yet, mathematical concepts, starting with Waismann and in the philosophical spirit of the Euclidian tradition, have historically been taken as unique exemplars of closed textured concepts. Only recently has literature emerged like Tanswell (2018), Vecht (2020) and Zayton (2022), which explore the possibility of applying Waismann's notion to mathematics, thus reconsidering some standard views of proof, rigour, and our mathematical foundations.

I conjecture that open-texture does not only occur in mathematics, but that it plays an indispensable role in characterising its epistemology, and particularly its resolution strategies for crises of non-trivial disagreement. In particular, it is the ability of mathematicians to post-hoc *axiomatise* and individuate mathematical structures, and the unique arbitrariness properties of mathematical concepts, that facilitate open-textured mathematical concepts to be consistently closed. I then propose that mathematical progress is the story of repeatedly opening and closing the texture of concepts, with the "imaginary" nature of mathematical concepts allowing mathematicians to repeatedly re-open their texture, under certain conditions.

I begin by examining various informal definitions of "open texture" from the literature, before subsuming them into a "quasiformal" logical characterisation. The quasiformal model suggests that open texture can be thought to result from (multiple) non-eliminative definitions of a concept, or definitions that violate certain "conservativeness" rules over theories. I consider several case studies from the history of mathematics to evince and explore the open texture of mathematcal concepts, such as the concepts of *size*, *polyhedron* and *continuous function*. Finally, I present an account in terms of quasiformal open texture of "crises" involving these concepts, and the mathematics they produced. Using resources from Lakatos (1976), I show how mathematicians ensure rigour through different strategies for textural closure, and fruitfulness through complementary stragies of textural opening. I conclude by tying these results back to the modern literature of mathematical pluralism, determinacy, and conventionalism.

Keywords: Concepts, Open Texture, Mathematics, Definitions.

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The Role of Anschauung in Kant's Conception of Geometry

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Far from being outdated, Kant's philosophy of mathematics continues to offer valuable insights for modern scholarship. Although recent studies have revitalized interest in Kantian mathematics (Posy (ed.) 1992; Posy and Rechter (eds.) 2019; Sutherland 2021; Shabel 2021), this interest primarily focuses on its relationship to Kant's overall theoretical philosophy. This paper aims to show that Kantian geometry can also contribute to modern philosophy of mathematics, particularly through Kant's concept of Anschauung (commonly translated as intuition). Contrary to a widely held view, it will be shown that invoking Kantian intuition remains necessary in certain contexts. After examining both historical and recent objections to Kant's arguments, a new interpretation of A 716-717/B 744-745 from the *Critique of Pure Reason* (*CPR*) will be proposed. The role that Kant assigns to intuition in this passage, which I term "revelatory," remains indispensable despite modern advancements in mathematics and logic.

The significance of intuition's revelatory role becomes clear when considering that most translations of Kant's *Critique of Pure Reason* — notably nearly all English versions, along with the official French and Greek translations — overlook this key aspect of intuition. This paper argues that such interpretations distort Kant's philosophy of mathematics and explains why recognizing the revelatory function of intuition is essential for addressing objections to the classical reading.

Scholars commenting on the role of *Anschauung* can be divided into two groups (Brittan 2006). The first group, represented by Frege and Hilbert, highlights the role of intuition in verifying the truth of the unproven first principles of mathematics. In contrast, the second group emphasizes intuition's role during the deductive process, arguing that intuition is needed when logical inference alone is insufficient to derive a seemingly true conclusion, as Michael Friedman (1992) has convincingly argued.

In modern mathematics, where the use of universal and existential quantifiers is considered fundamental, the existence of mathematical objects is established through purely logical means. This approach contrasts sharply with Kant's reliance on intuition, which is based on the spatio- temporal character of geometric objects. Given this difference, how, then, can Kantian Anschauung be considered not only relevant but even useful in today's mathematical context?

I propose an interpretation of passage A 716-717/ B 744-745 from the Doctrine of Method of the first *Critique*, where Kant compares the philosophical with the mathematical method in dealing with geometric objects. Kant's position is straightforward: purely analytic thinking, such as that used by philosophers when dealing with concepts, is ineffective in the field of geometry.

The difference between the philosopher and the geometer is not one of ability, since both are used to dealing with abstract concepts. Rather, the distinction lies in the faculties each employs: the philosopher relies solely on understanding (*Verstand*), whereas the geometer engages both understanding and intuition (*Anschauung*). The geometer does not merely contemplate the abstract concept of a triangle; rather, they engage with the problem through action (either empirical or a priori), constructing the figure and intuitively "seeing" the proof unfold. Kant describes the solution of the problem guided by intuition as "einleuchtenden," which could be literally translated as that which "sheds" light on something. In my opinion, this role of intuition, which I term "revelatory," aligns with the heuristic practices of modern geometers, remaining relevant in modern mathematics. Furthermore, it withstands objections to classical readings of Kant, reaffirming his contributions to the epistemology of mathematics.

Keywords: Kant, Intuition, Geometry, Methodology.

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Restoring the Analysis of the Ancients: Hugo de Omerique's Role in 17th Century Mathematical Debates

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Mathematics underwent a significant transformation throughout the seventeenth century in its objectives, practices and views on mathematical truth and proof. The renewed interest in the classical world of the early modern era would soon overflow the cultural, humanist scope and greatly influence the stream of mathematical development. Commandino's translation of Pappus' "Collection" into Latin was pivotal in encouraging the mathematical community into a collective endeavour to restore the "lost" or deliberately concealed method of the Ancients, that presumably had enabled them to achieve such notable geometrical results.

Just three years after Commandino's publication, François Viète would publish his "In Artem Analyticen Isagoge" (1591), in which he introduced a method for solving all mathematical problems. Additionally, Viète introduces a symbolic language to denote also known quantities and an operative algebra between *species* (either discrete or continuous quantities, i.e., arithmetical or geometrical objects), which marks a breakthrough in mathematical methodology. In the following years, Marino Ghetaldi and Viète would work on the relationship between algebraic and geometric resolutions, thereby paving the way for the emergence of a new field, the Analytic Geometry.

Viète's proposal raised some questions concerning the foundations of mathematics, such as the status of algebra in relation to arithmetic and geometry, as well as the source of the validity of mathematical propositions. Descartes' algebraic method for geometry was a restrictive interpretation of Viète's species: problems ought to be translated into algebraic relations between data, and have its algebraic resolution constructed geometrically a posteriori to achieve the status of true, proved results.

Over his lifetime, Newton developed an anti-Cartesian approach to mathematics with respect to methodology. Influenced by Barrow's views on mathematics, Newton though of Geometry as a model for reasoning and proof, and was interested in developing not an algebraic, but a geometrical analysis, which he believed to be more elegant and respectful with the nature of the problems. Furthermore, Descartes' analysis had became insufficient for scientific necessities: *mechanical curves*, that had been excluded in *La Géométrie*, resulted fundamental in engineering, astronomy and navigation. Analysis had to go beyond cartesianism and incorporate the use of the infinite and infinitesimals to be able to describe curves defined not by its position, but by the ratios of changes in them. Algebra was a valid tool not just for lengths, as Viète had proposed with his species.

According to Guicciardini, the mature Newton devoted the final decade of the 17th century to the *prisca sapientia*: his aim was to show that his youthful method of fluxions published in the *Principia* (1687) could be reformulated in terms acceptable by ancient standards. A missing piece of the puzzle in this narrative is Newton's discovery of the *Analysis Geometrica* (1698) by Antonio Hugo de Omerique, a Spanish geometer well versed in the mathematical developments on the continent due to his Jesuit training, that would be promptly reviewed in the *Philosophical Transactions*. In a letter, Newton would praise Omerique's treatise, considering it a simple and ingenious "foundation to restore the Analysis of the Ancients", which was "more fit for the Geometer than the Algebra of the Moderns".

It is my intention to explain in some detail Omerique's *Analysis Geometrica*, a Latin treatise strongly connected to both the Jesuit academic network and to the local development of mathematics in Spain. Omerique was able to integrate several geometrical propositions from a variety of mathematicians (van Schooten, Clavius, Viète, Reinhold, Tacquet, Grégoire de Saint-Vincent, Josep Saragossà, etc.) into a unified, coherent Euclidean-style system. Omerique's stance towards algebra is somewhat ambigous. On the one hand, he designates segments with letters and operates symbolically with them, according to the Euclidean rules for ratios. On the other hand, he is critical of the modern mathematical analysis based solely on algebra, and aims for a geometrical method for proving the results, which aligns with Newton's aspirations.

Keywords: History of Mathematics, Philosophy of Mathematics, Analytic Geometry, Analysis, 17th-century Spain.

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An Exploration on Motivations: The Case of Richard Dedekind's Ideal Theory

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In his paper for the International Congress of Mathematicians, Professor Anjing Qu suggested an alternative approach to the history of mathematics in China: while previous studies have mostly considered the what's and the how's, a new question concerning the "'why" (why this type of mathematics was done?) should be addressed ([Qu 2003]). The research paradigm associated to this "why" question is discussed thoroughly in [Qu 2021]. Such a change of paradigm is of course not restricted to mathematics in China, nor to the scale of studying a whole tradition. Indeed, the question of "why" clearly touches several levels of mathematical activity: we can wonder why there has been in in Wilhelmian Germany such an interest in developing so-called pure mathematics, that is mathematics with no apparent connection with the problems of the real world, be it that of physics and astronomy or that of economy? ([Mehrtens 1981] or [Pyenson 1983]). why certain problems were specifically studied, such as Fermat Last Theorem or higher reciprocity laws, or why specific techniques had been selected and used to solve a question? Human motivations, in particular, are complex and multiple.

As Richard Ryan explained in the introduction of the *Oxford Handbook on Human motivation*, he edited: "Humans are clearly motivated, goal-directed, creatures. They seek out specific ends, ranging from concrete goals such as obtaining food and shelter to abstract ones such as developing a sense of meaning or attaining aesthetic ideals.Sometimes people's motivation is explicit and conscious; at other times behavior is clearly energized and directed by nonconscious, implicit aims and attitudes. Finally, whether motives are implicit or explicit, the behavior organized by them will be variously successful." ([Ryan 2012]). These aspects seem also relevant for mathematics (if one replaces "food and shelter" by "position"); but they are particularly difficult to pinpoint for historical figures, first because historians of mathematics, contrarily to sociologists or psychologists, have to rely on written documents and also because mathematics can be written (and is written very often in modern times) as a succession of statements and proofs with few hints about what motivated them. Explicit research on the issue of the motivations of mathematicians are thus still scarce ([Ferreirós 2004] and [Ji and Wang 2020]).

Inspired by this program, we would like to explore here in more detail one case, that of the nineteenth-century German mathematician Richard Dedekind, and more specifically that of his theory of ideals, in order to study more thoroughly the issues raised by the "why" question.

Keywords: Ideal Theory, Motivation, Number Theory, Dedekind.

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