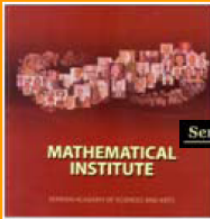
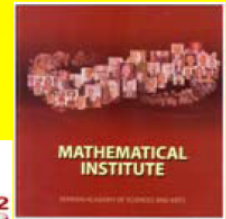


APPROXIMATIONS OF NONLINEAR DIFFERENTIAL EQUATION SOLUTIONS



Serbian Scientific Society



Hedrih R. (Stevanović) Katica

Mathematical Institute SANU Belgrade,
and Faculty of Mechanical Engineering University of Niš, Serbia

$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = \mp \tilde{\omega}_{N1}^2 x_1^3(t)$$

Paper ID: SM16-053
Poster: P-477

First asymptotic approximations starting from different analytical solutions

$$x_1(t) = a_o e^{-\delta_1 t} \cos \left[\omega_1 t - \frac{3}{16\delta_1 \omega_1} \omega_{N1}^2 a_o^2 (e^{-2\delta_1 t} - 1) + \psi_o \right]$$

Better first asymptotic approximations starting from analytical solution of damping linear vibrations

$$x_1(t) = R_{01} e^{-\delta_1 t} \cos \left(p_1 t \mp \frac{3}{16\delta_1 \omega_1} \omega_{N1}^2 a_o^2 (e^{-2\delta_1 t} - 1) + \phi_{01} \right) \quad p_1 = \sqrt{\omega_1^2 - \delta_1^2}$$



XXIII ICTAM, 19-24 August 2012, Beijing, China



$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = \mp \tilde{\omega}_{N1}^2 x_1^3(t)$$

Starting analytical solution for approximation

$$\ddot{x}_1(t) + \omega_1^2 x_1(t) = 0 \quad x_1(t) = R(t) \cos(\omega_1 t + \psi(t))$$

First asymptotic Approximation

$$x_1(t) = a_o e^{-\delta_1 t} \cos \left[\omega_1 t - \frac{3}{16\delta_1 \omega_1} \omega_{N1}^2 a_o^2 (e^{-2\delta_1 t} - 1) + \psi_o \right].$$

For limit case when nonlinearity is zero

$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = 0$$

$$\begin{array}{ll} \delta_1 \neq 0 & \varepsilon = 0 \\ \omega_1^2 > \delta_1^2 & \tilde{\omega}_{N1}^2 = 0 \\ \delta_1 \neq 0 & \omega_1^2 > \delta_1^2 \end{array}$$

Error solution

$$x_1(t) = R_{01} e^{-\delta_1 t} \cos(\omega_1 t + \psi_0)$$

Analytical solution

$$x_1(t) = R_{01} e^{-\delta_1 t} \cos(p_1 t + \alpha_{01})$$

$$p_1 = \sqrt{\omega_1^2 - \delta_1^2}$$

$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = \mp \tilde{\omega}_{N1}^2 x_1^3(t)$$

Starting analytical solution for approximation

$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = 0 \quad x_1(t) = R_{01} e^{-\delta_1 t} \cos(p_1 t + \alpha_{01})$$

First asymptotic approximation

$$x_1(t) = R(t) e^{-\delta_1 t} \cos(p_1 t + \phi(t)) \quad p_1 = \sqrt{\omega_1^2 - \delta_1^2}$$

$$x_1(t) = R_{01} e^{-\delta_1 t} \cos\left(p_1 t \mp \frac{3}{16\delta_1 \omega_1} \omega_{N1}^2 a_o^2 (e^{-2\delta_1 t} - 1) + \phi_{01}\right)$$

For limit case when nonlinearity is zero

$$\ddot{x}_1(t) + 2\delta_1 \dot{x}_1(t) + \omega_1^2 x_1(t) = 0$$

$$\begin{aligned} \delta_1 &\neq 0 & \varepsilon &= 0 \\ \omega_1^2 &> \delta_1^2 & \tilde{\omega}_{N1}^2 &= 0 \\ \delta_1 &\neq 0 & \omega_1^2 &> \delta_1^2 \end{aligned}$$

$$x_1(t) = R_{01} e^{-\delta_1 t} \cos(p_1 t + \alpha_{01})$$

Analytical solution

$$p_1 = \sqrt{\omega_1^2 - \delta_1^2}$$

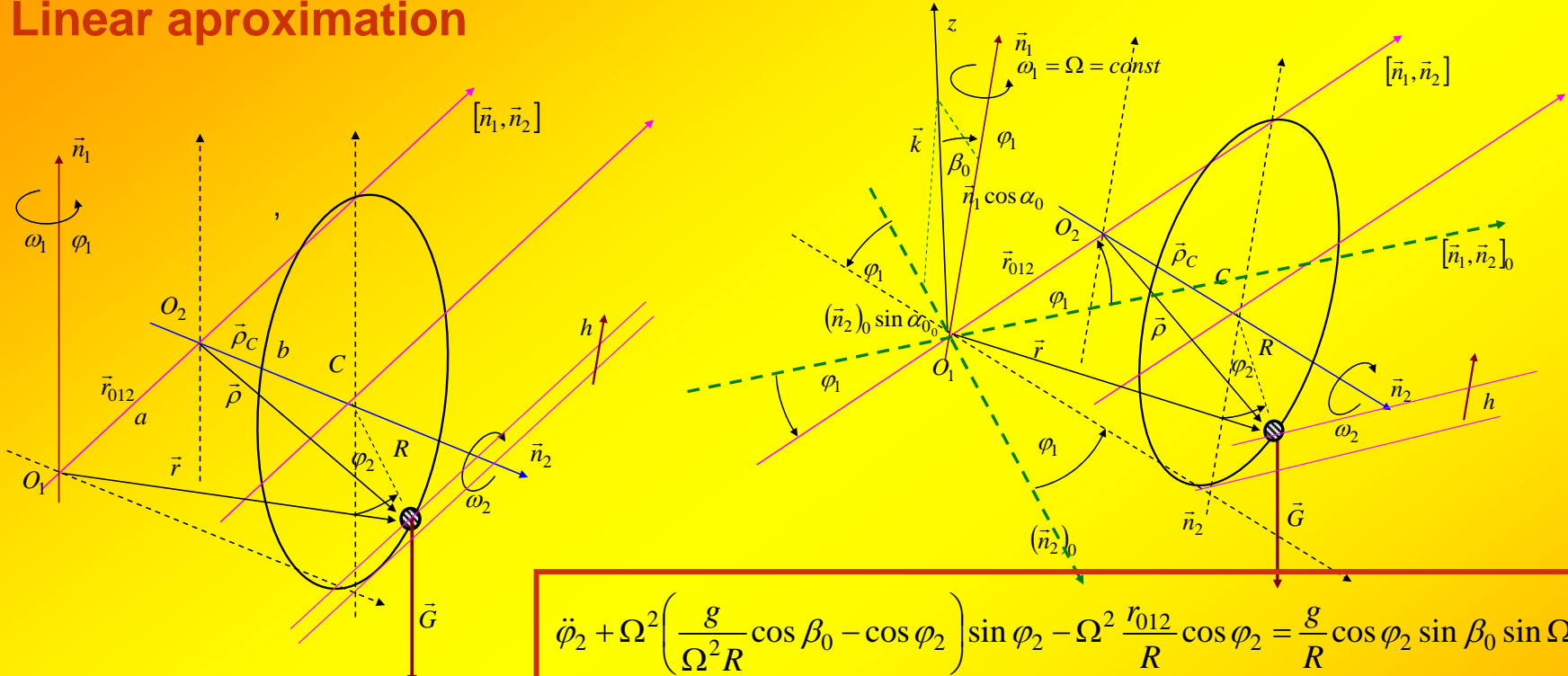
Nonlinear differential equations nonlinear approximations around stationary points

$$\frac{d\varphi}{dt} = f_1(\varphi, v) \qquad \frac{dv}{dt} = f_2(\varphi, v)$$

$$\begin{aligned} \frac{d\varphi}{dt} = & \left(\frac{\partial f_1(\varphi, v)}{\partial \varphi} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi + \left(\frac{\partial f_1(\varphi, v)}{\partial v} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v + \frac{1}{2!} \left(\frac{\partial^2 f_1(\varphi, v)}{\partial \varphi^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^2 + \frac{1}{2!} \left(\frac{\partial^2 f_1(\varphi, v)}{\partial v^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v^2 + 2 \frac{1}{2!} \left(\frac{\partial^2 f_1(\varphi, v)}{\partial v \partial \varphi} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi v + \\ & + \frac{1}{3!} \left[\left(\frac{\partial^3 f_1(\varphi, v)}{\partial \varphi^3} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^3 + 3 \left(\frac{\partial^3 f_1(\varphi, v)}{\partial \varphi^2 \partial v} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^2 v + 3 \left(\frac{\partial^3 f_1(\varphi, v)}{\partial \varphi \partial v^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi v^2 + \left(\frac{\partial^3 f_1(\varphi, v)}{\partial v^3} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v^3 \right] + \dots \end{aligned}$$

$$\begin{aligned} \frac{dv}{dt} = & \left(\frac{\partial f_2(\varphi, v)}{\partial \varphi} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi + \left(\frac{\partial f_2(\varphi, v)}{\partial v} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v + \left(\frac{\partial^2 f_2(\varphi, v)}{\partial \varphi^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^2 + \left(\frac{\partial^2 f_2(\varphi, v)}{\partial v^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v^2 + 2 \frac{1}{2!} \left(\frac{\partial^2 f_2(\varphi, v)}{\partial v \partial \varphi} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi v + \\ & + \frac{1}{3!} \left[\left(\frac{\partial^3 f_2(\varphi, v)}{\partial \varphi^3} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^3 + 3 \left(\frac{\partial^3 f_2(\varphi, v)}{\partial \varphi^2 \partial v} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi^2 v + 3 \left(\frac{\partial^3 f_2(\varphi, v)}{\partial \varphi \partial v^2} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} \varphi v^2 + \left(\frac{\partial^3 f_2(\varphi, v)}{\partial v^3} \right) \Big|_{\substack{\varphi=\varphi_s \\ v=v_s}} v^3 \right] + \dots \end{aligned}$$

Examples of nonlinear dynamics of heavy mass particle dynamics with coupled rotations about no intersecting axes- Linear approximation- Linear approximation



$$\ddot{\varphi}_2 + \Omega^2 \left(\frac{g}{\Omega^2 R} \cos \beta_0 - \cos \varphi_2 \right) \sin \varphi_2 - \Omega^2 \frac{r_{012}}{R} \cos \varphi_2 = \frac{g}{R} \cos \varphi_2 \sin \beta_0 \sin \Omega t_1$$

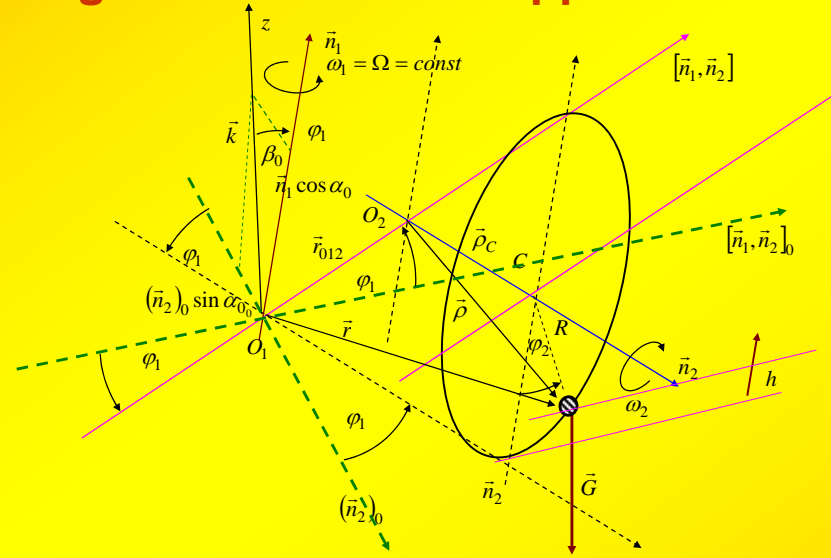
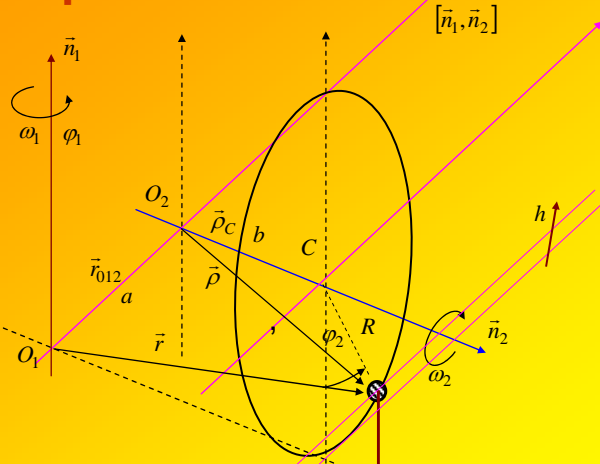
$$\ddot{\varphi}_2 + \Omega^2 \left\langle \frac{g}{R\Omega^2} - \cos \varphi_2 \right\rangle \sin \varphi_2 - \Omega^2 \frac{r_{012}}{R} \cos \varphi_2 = 0$$

$$\frac{d^2 \varphi}{dt^2} + \Omega^2 \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \varphi = 0$$

$$\ddot{\varphi}_2 + \varphi_2 \left[\omega_0^2 \cos^2 \alpha_0 + \Omega^2 \sin 2\varphi_{s2} - \Omega^2 \frac{r_{012}}{R} \sin \varphi_{s2} + \frac{g}{R} \sin \alpha_0 \sin \varphi_{s2} \cos \Omega t \right] + \omega_0^2 \cos \alpha_0 \sin \varphi_{s2} - \frac{\Omega^2}{2} \cos 2\varphi_{s2} - \Omega^2 \frac{r_{012}}{R} \cos \varphi_{s2} = \frac{g}{R} \sin \alpha_0 \cos \varphi_{s2} \cos \Omega t$$

$$\ddot{\varphi}_2 + \varphi_2 [\lambda + \gamma \sin \varphi_{s2} \cos \Omega t] = h_s \cos \Omega t$$

Examples of nonlinear dynamics of heavy mass particle dynamics with coupled rotations about non intersecting axes - Nonlinear approximation



$$\ddot{\varphi}_2 + \Omega^2 \left\langle \frac{g}{R\Omega^2} - \cos \varphi_2 \right\rangle \sin \varphi_2 - \Omega^2 \frac{r_{012}}{R} \cos \varphi_2 = 0$$

$$\ddot{\varphi}_2 + \Omega^2 \left(\frac{g}{\Omega^2 R} \cos \beta_0 - \cos \varphi_2 \right) \sin \varphi_2 - \Omega^2 \frac{r_{012}}{R} \cos \varphi_2 = \frac{g}{R} \cos \varphi_2 \sin \beta_0 \sin \Omega t_1$$

$$\frac{d^2 \varphi}{dt^2} + \Omega^2 \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1-\lambda_s^2} \right\rangle \varphi - \Omega^2 \left\langle \left\langle \frac{g}{R\Omega^2} \sqrt{1-\lambda_s^2} - 4\lambda_s \sqrt{1-\lambda_s^2} \right\rangle - \frac{r_{012}}{R} \lambda_s \right\rangle \varphi^2 - \frac{\Omega^2}{3!} \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1-\lambda_s^2} \right\rangle \varphi^3 + \dots = 0$$

$$\frac{d^2 \varphi}{dt^2} + \Omega^2 \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1-\lambda_s^2} + \frac{g}{R\Omega^2} \sin \alpha_0 \sqrt{1-\lambda_s^2} \cos \Omega t \right\rangle \varphi - \frac{\Omega^2}{2!} \left\langle \left\langle \frac{g}{R\Omega^2} \sqrt{1-\lambda_s^2} - 4\lambda_s \sqrt{1-\lambda_s^2} \right\rangle - \frac{r_{012}}{R} \lambda_s - \frac{g}{R\Omega^2} \lambda_s \sin \alpha_0 \cos \Omega t \right\rangle \varphi^2 - \frac{\Omega^2}{3!} \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1-\lambda_s^2} + \frac{g}{R\Omega^2} \sin \alpha_0 \sqrt{1-\lambda_s^2} \cos \Omega t \right\rangle \varphi^3 + \dots = \frac{g}{R} \lambda_s \sin \alpha_0 \cos \Omega t$$

$$\omega_{0,lin}^2 = \Omega^2 \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle \quad \kappa_2 = \Omega^2 \left\langle \left\langle \frac{g}{R\Omega^2} \sqrt{1 - \lambda_s^2} - 4\lambda_s \sqrt{1 - \lambda_s^2} \right\rangle - \frac{r_{012}}{R} \lambda_s \right\rangle$$

$$\kappa_3 = \frac{\Omega^2}{3!} \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle \quad \chi = \frac{g}{R} \sin \alpha_0$$

$$\ddot{\varphi} + \omega_{0,lin}^2 \varphi = \kappa_2 \varphi^2 + \kappa_3 \varphi^3$$

$$\varphi(t) = a(t) \cos \Phi(t)$$

$$\dot{a}(t) = 0$$

$$\dot{\phi}(t) = \frac{3\kappa_3}{8\omega_0} [a(t)]^2$$

$$a(t) = a_0 = \text{const}$$

$$\frac{d^2 \varphi}{dt^2} + \left\langle \omega_{0,lin}^2 + \chi \sqrt{1 - \lambda_s^2} \cos \Omega t \right\rangle \varphi = \left\langle \kappa_2 - \frac{\chi}{2} \lambda_s \cos \Omega t \right\rangle \varphi^2$$

$$+ \left\langle \kappa_3 + \frac{\chi}{6} \sqrt{1 - \lambda_s^2} \cos \Omega t \right\rangle \varphi^3 + \dots + \chi \lambda_s \cos \Omega t$$

$$\phi(t) = \frac{\Omega \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}{16 \sqrt{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}} [a(t)]^2 t + \phi(0)$$

$$\varphi(t) = a(t) \cos \Phi(t)$$

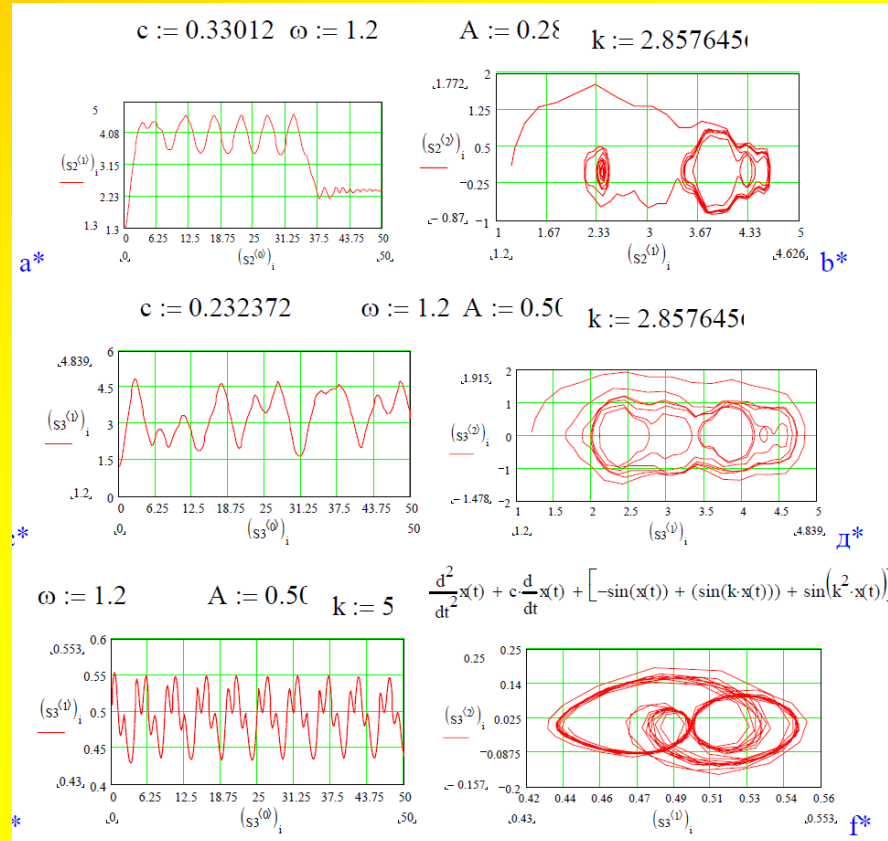
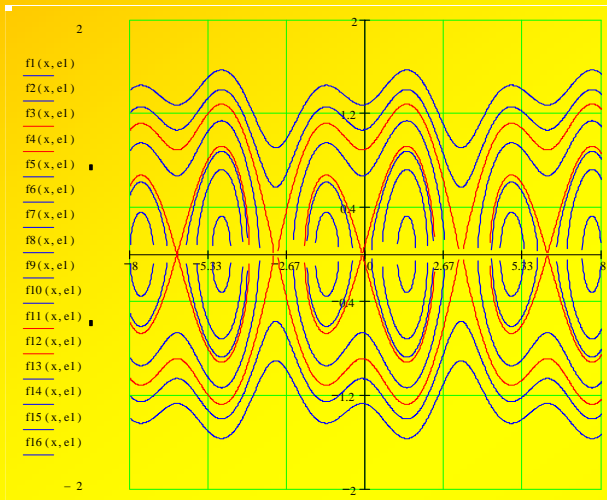
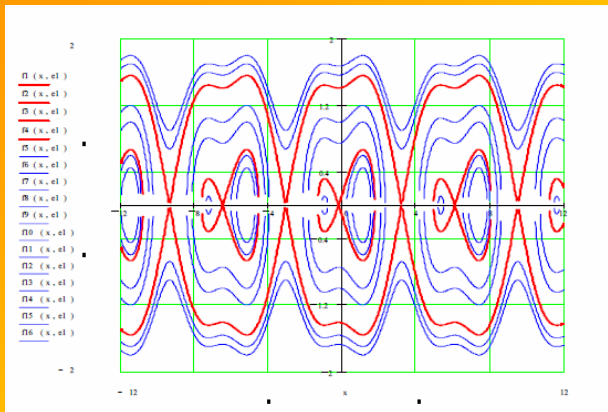
$$a(t) = a_0 = \text{const}$$

$$\dot{\phi}(t) = \frac{3\kappa_3}{8\omega_0} [a(t)]^2$$

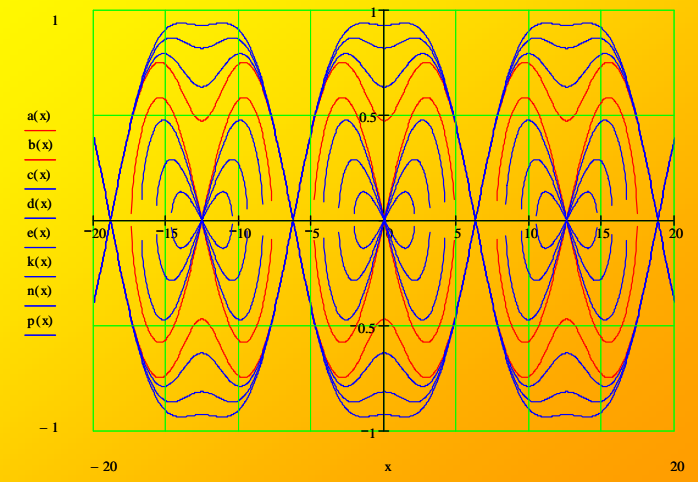
$$\phi(t) = \frac{\Omega \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}{16 \sqrt{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}} [a(t)]^2 t + \phi(0)$$

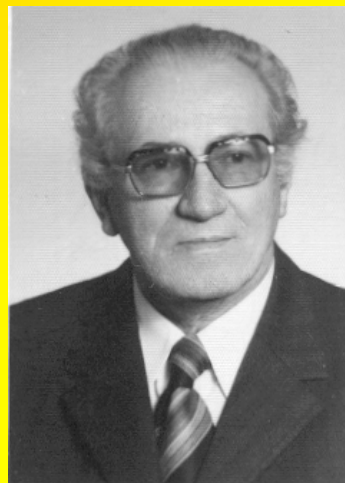
$$\Phi(t) = \left\langle \Omega \sqrt{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle} + \frac{\Omega \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}{16 \sqrt{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}} [a(t)]^2 \right\rangle t + \phi(0)$$

$$\omega_{nel} = \Omega \sqrt{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle} \left\{ 1 + \frac{\left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 8\lambda_s^2 + 4 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle}{16 \left\langle \left\langle \frac{g}{R\Omega^2} \lambda_s - 2\lambda_s^2 + 1 \right\rangle + \frac{r_{012}}{R} \sqrt{1 - \lambda_s^2} \right\rangle} a_0^2 \right\}$$



Paper ID: SM16-053 in the seminar session of SM16 Vibrations and control of structures
Poster board number: P-477



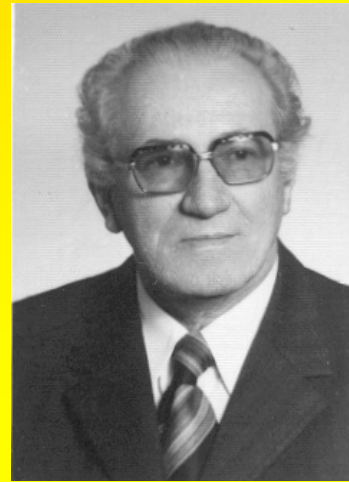


Serbian Scientific Society



Thank You for Attention!





Thank You for Attention!

