TAMNA MATERIJA I TAMNA ENERGIJA U NELOKALNOJ GRAVITACIJI

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1. Introduction: General relativity

- General Relativity (GR) is Einstein theory of gravity, which since 1915 serves as classical gravity theory.
- Nice theoretical properties and remarkable phenomenological achievements.
- Perfectly describes dynamics of Solar System.
- Many significant predictions: deflection of light by the Sun, gravitational redshift of light, gravitational waves, gravitational lensing, black holes, ...
- It also predicts Dark Energy (DE) and Dark Matter (DM).
- GR still serves as a starting point in understanding all gravitational phenomena.
- Modern cosmology is based on GR and started in 1917.

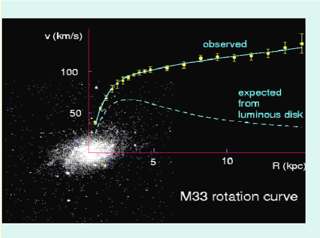
1. Introduction: Some problems

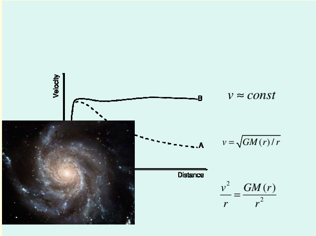
- There is no Quantum General Relativity: it is a non-renormalizabile quantum field theory.
- It predicts Dark Energy and Dark Matter which are mysterious and still without laboratory evidence.
- General Relativity has not been tested and confirmed at very large cosmic scales, hence its application to the Universe as a whole should be taken with caution.
- Cosmological solutions of GR mainly contain Big Bang singularity.
- It seems unnatural that Einstein GR is theory of gravity at all (spatial) scales: from Planck scale to the universe as a whole. (Physical theories depend on scales and complexity of systems.)
- There is a sense to search for generalization of Einstein theory of gravity.
- There are many attempts to modify GR motivated by theoretical and phenomenological reasons.
- Here we consider some modifications of GR with respect to cosmology.

1. Introduction: Dark matter or MOND

MOND (Modified Newtonian Dynamics), M. Milgrom 1983

$$F = ma\frac{a}{a_0} = \frac{GmM}{r^2}, \quad v^4 = GMa_0, \quad a_0 = 1.2 \times 10^{-8} cms^{-2}$$





Krive rotacione brzine u funkciji rastojanja zvezde (gasa) od centra galaksije

Some relevant references

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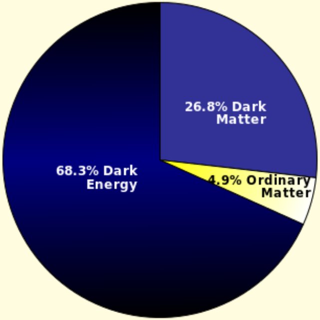
2. Modified Gravity: some motivations

- Einstein equation: $R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = 8\pi G\, T_{\mu\nu}.$
- FRW metric for homogeneous and isotropic space-time $ds^2 = -dt^2 + a^2(t)\big(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\big)$ and energy-momentum tensor $T_{\mu\nu} = diag(\rho, pg_{11}, pg_{22}, pg_{33})$.
- Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- In 1998: $\frac{\ddot{a}}{a} > 0 \Rightarrow \rho + 3p < 0$. Dark Energy. 68 %
- Big velocities in spiral galaxies: Dark matter. 27 %
- Visible matter. 5 %
- Standard cosmological model: ΛCDM, k = 0 (flat space).
- There is no experimental evidence of DE and DM
- Initial (Big Bang) singularity.


26.8% Dark Matter
4.9% Ordinary Matter
68.3% Dark Energy

2. Modified Gravity: Kinds of modification

- First modifications: Einstein 1917, ..., many modifications after 1998

Einstein-Hilbert action

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} R + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

modification

$$R \to f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, ...), \quad \Box = \nabla^{\mu}\nabla_{\mu} = \frac{1}{\sqrt{-g}}\partial_{\mu}\sqrt{-g}g^{\mu\nu}\partial_{\nu}$$

2. Modified Gravity: Kinds of modification

- There is no theoretical principle which could tell us in what direction to make modification of GR.
- $f(R)$ modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

- nonlocal modified gravity

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi G} f(R, \Box, ...) + \int d^4x \sqrt{-g}\, \mathcal{L}(matter)$$

3. Nonlocal Modified Gravity: Kinds of nonlocal gravity

- Nonlocal modified gravity with $\Box^{-n}$:

$$S = \frac{1}{16\pi G} \int \sqrt{-g}\,(R + L_{NL})\; d^4x,$$

with two typical examples:

1) $L_{NL} = R\, f(\Box^{-1} R),$

2) $L_{NL} = -\frac{1}{6} m^2 R \Box^{-2} R.$

3. Nonlocal Modified Gravity: Kinds of nonlocal gravity

The exact tree-level Lagrangian for effective scalar field φ which describes open *p-adic string tachyon* is

$$\mathcal{L}_p = \frac{m_p^D}{g_p^2}\,\frac{p^2}{p-1}\Big[-\frac{1}{2}\,\varphi\,p^{-\frac{\Box}{2m_p^2}}\,\varphi + \frac{1}{p+1}\,\varphi^{p+1}\Big]$$

where p is any prime number, $\Box = -\partial_t^2 + \nabla^2$ is the D-dimensional d'Alembertian and metric with signature $(- + ... +)$.

3. Nonlocal Modified Gravity: Analytic approach

Action for a class of models:

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + P(R)\mathcal{F}(\Box)Q(R) \Big) \sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$, $P(R)$ and $Q(R)$ are some differentiable functions of R, Λ is cosmological constant.

For simplicity, we consider nonlocal modification without matter.

3. Nonlocal Modified Gravity: Analytic approach

Equations of motion:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} P(R)\mathcal{F}(\Box)Q(R) + (R_{\mu\nu} - K_{\mu\nu})\,\Phi$$
$$+ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{\ell=0}^{n-1} \Big(g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \Box^\ell P(R) \partial_\beta \Box^{n-1-\ell} Q(R)$$
$$- 2\partial_\mu \Box^\ell P(R) \partial_\nu \Box^{n-1-\ell} Q(R) + g_{\mu\nu} \Box^\ell P(R) \Box^{n-\ell} Q(R) \Big) = 0,$$

where $K_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu}\Box$,
$\Phi = P'(R)\mathcal{F}(\Box)Q(R) + Q'(R)\mathcal{F}(\Box)P(R)$,
and $'$ denotes derivative on R.

3. Nonlocal Modified Gravity: Analytic approach

In homogeneous and isotropic spaces (FLRW metric) there are only two linearly independent equations of motion (Trace and 00-component):

$$4\Lambda - R - 2P(R)\mathcal{F}(\Box)Q(R) + (R\Phi + 3\Box\Phi)$$
$$+ \sum_{n=1}^{\infty} f_n \sum_{\ell=0}^{n-1} \left(\partial_\mu \Box^\ell P(R)\partial^\mu \Box^{n-1-\ell} Q(R) + 2\Box^\ell P(R)\Box^{n-\ell} Q(R)\right) = 0,$$

$$G_{00} - \Lambda + \frac{1}{2}P(R)\mathcal{F}(\Box)Q(R) + (R_{00}\Phi - K_{00}\Phi)$$
$$- \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{\ell=0}^{n-1} \left(g^{\alpha\beta}\partial_\alpha \Box^\ell P(R)\partial_\beta \Box^{n-1-\ell} Q(R)\right.$$
$$\left. + 2\partial_0 \Box^\ell P(R)\partial_0 \Box^{n-1-\ell} Q(R) + \Box^\ell P(R)\Box^{n-\ell} Q(R)\right) = 0.$$

4. Cosmological solutions: Case $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$

$$S = \frac{1}{16\pi G}\int_M \Big(R - 2\Lambda + \sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}\Big)\sqrt{-g}\; d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=1}^{\infty} f_n\,\Box^n$.

Cosmological solution: $a(t) = A\, t^{\frac{2}{3}} e^{\frac{\Lambda}{14} t^2}\,, \quad k = 0,\ \Lambda \neq 0$

- imitation of dark matter $t^{\frac{2}{3}}$ and dark energy $e^{\frac{\Lambda}{14} t^2}$
- $H(t) = \frac{\dot{a}}{a} = \frac{2}{3} t^{-1} + \frac{1}{7}\Lambda t$ – Hubble parameter
- $R(t) = \frac{4}{3} t^{-2} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^2 t^2$
- $\Box\sqrt{R-2\Lambda} = -\frac{3}{7}\Lambda\sqrt{R-2\Lambda}$
- $\mathcal{F}(-\frac{3}{7}\Lambda) = -1, \quad \mathcal{F}'(-\frac{3}{7}\Lambda) = 0$
- $\bar{\rho} = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}\,, \quad \bar{p} = -\frac{\Lambda}{56\pi G}\left(\frac{3}{7}\Lambda t^2 - 1\right).$
- $\bar{w} = \frac{\bar{p}}{\bar{\rho}} \to -1$ when $t \to \infty$

4. Cosmological solutions: Case $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$

The current Planck results for the ΛCDM universe are:

- $H_0 = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter;
- $\Omega_m = 0.315 \pm 0.007$ – matter density parameter;
- $\Omega_\Lambda = 0.685$ – Λ density parameter;
- $t_0 = (13.801 \pm 0.024) \cdot 10^9$ yr – age of the universe;
- $w_0 = -1.03 \pm 0.03$ – ratio of pressure to energy density.

4. Cosmological solutions: Case $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$

- Taking the above Planck results for t_0 and H_0 one obtains $\Lambda = 1.05 \cdot 10^{-35}\, s^{-2}$ (in $c = 1$ units). This is close to $\Lambda = 0.98 \cdot 10^{-35} s^{-2}$ calculated by standard formula $\Lambda = 3H_0^2 \Omega_\Lambda$.
- One can also calculate time (t_m) for which the Hubble parameter has minimum value H_m, i.e. $t_m = 21.1 \cdot 10^9$ yr and $H_m = 61.72$ km/s/Mpc.
- Beginning of the universe expansion acceleration was at $t_a = 7.84 \cdot 10^9$ yr, or in other words at 5.96 billion years ago.

$$\ddot{a}(t) = \Big(-\frac{2}{9} t^{-2} + \frac{\Lambda}{3} + \frac{\Lambda^2}{49} t^2 \Big) a(t)$$

4. Cosmological solutions: Case $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}$

- We calculated the critical energy density ρ_c and the energy density of the dark matter $\bar{\rho}$:

$$\rho_c = \frac{3}{8\pi G} H_0^2 = 8.51 \cdot 10^{-30} \frac{g}{cm^3} \tag{1}$$

$$\bar{\rho} = \Big(\frac{4}{9} t_0^{-2} - \frac{\Lambda}{7} + \frac{\Lambda^2}{49} t_0^2\Big) \frac{3}{8\pi G} = 2.26 \cdot 10^{-30} \frac{g}{cm^3}. \tag{2}$$

It follows that $\bar{\Omega} = \frac{\bar{\rho}}{\rho_c} = 0.27$. Since Ω_v for the visible matter is approximatively $\Omega_v = 0.05$, then $\bar{\Omega}_\Lambda = 1 - \bar{\Omega} - \Omega_v = 0.68$.

- After discovery of accelerating expansion of the universe appeared the 'coincidence problem' as a problem why, just in current period of the cosmic history, the densities of dark energy and dark matter are of the same order of magnitude. The answer within this cosmological solution consists in the fact that terms in expression for $\bar{\rho}$ are of the same order, because product $\Lambda t_0^2 = 2$.

4. Cosmological solutions: Case $a(t) = A\, e^{\frac{\Lambda}{6}t^2}$

$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4x$$

Cosmological solution: $a(t) = A\, e^{\frac{\Lambda}{6}t^2}\,, \quad k = 0,\ \Lambda \neq 0$

- $R(t) = 2\Lambda(1 + \frac{2}{3}\Lambda t^2), \qquad H(t) = \frac{1}{3}\Lambda t$
- $\Box \sqrt{R - 2\Lambda} = -\Lambda \sqrt{R - 2\Lambda}$
- $\mathcal{F}(-\Lambda) = -1, \quad \mathcal{F}'(-\Lambda) = 0$
- $\bar{\rho} = \frac{\Lambda}{8\pi G}\left(\frac{\Lambda}{3}t^2 - 1\right), \qquad \bar{p} = -\frac{\Lambda}{24\pi G}\left(\Lambda t^2 - 1\right)$
- $\bar{w} = \frac{\bar{p}}{\bar{\rho}} \to -1$ when $t \to \infty$

4. Cosmological solutions: Case $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}, \quad k = \pm 1,\ \Lambda > 0$

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \Big) \sqrt{-g}\, d^4x$$

Cosmological solution: $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}, \quad k = \pm 1,\ \Lambda > 0$

- $R(t) = \frac{6k}{A^2} e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t} + 2\Lambda, \qquad H = \pm\sqrt{\frac{\Lambda}{6}}$
- $\Box\sqrt{R - 2\Lambda} = \frac{\Lambda}{3}\sqrt{R - 2\Lambda},$
- $\mathcal{F}\left(\frac{\Lambda}{3}\right) = -1, \quad \mathcal{F}'\left(\frac{\Lambda}{3}\right) = 0$
- $\bar{\rho} = \frac{-\frac{\Lambda}{2} + \frac{3k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t}}{8\pi G}, \qquad \bar{p} = \frac{\frac{\Lambda}{2} - \frac{k}{A^2}\, e^{\mp\sqrt{\frac{2}{3}\Lambda}\, t}}{8\pi G}$

4. Cosmological solutions: Case $R = 4\Lambda > 0$

Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

- Cosmological solutions for $R = 4\Lambda > 0$
  Equations of motion are satisfied without conditions on function $\mathcal{F}(\Box)$, because $\Box\sqrt{R-2\Lambda} = 0$.

  (i) Solution $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{3}}\, t}$

  $k = 0$. One has $H(t) = \pm\sqrt{\frac{\Lambda}{3}}$.

  (ii) Solution $a(t) = \sqrt{\frac{3}{\Lambda}} \cosh \sqrt{\frac{\Lambda}{3}}\, t$

  $k = +1$. Now $H(t) = \sqrt{\frac{\Lambda}{3}} \tanh \sqrt{\frac{\Lambda}{3}}\, t$.

  (iii) Solution $a(t) = \sqrt{\frac{3}{\Lambda}} \left| \sinh \sqrt{\frac{\Lambda}{3}}\, t \right|$

  $k = -1$. Here $H(t) = \sqrt{\frac{\Lambda}{3}} \coth \sqrt{\frac{\Lambda}{3}}\, t$.

4. Cosmological solutions: Case $R = 4\Lambda < 0$

Model with nonlocal term $\sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}$

- Note that there is a problem with the Minkowski space solution, because $\Lambda = R = 0$ implies $\frac{1}{\sqrt{R-2\Lambda}} = \infty$.
- Cosmological solutions for $R = 4\Lambda < 0$
  The corresponding solution has the form
  $a(t) = \sqrt{-\frac{3}{\Lambda}}\left|\cos\sqrt{-\frac{\Lambda}{3}}\,t\right|$, where Λ is negative cosmological constant. In this case
  $H(t) = -\sqrt{-\frac{\Lambda}{3}}\tan\sqrt{-\frac{\Lambda}{3}}\,t$, and $k = -1$.

5. Concluding Remarks

- We have considered nonlocal gravity models of the form

$$S = \frac{1}{16\pi G} \int_M \Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R - 2\Lambda} \Big) \sqrt{-g}\, d^4x,$$

- We have found some new exact cosmological solutions which are not present in Einstein theory of gravity.
- All these cosmological solutions are valid for $\Lambda \neq 0$.
- There is no Minkowski space ($k = 0,\ R = 0,\ \Lambda = 0$).

5. Concluding Remarks

I would like to point out our new nonlocal gravity model

$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box)\, \sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4x$$

with solution

- $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$
- which mimics dark matter and dark energy!
- Computed cosmological parameters are in good agreement with observations!
- Requires existence of cosmological constant Λ!
- What is the next step? Add ordinary matter to the action!

HVALA ZA INTERES ZA OVO PREDAVANJE!