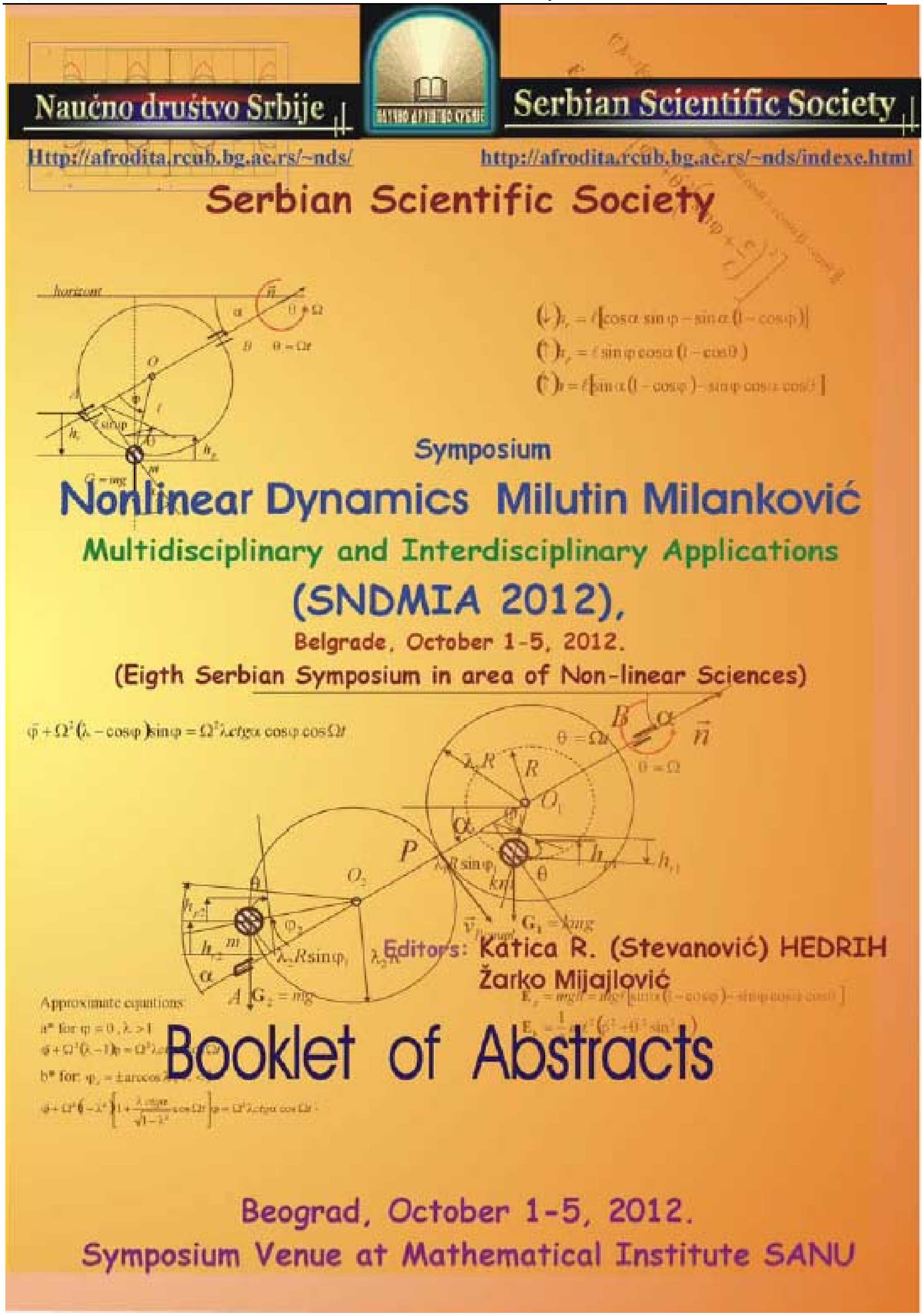

Naučno društvo Srbije
Serbian Scientific Society
Http://afrodita.rcub.bg.ac.rs/~nds/
http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Serbian Scientific Society
Symposium
Nonlinear Dynamics Milutin Milanković
Multidisciplinary and Interdisciplinary Applications
(SNDMIA 2012),
Belgrade, October 1-5, 2012.
(Eigth Serbian Symposium in area of Non-linear Sciences)
Editors: Katica R. (Stevanović) HEDRIH
Žarko Mijajlović
Booklet of Abstracts
Beograd, October 1-5, 2012.
Symposium Venue at Mathematical Institute SANU

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

### Multidisciplinary and Interdisciplinary Applications

### (SNDMIA 2012), Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Symposium

# Nonlinear Dynamics – Milutin Milanković

## Multidisciplinary and Interdisciplinary Applications

# (SNDMIA 2012),

Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Editors: Katica R. (Stevanović) HEDRIH
Žarko Mijajlović

# Booklet of Abstracts

Beograd, October 1-5, 2012.
Venue: Mathematical Institute SANU

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

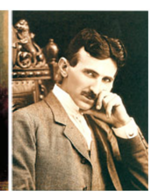
**Multidisciplinary and Interdisciplinary Applications**
**(SNDMIA 2012), Belgrade, October 1-5, 2012.**
**(Eigth Serbian Symposium in area of Non-linear Sciences**

Main organizers:

Department of natural-mathematical sciences of Serbian Scientific Society
and supported by Project ON174001 (2011-2014)
coordinated through
Department of Mechanics at Mathematical Institute SANU

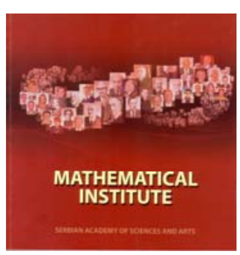
Support by
Ministry of Education and Science Republic of Serbia
http://www.mpn.gov.rs/sajt/



**Suppotyed by**
**Co-organizers scientists from**

Serbian Society of Mechanics,
Institute „Mihajlo Pupin" Belgrade,
Institute „Vincha",
The Society of Physical Chemists of Serbia
Faculty of Physical Chemistry University of Belgrade and
Faculty of Technical Sciences Kosovska Mitrovica, University of Pristina with allocated place in Kosovska Mitrovica.




Venue of Symposium – Mathematical Institute SANU
Ul. Knez Mihailova 36/III, Auditoria I

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

October 1, 2012, Belgrade, Serbia

Dear Participant,

The **Symposium Nonlinear Dynamics - Milutin Milanković, Multidisciplinary and Interdisciplinary Applications, (SNDMIA 2012), Belgrade, October 1-5, 2012., (Eigth Serbian Symposium in area of Non-linear Sciences), (Симпозијум Нелинеарна динамика - Мулти и интердисциплинарне примене)** is held in Belgrade, Sebia, and is organized by the Serbian Scientific Society and supported by Project ON174001 (2011-2014) coordinated through Department of Mechanics at Mathematical Institute SANU, in the framework of the scientific activities of the active researchers in area of Nonlinear Dynamics in Serbia and Russia.

Scientific support to the **Symposium Nonlinear Dynamics - Milutin Milanković, Multidisciplinary and Interdisciplinary Applications, (SNDMIA 2012), Belgrade, October 1-5, 2012 is given by following scientific institution in Serbia:** Serbian Society of Mechanics, Institute „Mihajilo Pupin" Belgrade, Institute „Vincha", The Society of Physical Chemists of Serbia, Faculty of Physical Chemistry University of Belgrade and Faculty of Technical Sciences Kosovska Mitrovica, University of Pristina with allocated place in Kosovska Mitrovica.

Financial support in part is given by Ministry of Education, Science and Technology Republic of Serbia_ http://www.mpn.gov.rs/sajt/.

The objective of the SYMPOSIUM is to bring together scientists and engineers working in different areas of science to present and discuss recent developments on different problems of nonlinear dynamics with multi and interdisciplinary applications.

**Technical Program**

- Plenary Lectures (13 Plenary Lectures by Leading Scientists in Nonlinear Dynamics - 40 minutes Lectures)
- Invited Lectures and Keynote Lectures (12 Invited Lectures or Keynote Lectures in sessions in different topics of natural-mathematical sciences and engineering – 30 minutes);
- Contributed Lectures and Presentations by Symposia participants (Contributed Lectures and Presentations accepted by Symposium Scientific Committee - 20 minutes; ).

**MAIN TOPICS OF THE SSYMPOSIUM**

A* Models and methods (analytical, numerical, geometrical, experimental) in nonlinear dynamics.
Qualitative and quantitative analysis of nonlinear dynamic systems.
B* Nonlinear dynamics of continuous, discontinuous and hybrid systems.
C* Bifurcations and chaos.
D* Nonlinear stochastic systems.
E* Nonlinear dynamic phenomena.
F* Control of oscillations and chaos.
G* Applications in mechanics at different scales, and real problems from any branch of engineering science including mechanical, civil, electronic, electrical, communication, medical, materials.
H* Cross-disciplinary topics from applied mathematics, physics, biophysics, genetics, nanotechnology, finance, medicine and earth sciences.

A ROUND TABLE: Research ethics and evaluation of scientific and technological research results.

Since 1992, the European Mechanics Society (EUROMECH) organizes *European Nonlinear Oscillations Conferences* (ENOCs) through its ENOC Committee. Actually, these events have a much longer tradition, since they are successors of the former ICNO (International Conference on Nonlinear Oscillations) series held from 1961 to 1990 in East-European countries. Starting with the 1st International Conference on Nonlinear Oscillations organized in Kiev, 1961, by Professor Yu. A. Mitropolsky, twelve ICNOs were held till 1990. Then, starting with the 1st European Nonlinear Oscillations Conference in Hamburg, 1992, six ENOCs were organized till 2008 (Prague, Copenhagen, Moscow, Eindhoven, St. Petersburg). Details are done in Appendix II.

Professor G. Rega was organizer last ENOC Rome 2012 and we poit out his sentences: "It is a great privilege to host the 50th Anniversary Conference of the ICNO-ENOC series in Rome, for the first time in a South-European country".

First Serbian Scientific meeting in area of Nonlinear mechanics (nonlinear oscillations and nonlinear dynamics) is organized by Serbian Society of Mechanics and Yugoslav society of Mechanics in 1984 in Arandjelovac. Academician Yu. Alekseevich Mitropolsky attend this Serbian Symposium and give one Plenary invited Lectures.Starting from this period Series of the scientific Symposia or Mini-Symposia is organized by Cher of Mechanics of Mecanical Engineering Faculty University of Niš supported bu Yougoslav or Serbian Society of Mechanics. Details are done in Appendix III.

We are happy to report that our **Symposium Nonlinear Dynamics – Milutin Milanković** has largely them previous with 83 accepted abstracts of the nonlinear dynamics contributions in different area of sciences from 12 countries are accepted and included in the program. A few Regular Sessions and one Special Session were organized.

We would like to thank all participants for their scientific contribution to **Symposium Nonlinear Dynamics – Milutin Milanković**, as well as colleagues and friends who meaningfully helped with the organization.

On behalf of the Serbian Scientific Society, welcome to Belgrade, the "Nice City", and enjoy a scientifically stimulating and socially pleasant **Symposium Nonlinear Dynamics – Milutin Milanković**!

We would like to wish all participants of this Symposium a warm welcome to our country and our Serbian Scientific Society and Venue Symposia place at Mathematical Institute SANU.

We would like to welcome you, hoping that creative efforts and achievements will prevail over preemption.

**Katica R. (Stevanović) HEDRIH and Žarko Mijajlović**
Chairs of **Symposium Nonlinear Dynamics – Milutin Milanković**
**Serbian Scientific Society**

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ | http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

### Multidisciplinary and Interdisciplinary Applications

### (SNDMIA 2012), Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Symposium

# Nonlinear Dynamics – Milutin Milanković

## With Multidisciplinary and Interdisciplinary Applications

## (SNDMIA 2012),

Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Editors: Katica R. (Stevanović) HEDRIH
Žarko Mijajlović

# PROGRAM

Beograd, October 1-5, 2012.
Venue: Mathematical Institute SANU

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

# PROGRAM

**Venue of Symposium**–Mathematical Institute SANU, Ul. Knez Mihailova 36/III, **Auditoria I**

**October 1, 2012 at 09,00h-09,40 h**

**Opening** of the Symposia Nonlinear Dynamics - Svečano otvaranje simpozijuma

## Speakers

* Members of Honorary Scientific Committee
* President of Serbian Scientific Society
* Invited Speakers - Scientists
*Chairman of Symposium: **SYMPOSIUMS ON NONLINEAR MECHANICS IN SERBIA, ICNO AND ENOC**

**October 1, 2012 at 9,45h-10,25 h, Plenary Lecture**

**Chairs:** H. Yabuno, Marina V. Shitikova and Dragan Milosavljević

**PL-1. ID-60. DYNAMICS OF MULTI-VALUED VECTOR FIELDS WITH BOUNCING FLOWS IN DISCONTINUOUS DYNAMICAL SYSTEMS, Albert C. J. Luo,** Mechanical and Industrial Engineering, Southern Illinois University Edwardsville, U. S. A.

**October 1, 2012 at 10,30h-11,10h, Plenary Lecture**

**Chairs:** H. Yabuno, Marina V. Shitikova and Dragan Milosavljević

**L-2. ID-6. SMALL NONLINEAR OSCILLATIONS OF THE SATELLITE IN AN WEAKLY ELLIPTICAL ORBIT PLANE (THE COMPARATIVE ANALYSIS OF ASYMPTOTIC METHODS OF RESEARCH)**

**P.S. Krasilnikov,** Department of Differential Equations, Moscow State Aviation Institute, Russia.

**Break 11,10h-12,00h**

**October 1, 2012 at 12,00h-12,40h, Plenary Lecture**

**Chairs:** Albert C. J. Luo, Ljiljana Kolar-Anić and Yury A. Rossikhin

**PL-3. ID-67. EXPERIMENTAL INVESTIGATESIONS OF ANALYSIS AND CONTROL OF NONLINEAR PEHNOMENA IN SOME MEHCANICAL SYSTEMS, H. Yabuno,** Department of Mechanical Engineering, Faculty of Science and Technology, Keio University, JAPAN

**Break 12,50-13,00h**

**October 1, 2012 at 13,00h-14,00h, Invited Lectures**

**Chairs:** Albert C. J. Luo, Ljiljana Kolar-Anić and Yury A. Rossikhin.

**IL-1. ID-5. GENERALIZED VAN DER POL OSCILLATORS: FROM A LIMIT CYCLE TO THE ENTRAINMENT PHENOMENON, I. Kovacić,** Department of Mechanics, Faculty of Technical Sciences, University of Novi Sad, 21 000 Novi Sad, Serbia.

**IL-2. ID-34. WALL FUNCTION CONCEPT FOR MODELING TURBULENT CHANNEL FLOW,** B. Stankovic[1], M. Sijercic[1], S. Belosevic[1], **S. Cantrak[2],** [1] Institute of Nuclear Sciences "Vinca", Belgrade University, Laboratory for Thermal Engineering and Energy, Belgrade , [2] Faculty of Mechanical Engineering, Belgrade University, Belgrade, Serbia.

**October 1, 2012 at 14,00h-14,40 h, Contributed Lectures**

**Chairs:** Ivana Kovačić, O. Kholostova and I. Ryabicova

**CL-1. ID-1. NONLINEAR DYNAMIC EFFECTS IN CYCLIC MACHINES** , **I. Vulfson,** St.-Petersburg State University of Technology and Design, Bolshaya Morskaya,18, 191186, St-Petersburg , Russia.

**Cl-2. ID-9. UNEXPECTED FEATURE OF NEW OSCILLATING HOMOGENEOUS REACTIONS: ALKYNES CARBONYLATION IN Pd COMPLEXES SOLUTIONS, Sergey N. Gorodsky,** Department of Basic Organic Synthesis, Moscow State University of Fine Chemical Technologies (MITHT), Moscow, Russia

**Break 14,40-16,00h**

**October 1, 2012 at 16,00h-19,30h, Invited Lectures – Special Session**

**Chairs:** Boris A. Malomed, Milivoj Belić and A. Maluckov.

**IL-3. ID-52. EXACT SOLUTIONS TO THE MULTIDIMENSIONAL GENERALIZED NONLINEAR SCHRÖDINGER EQUATION AND THEIR STABILITY, Milivoj Belić[a],** Wei-Ping Zhong[b], Nikola Petrović[c], Najdan Aleksić[c, a]Texas A&M University at Qatar, 23974 Doha, Qatar, [b]Shunde Polytechnic, Shunde 528300, China, [c]Institute of Physics, University of Belgrade, Serbia

**IL-4. ID-53. BIFURCATIONS AND SYNCHRONIZATION IN SYSTEMS OF REALISTIC NEURONAL MODELS, Nikola Buric,** Institute of Physics, University of Belgrade

**IL-5. ID-51. GEOMETRIC RESONANCES IN BOSE-EINSTEIN CONDENSATES WITH TWO-AND THREE-BODY INTERACTIONS, A. Balaž[1],** H. Al Jibbouri[2], I. Vidanović[3], Axel Pelster[4], ([1] Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade, Serbia, [2] Institute for Theoretical Physics, Free University of Berlin, Germany, [3] Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade, Serbia, [4] Department of Physics and Research Center OPTIMUS, Technical University of Kaiserslautern, Germany)

**IL-6. ID-56. NONLINEAR IONIC PULSES ALONG MICROTUBULES, Miljko V. Satarić,** Faculty of Technical Sciences, University of Novi Sad, Novi Sad, Serbia

**IL-7. ID-57. SOLITON DYNAMICS IN ONE-DIMENSIONAL WAVEGUIDE ARRAYS WITH SATURABLE, SELF-DEFOCUSING NONLINEARITY, M. Stepić[1],** P. P. Beličev[1], A. Maluckov[1], I. Ilić[1], D. Kip[2], A. Kanshu[2], C.E. Rüter[2], V. Shandarov[3, 1] Vinča Institute of Nuclear Sciences, Belgrade, Serbia, [2] Faculty of Electrical Engineering, Helmut Schmidt University, Hamburg, Germany, [3] State University of Control Systems and Radioelectronics, Tomsk, Russia

**IL-8. ID-54. ULTRAFAST-LASER FABRICATION OF PHOTONIC COMPONENTS, J. Petrović[1],** V. Mezentsev [2], [1] Institute of Nuclear Sciencies Vinca, Beograd, Serbia

**IL-9. ID-55. SELF-ORGANIZATION IN THE HIDDEN MARKOV MODEL OF WAVELET SIGNAL PROCESSING, M. Milovanović[1],** M. Rajković[2], [1] Mathematical Institute of the Serbian Academy of Arts and Sciences, Belgrade, [2] Vinca Institute of Nuclear Sciences, Belgrade, Serbia,

* * * * *

**October 2, 2012 at 8,30h-09,40 h, Contributed Lectures**

**Chairs:** Ilya B. Simanovskii, I. Vulfson and Ljiljana Kolar-Anić.

**CL-14. ID-8. CHAOTIC OCCURRENCE OF THE RECORDED EARTHQUAKE MAGNITUDES IN SERBIA , S. Kostić[1], N. Vasović[2]** , [1] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia,, [2] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia, e-mail:

**CL-15 ID-7. A SIMPLE MODEL OF EARTHQUAKE NUCLEATION WITH TIME-DELAY , S. Kostić[1],** I. Franović[2], K. Todorović[3], N. Vasović[4] , [1] University of Belgrade Faculty of Mining and Geology, Belgrade, Serbia, [2] University of Belgrade Faculty of Physics, Belgrade, Serbia, [3] University of Belgrade Faculty of Pharmacy, Belgrade, Serbia, [4] University of Belgrade Faculty of Mining and Geology, Belgrade, Serbia.

**CL-9. ID-74. NONLINEAR OSCILLATIORY FLOWS WITH DIFFERENT SYMMETRY PROPERTIES IN TWO-LAYER SYSTEMS, Ilya B. Simanovskii**, Department of Mathematics, Technion, Israel Institute of Technology, Haifa, Israel

**October 2, 2012 at 9,45h-10,25 h, Plenary Lecture**

**Chairs:** **A. Markeev,** Žarko Mijajlović and Pavel Krasilnikov.

**PL-4. ID-21. BRIGHT SOLITONS IN SELF-DEFOCUSING MEDIA,** Olga V. Borovkova,[1] Yaroslav V. Kartashov,[1] Valery E. Lobanov,[1] **Boris A. Malomed,**[2,1] Lluis Torner,[1] and Victor A. Vysloukh[3,1,] [1]ICFO-Institut de Ciencies Fotoniques, and Universitat Politecnica de Catalunya, 08860 Castelldefels (Barcelona), Spain, [2]Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel, [3]Departamento de Fisica y Matematicas, Universidad de las Americas – Puebla, Mexico

**October 2, 2012 at 10,30h-11,10h, Plenary Lecture**

**Chairs:** A. Markeev**,** Žarko Mijajlović and Pavel Krasilnikov.

**PL-5. ID-30. FURTHER RESULTS ON APLICATIONS OF FRACTIONAL CALCULUS IN NONLINEAR DYNAMICS - STABILITY AND CONTROL ISSUES, M. Lazarević**, Department of Mechanics,Faculty of Mechanical Engineering, Serbia.

**Break 11,10h-12,00h**

**October 2, 2012 at 12,00h-12,40h, Plenary Lecture**

**Chairs:** Mihailo Lazarević, Albert Luo and Stevan Maksimović

**PL-6. ID-18. CHAOS IN MULTIPLE-TIME-SCALE DYNAMICS OF THE BRAY-LIEBHAFSKY OSCILLATORY REACTION, Ž. Čupić[1] and Lj. Kolar-Anić[2,]** [1] Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Center of Catalysis and Chemical Engineering, Belgrade, Serbia, [2] Faculty of Physical Chemistry, University of Belgrade, Serbia

**Break 12,50-13,00h**

**October 2, 2012 at 13,00h-15,00h, Invited Lectures**

**Chairs:** Alexander V. Rodnikov, Sergey N. Gorodsky and Stevan Maksimović

**IL-10. ID-36. DIFFERENTIAL EQUATIONS OF MOTION OF MECHANICAL SYSTEMS WITH NONLINEAR NONHOLONOMIC CONSTRAINTS – VARIOUS FORMS AND THEIR EQUIVALENCE, Zekovic N. Dragomir,** University of Belgrade,Faculty of Mechanical Engineering, Belgrade, Serbia.

**IL-11. ID-3. SOME ASPECTS OF BIRD IMPACT THEORY, M. Ugrčić[1]**, [1] Economics Institute, Belgrade, Serbia.

**IL-12. ID-73. THE USE OF FINITE ELEMENTS METHOD IN VIBRATIONAL PROPERTIES CHARACTERIZATION OF MOUSE EMBRYO IN ICSI,** A. Hedrih[1]**, M. Ugrčić[2]**, [1] State University in Novi Pazar, Novi Pazar, Serbia, [2] Economics Institute, Belgrade, Serbia.

**IL – 13. ID-15. RAYLEIGH-BENARD CONVECTIVE INSTABILITY IN THE PRESENCE OF TEMPERATURE MODULATION, M. Jovanović**[1], D. Živković[2], J. Nikocijević[3], [1,2,3] University of Niš, The Faculty of Mechanical Engineering, Niš, Serbia.

**Break 15,00-16,30h**

**October 2, 2012 at 16,30h-19,10h, Contributed Lectures**

**Chairs:** S. Cantrak , M. Jovanović and A. Zlenko.

**CL-4. ID- 27. STABILITY INVESTIGATION OF A RIGID BODY AND A SYSTEM OF RIGID BODIES UNDER RAPID VIBRATIONS, O. Kholostova,** Faculty of Applied Mathematics and Physics, Moscow Aviation Institute (National Research University), Moscow, Russia.

**CL-5. ID- 32. THE NEW FORM OF FORCE FUNCTION OF TWO FINITE BODIES IN TERMS OF MODIFIED DELAUNAY'S AND ANDOYER'S ANGLE VARIABLES, A. Zlenko,** Department of high matematics of Moscow automobile and road construction state technical university (MADI), Moscow, Russia.

**CL-6. ID-35. SOME DYNAMICAL PROBLEMS FOR A PARTICLE TETHERED TO A RIGID BODY, Alexander V. Rodnikov[1]**,[1] Bauman Moscow State Technical University, Moscow, Russia.

**CL-7. ID-37. THE DEFINITION OF OPTIMAL MASS OF LOADER WITH A STOCHASTIC MODEL OF WORKING PARAMETERS, I. Ryabicova,** Department of descriptive geometry of Moscow automobile and road construction state technical university (MADI), Moscow, Russia,

**CL-8. ID-66. ELEMENTS OF MODIFICATIONS AND SENSITIVITY OF DYNAMIC PARAMETERS, Nataša Trišović**, Faculty of Mechanical Engineering, Belgrade, Serbia.

**CL-10. ID-12. ON STABILITY OF SINGULAR TIME DELAY SYSTEMS OVER THE FINITE TIME INTERVAL: CLASSICAL AND LMI APPROACH, D. Lj. Debeljkovic[1],** S. B. Stojanovic[2], G. V. Simeunovic[1], N. J. Dimitrijevic[1], [1] Faculty of Mechanical Engineering,University of Belgrade, Kraljice Marije 16, 11120 Belgrade, [2]Faculty of Technology, University of Nis, Leskovac, Serbia.

**CL-11. ID-16. MATHEMATICAL MODEL OF AERIAL ROBOTS AS THE BASIS FOR NEW RESEARCH, M. Filipovic[1]**,[1] Mihajlo Pupin Institute, The University of Belgrade, Volgina 15, 11060 Belgrade, Serbia, e-mails: mirjana.filipovic@pupin.rs

**CL-12. ID-11. PRECISE TRAJECTORY TRACKING OF ROBOTIC MECHANISM, Lj. Kevac[1],** M. Filipović[2] ,[1] School of Electrical Engineering, The University of Belgrade, Belgrade,[2] Mihajlo Pupin Institute, The University of Belgrade, Belgrade.

* * * * *

**October 3, 2012 at 8,30h-09,40 h, Contributed Lectures**

**Chairs:** I. Ryabicova, Ljiljana Kolar-Anić and V. Dragičević.

**CL-16. ID-10, REGRESION AND WEIBULL ANALYSIS OF CROPPING PRACTICES IN CHANGING CLIMATE, V. Dragičević,** Ž. Videnović, B.Kresović, M. Simić, Z. Dumanović, I. Spasojević, [1] Maize Research Institute "Zemun Polje", Serbia.

**CL17. ID-20. THE COMPARATIVE ANALYSIS OF NONLINEAR ALGORITHMS FOR TWO TANK SYSTEM CONTROL, A. Ćosić,** M. Šušić, Mihajlo Pupin Institute, Robotics Laboratory,The University of Belgrade, Belgrade, Serbia,

**CL-18. ID-23. $H_2$ AND $H_\infty$ BASED OPTIMAL PLACEMENT OF ACTUATORS AND SENSORS FOR ACTIVE VIBRATION CONTROL,** Tamara Nestorović, **Miroslav Trajkov,** Institute of Computational Engineering, Mechanics of Adaptive Systems, Ruhr-Universität Bochum, Germany

**October 3, 2012 at 9,45h-10,25 h, Plenary Lecture**

**Chairs:** A. Markeev and Dragutin Debeljković.

**PL-7. ID-25. MODERN CONCEPTS OF ACTIVELY CONTROLLED SMART STRUCTURES – AN OVERALL DESIGN APPROACH, Tamara Nestorović,** Institute of Computational Engineering, Mechanics of Adaptive Systems, Ruhr-Universität Bochum, Universitätsstr. 150, D-44801 Bochum, Germany

**October 3, 2012 at 10,30h-11,10h, Plenary Lecture**

**Chairs:** E. Grebenikov, Zekovic N. Dragomir and Nataša Trišovićć

**PL-8. ID-50. WAVE PROPAGATION IN FIBRE REINFORCED COMPOSITE LAMINATES D. Milosavljević, G. Bogdanović**Faculty of Engineering The University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia

**Break 11,10h-11,30h**

**October 3, 2012 at 11,30h-12,10h, Plenary Lecture**

**Chairs:** E. Grebenikov, Zekovic N. Dragomir and Nataša Trišovićć

**PL- 9. ID-28. SOME PROBLEMS OF DYNAMICS OF A HEAVY RIGID BODY CARRYING A MATERIAL POINT, A. Markeev,** Ishlinski Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, Russia.

**Break 12,10-12,40h**

**October 3, 2012 at 12,40h-13,20h, Plenary Lectures**

**Chairs:** A. Markeev, Zekovic N. Dragomir and Ljiljana Kolar-Anić

**PL-10. ID-69**. **ONE APPLICATION OF REGULARLY VARYING FUNCTIONS TO FRIEDMANN EQUATIONS, Žarko Mijajlović,** University of Belgrade – Faculty of Mathematics, Belgrade, Serbia

**October 3, 2012 at 13,20h-15,00h, Contributed Lectures**

**Chairs:** Tamara Nestorović, Ivana Atanasovska and Marinko Ugrčić

**CL-19. ID-26. OPTIMAL PLACEMENT OF PIEZOELECTRIC ACTUATORS AND SENSORS FOR SMART STRUCTURES USING GENETIC ALGORITHM, Seyed Mehdi Garmabi [1],** Miroslav Trajkov [2] and Tamara Nestorović [3], [1, 2, 3] Institute of Computational Engineering, Mechanics of Adaptive Systems, Ruhr-Universität Bochum, Universitätsstr. Bochum, Germany

**CL-20, ID-29. FURTHER RESULTS ON MODELING OF BIOIMPEDANCE OF THE HUMAN SKIN: CALCULUS OF NON-INTEGER ORDER APPROACH, M. Lazarević**[1,2], Z.Vosika,PhD[2] ,G. Lazović [3] , J Simić-Krstić[2], Đ. Koruga[2], [1] Department of Mechanics,[2] Module of Biomedical Engineering, Faculty of Mechanical Engineering, [2]The University of Belgrade, [3] Department of Mathematics, Faculty of Mechanical Engineering,

**CL-21. ID-2. DYNAMICS OF GEAR-PAIR SYSTEMS WITH PERIODIC VARYING MESH STIFFNESS - SPUR GEARS VS HELICAL GEARS,I. Atanasovska, M. Vukšić Popović,** Institute Kirilo Savić, Belgrade, Serbia.

**CL-22. ID. 4. FINITE ELEMENT MODELING OF WINDSHIELD- AND WING-BIRD STRIKES, M. Ugrčić,** Economics Institute, Belgrade, Serbia.

**CL-23. ID-19. THE POTENTIAL ROLE OF BULK WATER IN BRAY–LIEBHAFSKY OSCILLATORY REACTION, D. Stanisavljev,** M. Milenković, Faculty of physical chemistry, The University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia

**Break 15,00-16,30h**

**October 3, 2012 at 16,30h-19,10h, Contributed Lectures – Special Session**

**Chairs:** Ž. Čupić, Ana Ivanović-Šašić and S. Anić

**CL-24. ID-31. APPLICATION OF NON-LINEAR FREQUENCY RESPONSE METHOD FOR INVESTIGATION OF PERIODICALLY OPERATED CHEMICAL REACTORS, D. Nikolic Paunic**[1] , M. Petkovska[2], [1] Institute of Chemistry, Technology and Metallurgy, The University of Belgrade, Belgrade, [2] Faculty of Technology and Metallurgy, Department of Chemical Engineering , The University of Belgrade, Belgrade.

**CL-25. ID-38. APPLICATION OF NONLINEAR FREQUENCY RESPONSE METHOD FOR INVESTIGATION OF GAS ADSORPTION,** D. Brzić**, M. Petkovska,** Faculty of Technology and Metallurgy, University of Belgrade, Belgrade

**CL-26. ID-40. NUMERICAL EVIDENCE OF COMPLEX NONLINEAR PHENOMENA OF THE BELOUSOV–ZHABOTINSKY OSCILLATORY REACTION UNDER BATCH CONDITIONS, S. M. Blagojević**[1], S. N. Blagojević[2] and S. Anić[3], [1]Faculty of Pharmacy, University of Belgrade, Belgrade, [2]Institute of General and Physical chemistry, Belgrade, [3]Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia

**CL-27. ID-41. BIFURCATION ANALYSIS OF THE OSCILLATORY REGION OF A HYPOTHALAMIC-PITUITARY (HPA) AXIS MODEL, S. Maćešić[1],** V. M. Marković[1], A. Ivanović-Šašić[2], Ž. Čupić[2] and Lj. Kolar-Anić[1], [1]Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia , [2]Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Belgrade, Serbia.

**CL-28. ID-42. A NEW STRUCTURE OF CHAOS IN THE BRAY LIEBHAFSKY OSCILLATORY REACTION, Ana Ivanović-Šašić[1],** Vladimir Marković[2], Željko Čupić[1], Ljiljana Kolar-Anić[2], Slobodan Anić[1] **,** [1]Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia, [2]Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia.

**CL-29. ID-43. QUALITATIVE AND QUANTITATIVE ANALYSIS OF THE CHAOTIC SEQUENCE IN THE BRAY-LIEBHAFSY REACTION, Ana Ivanović-Šašić[1],** Marija Janković[2], Stevan Blagojević[3] and Nataša Pejić[4],[1]*I*nstitute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical

Engineering, Belgrade, Serbia, [2]University of Belgrade, Institute Vinča, Radiation and Environmental Protection Department, Belgrade, Serbia , [3]Departmant of Electrochemistry, Institute of General and Physical Chemistry, Belgrade, Serbia, [4]Faculty of Pharmacy, University of Belgrade, Belgrade, Serbia.

**CL-30. ID-44. BRAY-LIEBHAFSKY REACTION. DYNAMIC STATES WHEN TEMPERATURE IS THE CONTROL PARAMETER, J. Maksimovć[1],** M. Milenković[1], N. Pejić[2], D. Stanisavljev[1], S. Anić[3], [1] Faculty of Physical Chemistry, University of Belgrade, Belgrade, [2] Faculty of Pharmacy, University of Belgrade, Belgrade, [3]IChTM – Department of Catalysis and Chemical Engineering, Belgrade.

**CL-31. ID-45. HYPOTHALAMIC-PITUITARY-ADRENAL (HPA) AXIS AS NONLINEAR SYSTEM WITH FEEDBACK, Lj. Kolar-Anić[1],** Ž. Čupić[2], S. Jelić[3], V. Marković[1], S. Maćešić[1], V. Vukojević[4], [1] Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia, [2] Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Center of Catalysis and Chemical Engineering, Belgrade, Serbia, [3] Department of Theoretical Physics and Physics of Condensed Matter 020/2, Vinča Institute of Nuclear Sciences, University of Belgrade, Belgrade, Serbia, [4] Department of Clinical Neuroscience, Karolinska Institutet, CMM L8:01, 17176 Stockholm, Sweden

**CL-32. ID-46.STOHASTIC ANALYSIS OF MONOLAYER GAS ADSORPTION: THE USE OF BI-VARIATE AND MONOVARIATE PROBABILITY GENERATING FUNCTION, O. Jakšić[1],** Z. Jakšić[1], D. Ranđelović[1], Lj. Kolar-Anić[2], [1] Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Belgrade, Serbia, [2] Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia.

* * * * *

## October 4, 2012 at 8,30h-09,40 h, Contributed Lectures

**Chairs:** Ž. Čupić, Ana Ivanović-Šašić and S. Anić

**CL-33. ID-47. ANALYSIS OF REAL SAMPLES BY PERTURBATION OF NON-EQUILIBRIUM STATIONARY STATES IN AN OSCILLATING REACTION, N. Pejić[1],** S. Anić[3], J. Maksimović[2], N. Sarap[3], [1]Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, 11000 Belgrade, Serbia, [2]Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia, [3]IHTM, University of Belgrade – Department of Catalysis and Chemical Engineering, Belgrade, Serbia, [4]Institute of Nuclear Science Vinča, Radiation and Environmental Protection Department, University of Belgrade, Serbia

**CL-34. ID-48. EMULSIONS AND DOUBLE EMULSIONS AS PARTICULAR EXAMPLES OF MEMRISTIVE SYSTEMS, A.M. Spasic,** Institute for Technology of Nuclear and Other Mineral Raw Materials, Dept. of Chem.Engng. Belgrade, Serbia.

**CL-35. ID-75. TYPES OF BIFURCATION POINTS IN BRAY-LIEBHAFSKY OSCILLATORY REACTION, B. Stanković[1],** Ž. Čupić[2] and Lj. Kolar-Anić[1],[1]Faculty of Physical Chemistry, University of Belgrade, Belgrade, Serbia, [2]Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Belgrade, Serbia.

## October 4, 2012 at 9,45h-10,25 h, Plenary Lecture

**Chairs:** Albert Luo, Mihailo Lazarević and K. (Stevanović) Hedrih

**PL-11. ID-84. FRACTIONAL DERIVATIVE VISCOELASTIC MODELS IN THE WAVE THEORY OF IMPACT, Yury A. Rossikhin and Marina V. Shitikova,** Research Center on Dynamics of Solids and Structures, Voronezh State University of Architecture and Civil Engineering, Voronezh, Russia

## October 4, 2012 at 10,30h-11,10h, Special Presentation

**Chairs:** Tamara Nestorović, Ljiljana Kolar-Anić and Stevan Maksimović

**SL-1. ID-70. DIGITIZATION OF SCIENTIFIC AND CULTURAL HERITAGE IN SERBIA, Žarko Mijajlović,** University of Belgrade – Faculty of Mathematics, Belgrade, Serbia

**Break 11,10h-12,00h**

## October 4, 2012 at 12,00h-12,40h, Contributed Lectures

**Chairs:** Alexander V. Rodnikov, Šefik M.Bajmak and Srdjan Jović.

**CL-49. ID-61. COMPARATIVE ANALYSIS OF OPTIMAL SIZE WITH THE RESULTS EJECTORS EXPERIMENTAL EJECTORS TESTED IN DISTRICT HETING , Šefik M.Bajmak,** University of Pristine, Faculty of Technical Sciences

**CL-50. ID-68. NUMERICAL EVIDENCE OF COMPLEX NONLINEAR PHENOMENA OF THE BRAY–LIEBHAFSKY OSCILLATORY REACTION UNDER CSTR CONDITIONS, S. N. Blagojević[1], S. M. Blagojević[2], Ž. Čupić[3] ,** [1]Institute of General and Physical chemistry, Belgrade, [2]Faculty of Pharmacy, University of Belgrade, Belgrade, [3]IChTM –Department of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade

**Break 12,50-13,00h**

**October 4, 2012 at 13,00h-13,40h, Round Table**

**Invitataion for Chairs and Contribytion Discussions:** Slobodan Perović, Jaroslav Labat, Zoran Marković, Živorad Čekovićć, Hiroshi Yabuno, Pavel Krasilnikov, Marina V. Shitikova, Gradimir Milovanović, Žarko Mijajlović, K. (Stevanović) Hedrih.

**Round Table: "Ethic of Scientists and Evaluation of Scientific Research Results".**

**October 4, 2012 at 13,40h-15,00h, Contributed Lectures**

**Chairs:** Srdjan Jović, Ljiljana Veljović and Julijana D. Simonović

**CL-36. ID-80. STOCHASTIC PARAMETRICALLY EXCITED HEREDITARY SANDWICH MULTI BEAM DYNAMICAL SYSTEMS, K. (Stevanović) Hedrih'** Mathematical Institute SANU, Department of Mechanics, and Faculty of Mechanical Engineering University of Niš, Serbia.

**CL-37. ID-79. HYBRID SYSTEM DYNAMICS ON LAYER WITH NONLINEAR ELASTIC AND INERTIA PROPERTIES,** K. (Stevanović) Hedrih [1]**, M. Stamenković** [2]**, N. Nešić** [2]**,** [1] Mathematical Institute SANU, Department of Mechanics, and Faculty of Mechanical Engineering University of Niš, Serbia,[2] Mathematical Institute SANU, Department of Mechanics, Kneza Mihaila 36/III, Serbia,

**CL-38. ID-22. SYSTEM OF DOUBLE THIN PLATES CONNECTED WITH LAYER OF ROLLING VISCO-ELASTIC NONLINEAR PROPERTIES, Julijana D. Simonović,** Faculty of Mechanical Engineering, University of Niš, Niš, Serbia.

**CL-39. ID-78. FORCED OSCILLATIONS OF A MEMBRANE ON NONLINEAR ELASTIC FOUNDATION, Nikola Nešić**, Mathematical Institute SANU, Department of Mechanics, Belgrade, Serbia,

**Break 15,00-16,30h**

**October 4, 2012 at 16,30h-19,10h, Contributed Lectures**

**Chairs:** Srdjan Jović, Ljiljana Veljović and Julijana D. Simonović

**CL-40. ID-59. VIBRO-IMPACT SYSTEM BASED ON OSCILLATOR WITH TWO HEAVY MASS PARTICLES ALONG HORIZONTAL ROUGH LINE ,** Srdjan Jović, **Vladimir Raičević,** Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština- Kosovska Mitrovica, Serbia.

**CL-41. ID-71. THE PHASE PORTRAIT OF THE VIBRO-IMPACT SYSTEM BASED ON OSCILLATOR, WITH THREE HEAVY MASS PARTICLES MOVING ALONG A ROUGH CIRCLE, Srdjan Jović,** Vladimir Raičević, Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština-Kosovska Mitrovica, Serbia,

**CL-42, ID-39. ANALYSIS OF A RIGID BODY ROTATION AROUND TWO NO INTERSECTING AXES – VECTOR METHOD AND PARAMETER ANALYSIS OF PHASE TRAJECTORIES, Ljiljana Veljović,** Faculty of Mechanical Engineering University of Kragujevac, Kragujevac, Serbia.

**CL-43. ID-33. THREE-PARAMETRIC TESTING OF SINGULARITY AND POSITION OF NON-LINEAR DYNAMICS RELATIVE BALANCE OF HEAVY MATERIAL PARTICLE ON ECCENTRICALLY ROTATING SMOOTH CIRCLE LINE, Marija Stamenković***, Marija Mikić**, *Mathematical Institute SANU, Department of Mechanics, Belgrade, Serbia,**Mathematical Institute SANU, Department of Mechanics, Belgrade, Serbia,and Faculty of Mathematics, Belgrade, Serbia.

**CL-44. ID-49. OPTIMAL DESIGN OF THIN-WALLED AIRCRAFT STRUCTURES USING TWO-LEVEL OPTIMIZATION APPROACH, S. Maksimovic[1],** K. Maksimovic[2] I. Vasovic[3], [1] Military Technical Institute,Belgrade, Serbia, [2]Republic Serbia, City Administration of City of Belgrade, Secretariat for Utilities and Housing Services Water Management, Belgrade, Serbia, [3] Institute Goša, Milana Rakica 35, Belgrade, Serbia

**CL-45. ID-18. OPTIMIZATION OF SANDWICH PLATES WITH PRISMATIC CORES, J. Djoković[1],** K.Veljković[2], [1] Technical Faculty of Bor, University of Belgrade, Serbia, [2] Polytechnical school Kragujevac, Kragujevac, Serbia

**CL-46. ID-76. NONLINEAR FREE VIBRATIONS OF AN ELASTICALLY CONNECTED CIRCULAR DOUBLE-MEMBRANE COMPOUND SYSTEM (PART I), Milan Cajić**, Danilo Karličić**,** Mathematical Institute SANU, Department of Mechanics. Belgrade, Serbia.

**CL-47. ID-77. NONLINEAR FORCED VIBRATIONS OF AN ELASTICALLY CONNECTED CIRCULAR DOUBLE-MEMBRANE COMPOUND SYSTEM, Danilo Karličić,** Milan Cajić ,Mathematical Institute SANU, Department of Mechanics. Belgrade, Serbia.

* * * * *

## October 5, 2012 at 8,30h-09,40 h, Contributed Lectures

**Chairs:** Stevan Maksimović and Zijah Burzić.

**CL-51. ID-62. COMPATIBILITY OF ENDURANCE LIMIT AND FATIGUE CRACK GROWTH PARAMETERS IN BEHAVIOR OF WELDED JOINT OF LOW ALLOYED STEELS, Ivica Čamagić[1],** Zijah Burzić[2], Nemanja Vasić[1], Srđan Jović[1], Slobodan Makragić[1], Predrag Živković[1],[1] Faculty of Technical Sciences, University of Pristina, - Kosovska Mitrovica, Serbia, [2] Military Institute of Techniques, 1 Ratka Resanovića Street, Belgrade, Serbia

**CL-52. ID-64. CHARACTERIZATION OF ALLOYS AND LIQUIDUS PROJECTIONS OF THE TERNARY Bi–Cu–In SYSTEM, M. Premović** and D. Minić,[1] The University of Pristina, Faculty of Technical Sciences, Kosovska Mitrovica, Serbia .

**CL-53.ID-65. CHARACTERIZATION OF ALLOYS OF THE TERNARY Ag–Bi–Zn SYSTEM, D. Minić and M. Premović,** The University of Pristina, Faculty of Technical Sciences, -Kosovska Mitrovica, Serbia

## October 5, 2012 at 9,45h-10,25 h, Plenary Lecture

**Chairs:** H. Yabuno, Pavel Krasilnikov and Marina V. Shitikova

**PL-12. PL-11. IF-14. HOMOGRAPHICAL SOLUTIONS OF HAMILTONIAN SYSTEMS AND COMPUTER ALGEBRA, E. Grebenikov[1] , N. Zemtsova[2],** Institution of Russian Academy of Sciences Dorodnicyn Computing Centre of RAS, Moscow, Russia

## October 5, 2012 at 10,30h-11,10h, Contributed Lectures

**Chairs:** Stevan Maksimović and Zijah Burzić.

**CL-54. ID-63. HIGH- TECH ARCHITECTURE AND SYNONYMS FOR A PREFABRICATED MODEL, Julija Aleksić,** Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština - Kosovska Mitrovica, Serbia.

**CL-55. ID-24. EXPERIMENTAL DEMONSTRATION OF HOMOCLIC CHAOS AND MIXED-MODE OSCILLATION, Syamal K.Dana,** S.Chakraborty Central Instrumentation,CSIR-Indian Institute of Chemical Biology Jadavpur, Kolkata 700032, India e-mail: syamaldana@gmail.com/

**CL- 56. ID-83.. METHODS OF RELIABILITY ASSESSMENT OF DAMAGED PIPELINE CORROSIO, Živče Šarkoćević[1], Miodrag Arsić[2], Srdjan Jović[3], Dragan Lazarević[1],[1]**High Technical School of Professional Studies, Zvečan, Serbia, [2]Institute for Testing of Materials (IMS), Belgrade, Serbia, [3] Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština, Serbia,

**Break 11,30h-12,00h**

## October 5, 2012 at 12,00h-12,40h, Plenary Lecture

**Chairs:** H. Yabuno, Pavel Krasilnikov and Marina V. Shitikova

**PL-13. ID-81. PHENOMENOLOGICAL MAPPING AND MATHEMATICAL ANALOGY IN NONLINEAR DYNAMICAL SYSTEMS, K. (Stevanović) Hedrih [1,] [1]** Mathematical Institute SANU, Department of Mechanics, and Faculty of Mechanical Engineering University of Niš, Serbia,

## October 5, 2012 at 12,40h-13,20h

**Chairs:** H. Yabuno, Albert Luo, Pavel Krasilnikov, Marina V. Shitikova , K. (Stevanović) Hedrih and Žarko Mijajlović.

## Closing ceremony – Future in Nonlinear dynamics

## Serbian Scientific Society

**Symposium Nonlinear Dynamics – Milutin Milanković**

**Multidisciplinary and Interdisciplinary Applications**

**(SNDMIA 2012), Belgrade, October 1-5, 2012.**

**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Symposium**

# Nonlinear Dynamics – Milutin Milanković

**Multidisciplinary and Interdisciplinary Applications**

## (SNDMIA 2012),

**Belgrade, October 1-5, 2012.**

**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Editors: Katica R. (Stevanović) HEDRIH**
**Žarko Mijajlović**

# Booklet of Abstracts

**Beograd, October 1-5, 2012.**
**Venue: Mathematical Institute SANU**

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

**Multidisciplinary and Interdisciplinary Applications**
**(SNDMIA 2012), Belgrade, October 1-5, 2012.**
**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Symposium**

# Nonlinear Dynamics – Milutin Milanković

## Multidisciplinary and Interdisciplinary Applications

# (SNDMIA 2012),

**Belgrade, October 1-5, 2012.**
**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Editors: Katica R. (Stevanović) HEDRIH**
**Žarko Mijajlović**

# Plenary Lectures

**Beograd, October 1-5, 2012.**
**Venue: Mathematical Institute SANU**

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-1. ID-60.**

# DYNAMICS OF MULTI-VALUED VECTOR FIELDS WITH BOUNCING FLOWS IN DISCONTINUOUS DYNAMICAL SYSTEMS

**Albert C. J. Luo**
Department of Mechanical and Industrial Engineering, Southern Illinois University Edwardsville
Edwardsville, IL62026-1805, USA
Tel: 1-618-650-5389, Fax: 1-618-650-2555
Email: aluo@siue.edu

In this presentation, a classification of discontinuity in discontinuous dynamical systems will be discussed first. To discuss the singularity to the boundary, the grazing and inflexional singular sets on the boundary will be presented, and the real and imaginary singular sets will be also discussed. With permanent flow barriers, the forbidden boundary and the boundary channel will be presented. The forbidden boundary will not allow any flows passing through the boundary, and the boundary channel will not allow any boundary flows getting into the corresponding domains. Further, the domain and boundary classification will be addressed. Sink and source domains will be discussed. Similarly, the sink and source boundary will be also presented. Because of $C^0$-discontinuity, the flow barriers, the isolated domains and the boundary channels, the transport laws are needed to continue the flow in discontinuous dynamical systems. Multi-valued vector fields in a single domain will be introduced. With the simplest transport law (i.e., the switching rule), the bouncing flow on the boundary will be presented, and the extendable flows will be discussed as well. A controlled piecewise linear system will be presented as an application, and the vector fields on both sides of the boundary will be switched at the boundary. The bouncing flows will be illustrated in such a controlled piecewise linear system.

## Brief CV

Albert C. J. Luo, Ph.D., ASME Fellow, Full Professor of Mechanical Engineering at Southern Illinois University Edwardsville, Edwardsville, IL62026. Dr. Luo received his BS in Mechanical Engineering and MS in Engineering Mechanics from China, and Ph.D. in applied mechanics from University of Manitoba, Canada. His main research interests are involved in fundamental theory for: (i) chaos in nonlinear Hamiltonian systems, (ii) nonlinear deformable-body dynamics, and (iii) complexity in discontinuous dynamical systems. Dr. Luo has published four monograph: Singularity and Dynamics on Discontinuous Vector Fields (Elsevier), Global Transversality, Resonance and Chaotic Dynamics (World Scientific), Discontinuous Dynamical Systems on Time-varying Domains (Springer-HEP), and Nonlinear Deformable-body Dynamics (Springer-HEP). He has also published over 100 peer-reviewed journal articles. Editors for book series on "Nonlinear Physical Science" (Springer-HEP), and "Mathematical Methods and Modeling for Complex Phenomena" (Springer-HEP), Complexity, Nonlinearity and Chaos (World-Scientific). Editor for the Journal "Communications in Nonlinear Science and Numerical Simulation".

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-2. ID-6.**

# SMALL NONLINEAR OSCILLATIONS OF THE SATELLITE IN AN WEAKLY ELLIPTICAL ORBIT PLANE (THE COMPARATIVE ANALYSIS OF ASYMPTOTIC METHODS OF RESEARCH)

**P.S. Krasilnikov**
Department of Differential Equations, Moscow State Aviation Institute,
Volokolamskoe st. 4, Moscow, 125871, Russia
e-mail: krasil06@rambler.ru

Small plane oscillations of the satellite in an weak elliptical orbit are investigated. The equation of oscillations of the satellite

$$\varepsilon(1+e\cos\nu)\frac{d^2\bar{\delta}}{d\nu^2}-2e\varepsilon\sin\nu\frac{d\bar{\delta}}{d\nu}+\omega^2\sin(\varepsilon\bar{\delta})=4e\sin\nu \quad \left(\bar{\delta},\bar{\delta}' : \ 1\right)$$

contains two small parameters: $e$ (an orbit eccentricity) and $\varepsilon$ (a measure of a deviation of a phase point from zero values of co-ordinates and velocities). The various kinds of procedure reducing an input problem to a case of one parameter is analyzed: as linear and non-linear explicit reductions, implicit one-parameter reductions having one of forms

$$e=\varepsilon\left(e^{2/3}+D\varepsilon^{2/3}\right), \quad e=\varepsilon\left(e^{\alpha}+\varepsilon^{\alpha}\right)$$

setting various orders of smallness of the singular parametre $e/\varepsilon$. It is shown that the reduction is a little effective as it conducts to a set of the shortened equations and the subsequent joining of solutions of these equations with each other. The nonlinear reduction complicates the problem of the research because the right hand side of equation has not Taylor expansions on small parameter, as a rule. The reduction does not allow to investigate a bifurcation of solutions of the equation, except for a special case when the set of curves of a reduction covers a bifurcation surface. The reduction does not give also completeness of a picture of oscillations as does not allow to investigate oscillations along arbitrary curves of space of small parameters.

The equation of small oscillations of the satellite is investigated with the help of the generalized averaging method with small independent parameters [2]. The picture of oscillations in the first and second approximations of averaging method is described. It is shown that outcomes of researches are free from reduction shortages.

**Keywords**: small oscillations, reductions, averaging method, small independent parameters

**References**

[1]. V.V. Beletskii The attitude motions of the satellite around the centre of mass. *Moscow State University, Moscow*, 1975, 308 p.
[2]. P.S. Krasilnikov The non-linear oscillations of a pendulum of variable length on a vibrating base *J. Appl. Math. Mech.* V. 76 Is. 1 (2012), pp. 25-35

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-3. ID-67**.

# EXPERIMENTAL INVESTIGATESIONS OF ANALYSIS AND CONTROL OF NONLINEAR PEHNOMENA IN SOME MEHCANICAL SYSTEMS

**H. Yabuno**
Faculty of Science and Technology
Keio University 223-8522, JAPAN
e-mail: yabuno@mech.keio.ac.jp

In this presentation, positive utilizations of nonlinear phenomena are considered for establishing high performance mechanical systems [1]. Because nonlinear phenomena are generally very complex and not predictable of their occurrences, many control methods to avoid the occurrences have been proposed. On the other hand, most mechanical systems have inherently nonlinear characteristics in their inertia or restoring forces. In this presentation, we do not suppress the nonlinear phenomena due to the nonlinearity, but try to positively utilize nonlinear phenomena produced due their nonlinearity. First, we deal with an under actuated manipulator of which fist joint has actuator and sensor but second joint does not have actuator nor sensor. In most researches on under actuated manipulators, the assumption that the second joint does not have actuator but has sensor is given, i.e., the feedback with respect to the angle of the free link connected to the second joint is possible, are given. On the other hand, we consider the case when the angle of the free link is not measurable, i.e. the feedback control with respect to the angle of the free link cannot be applied [2]. By high-frequency excitation of the first link to the second joint, the supercritical and subcritical, and their perturbed pitchfork bifurcations can be produced and many kinds of stable steady states can appear. We analytically show the motion control strategy by using the amplitude equation and confirm the validity through experiments. The second topic is related to the realization of a high performance atomic force microscope (AFM) [3]. For measuring soft materials as biological samples, we have to keep small amplitude in the self-excited micro-cantilever probe not to give the damage to the sample due to the contact between the cantilever and the samples. To this end, we utilize the nonlinear dynamics of van der Pol oscillator which has a limit cycle depending on the magnitude of the nonlinearity. We apply the nonlinear feedback proportional to the velocity and the deflection squared to realize the dynamics of van der Pol oscillator in the micro-cantilever probe in AFM. The validity is experimentally confirmed from the practical sample images.

**Keywords**: Bifurcation, bifurcation control, amplitude equation, underactuated manipulator, Atomic force microscope.

**References**

[1] Yabunho, H., Stabilization and utilization of nonlinear phenomane based on bifurcation control of slow dynamics, *Journal of Sound and Vibration*, **315** (2008), pp. 766-780.
[2] Yabuno, H. Matsuda, T., and Aoshima, N., Reachable and stabilizable area of an underactuated manipulator without state feedback control, *IEEE/ASME Transactions on Mechatronics*, **10** (2005), pp. 397-403.
[3] Yabuno, H., Kuroda, M., Someya, T., Nishimura, K., Hayashi, K., and Ashida, K., *Japanese Journal of Applied Physics*, **50** (2011), 076601.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-4. ID-21.**

# BRIGHT SOLITONS IN SELF-DEFOCUSING MEDIA

**Olga V. Borovkova,**[1] **Yaroslav V. Kartashov,**[1] **Valery E. Lobanov,**[1] ***Boris A. Malomed*,**[2,1] **Lluis Torner,**[1] **and Victor A. Vysloukh**[3,1]
[1]ICFO-Institut de Ciencies Fotoniques, and Universitat Politecnica de Catalunya, 08860 Castelldefels (Barcelona), Spain
[2]Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel
[3]Departamento de Fisica y Matematicas, Universidad de las Americas – Puebla, Mexico

**Summary.**

We present a survey of recent numerical and analytical results which demonstrate a new setting for the creation of stable bright solitons and solitary vortices in any spatial dimension ($D$ = 1,2,3): the use of the purely self-defocusing (SDF) nonlinearity whose local strength grows with the radial coordinate, $r$ , faster than $r^D$ , without adding any linear potential.

## Introduction

A commonly adopted principle underlying the studies of self-sustained localized modes (bright solitons) in diverse physical settings is that they are supported either by the self-focusing nonlinearity, or, in the form of gap solitons, by the defocusing nonlinearity combined with periodic linear potentials. The formation of bright solitons was also studied in more sophisticated settings, in the form of *nonlinear lattices* [1], where the nonlinearity periodically changes its magnitude in space. Such lattices readily support stable solitons in 1D, while it is much harder to employ them for the stabilization of multidimensional solitons.

Guiding bright solitons by pure self-defocusing (SDF) nonlinearities, without the help of a linear potential, was until recently considered impossible. Nevertheless, recent works [2-4] have demonstrated that this is *possible* (and, in fact, quite easy), if the local strength of the SDF term is modulated in space, growing towards the periphery faster than $r^D$ , where $r$ is the radial coordinate in the $D$ -dimensional space. In terms of the theoretical analysis, the existence of bright solitons in this setting is a consequence of the fact that, in contrast to homogeneous media, where the necessity to support decaying tails of the solitons places them into the semi-infinite spectral gap of the linearized system, in which SDF nonlinearities cannot support any self-localization, in the present case the growth of the nonlinearity coefficient makes the underlying equations *non-linearizable* for the decaying tails.

## The model and basic results

The simplest model equations which demonstrate the new mechanism of the soliton formation are 1D and 2D nonlinear Schrödinger equations for optical wave field $q$, or the wave function of a Bose-Einstein condensate, with modulation index $a > D$ [2]:

$$i¶q/¶x = -(1/2)Ñ^2 q + (1 + r^a)|q|^2 q. \qquad (1)$$

Soliton solutions to Eq. (1) with propagation constant $b$ are looked for as $q(r,x) = w(r)\exp(ibx + imf)$, where $m$ is the topological charge of 2D vortices and $f$ the angular coordinate. Typical examples of 1D and 2D fundamental and vortical solitons are shown in Fig. 1.

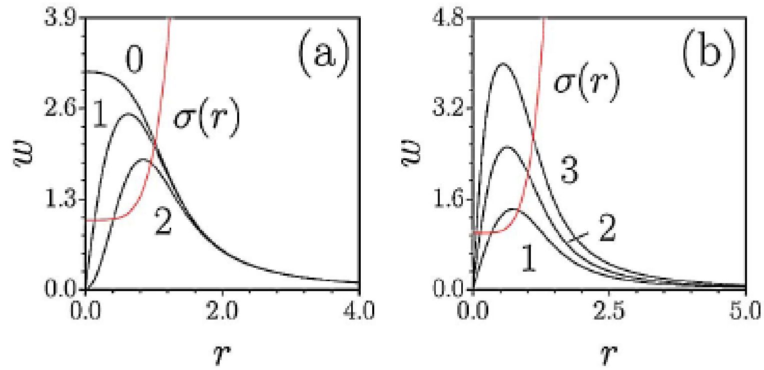



**Figure 1.** (a) Profiles of 2D solitons with different vorticities ($m = 0,1,2$) corresponding to $\alpha = 5, b = 10$. (b) Vortices with $m = 1$ corresponding to $b = -5, -10, -20$ (curves 1, 2, 3). The nonlinearity-modulation profile is shown too.

The shapes of the fundamental solitons are accurately produced by the Thomas-Fermi approximation, which neglects the diffraction term in Eq. (1): $w_{\text{TF}}^2(r) = -b/\left(1 + r^2\right)$. These shapes are virtually indistinguishable from their numerically found counterparts. Further, the fundamental solitons are completely stable, irrespective of the dimension and modulation index $\alpha$ in Eq. (1), while 2D vortices, with both $m = 1$ and $m \geq 2$, are stable unless $\alpha$ is too close to the minimal value, $\alpha = 4$. The same pertains to higher-order 1D solitons (multipoles).

## Conclusions

In models of the same type, but with steeper modulation of the SDF nonlinearity, the stability region is still broader (in particular, all vortices with $m = 1$ may be stable) [3]. The analysis was also extended to bimodal systems [4] and to similarly structured dissipative media, with the uniform linear gain and cubic loss growing at $r \to \infty$ faster than $r^D$ [5].

## References


[1] Y.V. Kartashov, B.A. Malomed, and L. Torner, *Rev. Mod. Phys*., **83**, 247, 2011
[2] O.V. Borovkova, Y. V. Kartashov, B. A. Malomed, and L. Torner, *Opt. Lett*. **36**, 3088, 2011
[3] O. V. Borovkova, Y. V. Kartashov, L. Torner, and B. A. Malomed, *Phys. Rev*. E (R), **84**, 035602, 2011
[4] Y.V. Kartashov, V. A. Vysloukh, L. Torner, and B. A. Malomed, *Opt. Lett*. **36**, 4587 (2011).
[5] O. V. Borovkova, Y. V. Kartashov, V. A. Vysloukh, V. E. Lobanov, B. A. Malomed, and L. Torner, *Opt. Exp*. **20**, 2657, 2012.

Naučno društvo Srbije | Serbian Scientific Society

http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-5. ID-30.**

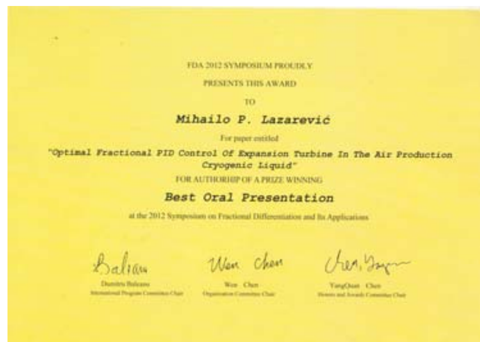
# FURTHER RESULTS ON APLICATIONS OF FRACTIONAL CALCULUS IN NONLINEAR DYNAMICS - STABILITY AND CONTROL ISSUES

**Mihailo Lazarević[1]**
[1] Department of Mechanics,Faculty of Mechanical Engineering,
The University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35,Serbia
e-mail: mlazarevic@mas.bg.ac.rs

**ABSTRACT.** In this paper, they are presented recently obtained results which are related to applications of fractional calculus in dynamics- specially stability and control issues. Some of these results [1-4] are presented at the fifth symposium of fractional differentiation and its applications was held a t the Hohai University, Nanjing, China in the period of May 14-May 17, 2012. A number of 262 participants from 26 different countries have attended the symposium. A number of 12 symposia were organized, six plenary lecturers and 10 semi-plenary lecturers were presented. After a peer review 208 articles were accepted as oral presentations and 73 as posters. Also, I received awards *for the Best poster* for the paper *Finite Time Partial Stability of Fractional Order Time Delay Systems*,[1] as well as and *the Best oral presentation* for the *paper Optimal Fractional Order PID Control Of Expansion Turbine In The Air Production Cryogenic Liquid,* [3](*see 2page).*
In recent years, there have been extensive research activities related to applications of fractional calculus (FC), [5] in nonlinear dynamics, mechatronics as well as control theory. First, we propose sufficient conditions for finite time stability for the (non)homogeneous fractional order systems with time delay. Specially, the problem of finite time stability with respect to some of the variables (partial stability) is considered. Namely, along with the formulation of the problem of stability to all variables, Lyapunov also formulated a more general problem on the stability to a given part of variables (but not all variables) determining the state of a system,[6].The problem of the stability of motion with respect to some of the variables also known as partial stability arises naturally in applications. So, in this presentation, it will be proposed finite time partial stability test procedure of linear (non)autonomous unknown time delay fractional order systems. Time-delay is assumed to be unknown but its upper bound is assumed to be known. New stability criteria for this class of fractional order systems will be derived using a recently obtained generalized Gronwall inequality as well as "*classical"* Bellman-Gronwall inequality,[7]. Last,a numerical example is provided to illustrate the application of the proposed stability procedure.

Second, it is suggested a robust fractional-order sliding mode control of a 3-DOF`s robot system driven by DC motors. Primarily, a conventional sliding mode controller based on $PD^{\alpha}$ sliding surface is designed. Sliding-mode controller is a powerful tool to robustly control incompletely modeled or uncertain systems which has many attractive features such as fast response, good transient response and asymptotic stability, [8]. Numerical simulations have been carried out to verify and compare the significance of the proposed control which resulted in reducing high switching amplitude where fractional-order sliding mode controller has better performance in the comparison with the conventional sliding mode controller. Third, it will be presented here a new algorithms of PID control based on applying FC in control of given

mechatronic system for the producing of technical gases, i.e air production cryogenic. Objective is to find out optimum settings using genetic algorithms for a fractional $PI^{\alpha}D^{\beta}$ controller in order to fulfill different design specifications for the closed-loop system, taking advantage of the fractional orders and properties of liquid. This method allows the optimal design of all major parameters of a fractional PID controller and then enhances the flexibility and capability of the PID controller. Then, in simulations they are compared step responses of these two optimal controllers. Last, it will be shown that FOPID controller improves transient response as well as provides more robustness in than conventional *PID,* particularly in the case of disturbance rejection. Last, this paper addresses the problem of iterative learning control (ILC) for fractional (non)linear time delay system. Here, we extends results obtained in [9], to consider more general systems i.e. fractional time delay systems described in the form of state space and output equations with uncertain constant delay as well as time-varying delay. Sufficient convergent conditions of a proposed ILC will be derived in time-domain and formulated by the theorems.


**Keywords**: robust control, fractional calculus, sliding-mode, chattering-free, stability criteria, partial stability, time delay, finite-time stability, optimal settings, iterative learning control

***Acknowledgement.*** This work is supported by the Ministry of Education and Science of Republic of Serbia, Projects No.41006 and 35006, as well as EUREKA!4930 project.

**References**

[1] M.Lazarević, Finite Time Partial Stability of Fractional Order Time Delay Systems, No147, (2012),*FDA2012,Nanjing*,May,China.

[2] S.A. Batalov, M.P. Lazarević,A. Hace, K.Jezernik, A Chattering-free Fractional PDa Sliding-mode Control of a 3-DOF Robot System Driven by DC motors,No130,(2012) *FDA2012,Nanjing*,May,China.

[3] Lj. Bučanović, M. Lazarević,S. Batalov,Optimal Fractional Order PID Control Of Expansion Turbine In The Air Production Cryogenic Liquid,No128, (2012),*FDA2012, Nanjing*,May,China.

[4] M.Lazarević,Iterative Learning Control for Fractional Order Time Delay System - *PD*a Type,No 259,*(2012),FDA2012, Nanjing*,May,China..

[5] Sabatier J., O. P. Agrawal, J.A. Tenreiro Machado:*Advances in Fractional Calculus*:Theoretical Developments and Appl. in Physics and Engineering, Springer,(2007),p.55.

[6] Vorotnikov, V., Partial Stability and Control: The State-of-the-art and Development Prospects, *Automation and Remote Control*, Vol. 66, No. 4, (2005).pp. 511-561.

[7] Lazarević, M.,A.SpasićFinite-time stability analysis of fractional order time delay systems: Gronwall`s Approach, *Mathematical and Computer Modelling*,(2009),49, pp.475-481.

[8] Edwards, C., and Spurgeon, S.K.(1998). *Sliding mode control theory and applications,* Taylor & Fran., NewYork

[9] Lazarević M., $PD^{\alpha}$ -type iterative learning control for fractional LTI system-*16th International Congress of Chemical and Process Engineering, CHISA* 2004, (2004),22-26, August, Praha,Czech Republic.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-6. ID-18.**

# CHAOS IN MULTIPLE-TIME-SCALE DYNAMICS OF THE BRAY-LIEBHAFSKY OSCILLATORY REACTION

**Ž. Čupić[1] and Lj. Kolar-Anić[2]**

[1] Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Center of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade, Serbia
e-mail: zcupic@nanosys.ihtm.bg.ac.rs
[2] Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, Belgrade, Serbia
e-mail: lkolar@ffh.bg.ac.rs

**ABSTRACT.** Any reaction system starting from some arbitrary initial conditions tends to final steady or equilibrium state that plays the role of an attractor, passing through distinct regions of the phase space while its different chemical species simultaneously transform through the reaction network. Since, almost all complex many variable dynamical systems are characterized by multiple-time-scales, various forms of attractors and transitions between different dynamical states were studied in such systems. Typical example is an oscillatory reaction, known as the Bray-Liebhafsky (BL) one, that consists of a complex homogeneous catalytic oscillatory process involving numerous iodine intermediates such as $I_2$, $I^-$, HIO, $HIO_2$ and $I_2O$, that all oscillates [1]-[4]. The concentrations of mentioned species in the considered process differ for several orders of magnitudes among themselves. Thus, typical concentration of hydrogen peroxide during oscillatory state of the system is between $10^{-2}$ and $10^{-1}$ mol dm$^{-3}$, the concentration of iodine is between $10^{-5}$ and $10^{-4}$ mol dm$^{-3}$, whereas the concentrations of other species are much lower, between $10^{-9}$ and $10^{-6}$ mol dm$^{-3}$. Consequently, their simultaneous time variations are different, resulting in dissimilar behaviors characteristic for multiple-time-scale systems with, at least, slow (large-concentration) and fast (low-concentration) species. In systems, where concentrations of crucial species differ significantly, the relaxation oscillations are common. Furthermore, in multiple-time-scale systems with more than one slow variable, mixed-mode oscillations may appear in the region with simple sustained oscillations. [5] They generally consist of two types of oscillations with distinct amplitudes: large-amplitude oscillations (LAO-s) and small-amplitude oscillations (SAO-s). Depending on numbers of small (S) and large (L) oscillations in a period, different periodic dynamical states can be identified and assigned by state enumeration $L^S$. Between every two successive periodic states with different dynamical assignation, the chaotic states appear. [6] Here we analyze emerging of chaotic atractor in the model [7] of BL reaction with multiple-time-scale dynamics. With aim to explain the mixed-mode oscillations obtained by numerical simulations of the various dynamical states of a model for the Bray-Liebhafsky reaction under CSTR conditions, the folded singularity points on the critical manifold of the full system and Andronov-Hopf

bifurcation of the fast subsystem are calculated. The interaction between those singularities causes occurrence of tourbillion structure and canard solutions.

**Keywords**: Bray-Liebhafsky oscillatory reaction, mixed mode oscillations, critical manifold, multiple time scale dynamics

**Acknowledgement.** This work was funded by Serbian Ministry of Education and Science through the projects III 45001 and ON 172015.

**References**

[1] W.C. Bray, A periodic reaction in homogeneous solution and its relation to catalysis, J. Am. Chem. Soc. 43 (1921) 1262-1267.

[2] W.C. Bray, H.A. Liebhafsky, Reaction involving hydrogen peroxide, iodine and iodate ion. I. Introduction, J. Am. Chem. Soc. 53 (1931) 38.

[3] Lj. Kolar-Anić, Ž. Čupić, S. Anić, G. Schmitz, Pseudo-steady states in the model of the Bray-Liebhafsky oscillatory reaction, J. Chem. Faraday Trans. 93 (1997) 2147-2152.

[4] G. Schmitz, Models for the oscillating reactions nullclines and steady states, in: Physical Chemistry '98, S. Ribnikar, S. Anić (Eds.), Soc. Phys. Chemists of Serbia, Belgrade, 1998, pp.173-179.

[5] M. Desroches, J. Guckenheimer, B. Krauskopf, C. Kuehn, H. Osinga, M. Wechselberger, Mixed-mode oscillations with multiple time-scales, SIAM Review 54 (2012) 211-288.

[6] A.Z. Ivanović-Šašić, V.M. Marković, S.R. Anić, Lj.Z. Kolar-Anić, Ž.D. Čupić, Structures of chaos in open reaction systems, Phys. Chem. Chem. Phys. 13 (2011) 20162-20171.

[7] Lj. Kolar-Anić, Ž. Čupić, G. Schmitz, S. Anić, Improvement of the stoichiometric network analysis for determination of instability conditions of complex nonlinear reaction systems, Chem. Eng. Sci. 65 (2010) 3718-3728.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-7. ID-25.**

# MODERN CONCEPTS OF ACTIVELY CONTROLLED SMART STRUCTURES – AN OVERALL DESIGN APPROACH

**Tamara Nestorović**

Institute of Computational Engineering, Mechanics of Adaptive Systems
Ruhr-Universität Bochum, Universitätsstr. 150, D-44801 Bochum, Germany
e-mail: tamara.nestorovic@rub.de

**ABSTRACT.** Mechanical lightweight structures often tend to unwanted vibrations due to disturbances. Passive methods for increasing the structural damping are often inadequate for the vibration suppression, since they include additional mass in the form of damping materials, additional stiffening elements or mass damper. This paper presents a modern concept for active control of smart structures using piezoelectric materials. The approach is especially well suited for light weight structures and it is presented through several subsequent steps: modeling (model identification and numerical modeling), optimization, controller design, simulation and experimental verification/testing.

## MODELLING OF SMART STRUCTURES

In the overall approach to design of smart structures modeling can be performed either numerically, using the finite element analysis (FEA) and modal reduction, or experimentally through model identification. Both methods result in models convenient for the controller design. The FEA based procedure results in a reduced-order state-space model in the form:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega} & -\mathbf{\Delta} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\overline{\mathbf{B}} \end{bmatrix} \mathbf{u}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{\Phi}^{\mathrm{T}}\overline{\mathbf{E}} \end{bmatrix} \mathbf{f}(t) \tag{1}$$

with the state vector $\mathbf{x}(t) = [\mathbf{z} \quad \dot{\mathbf{z}}]^{\mathrm{T}}$ formed from modally reduced states $\mathbf{z}$, $\mathbf{\Phi}$ being the modal matrix, $\mathbf{\Delta}$ - the damping matrix, $\mathbf{\Omega}$ - diagonal spectral matrix of eigenfrequencies, and $\overline{\mathbf{B}}$ and $\overline{\mathbf{E}}$ represent the coupling matrices related to the system control input and disturbances, respectively.

The model identification, which results in a state space model as well, is based on the subspace based parameter identification. It determines the state space model of a smart structure form the input-to-output relation:

$$\mathbf{Y}[k] = \mathbf{\Gamma}_{\alpha}\mathbf{x}[k] + \mathbf{\Phi}_{\alpha}\mathbf{U}[k] \tag{2}$$

by extracting the model matrices as sub-matrices through the subspace-based algorithm.

## STRUCTURE OPTIMIZATION

Optimization procedures play a crucial role in the overall development of smart structures. Here they will be explained with the purpose of optimal placement of actuators and sensors. In order to achieve maximal efficiency of the structures, with lowest possible production costs, it is necessary to evaluate the structural behavior in early development phases, even before building prototypes.

For the optimization of the actuator/sensor placement, the criteria based on the system controllability and observability are used, as well as criteria based on the norms derived from reduced models. For complex structures the genetic algorithm is applied.

## CONTROLLER DESIGN AND IMPLEMENTATIONS

Applying different control strategies (optimal, adaptive), active control of piezoelectric smart structures can be successfully implemented for the vibration suppression or in the active structural acoustic control. Several examples will be shown, demonstrating significant reduction of vibration amplitudes in the presence of dangerous disturbances, which would lead to resonant states without control.

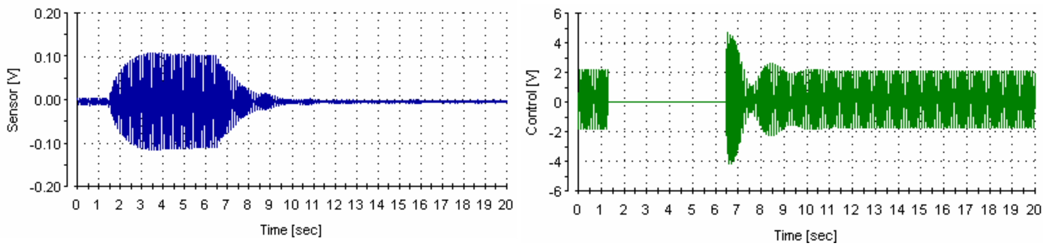



**Fig.1** Vibration reduction of a magnetic resonance image tomograph
(sensor and actuator signal – voltages on piezoelectric patches)

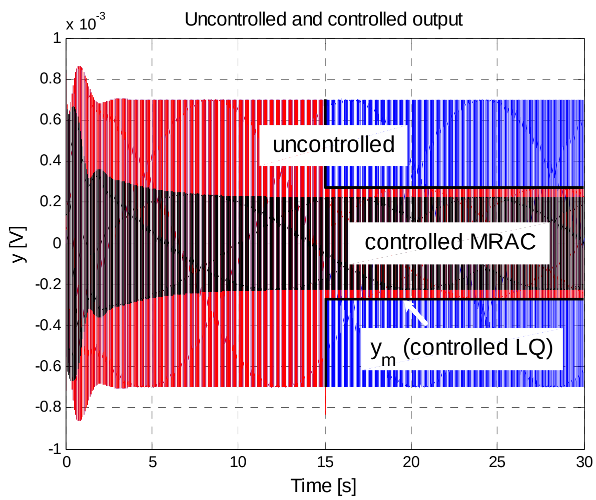



**Fig.2** Active noise reduction of an acoustic box applying optimal LQ
and model reference adaptive control (MRAC)

## References

[1] Nestorović Trajkov T., Köppe H., Gabbert U.: Vibration control of a funnel-shaped shell structure with distributed piezoelectric actuators and sensors, Smart Materials and Structures, **15** (2006), 1119-1132

[2] Gabbert U., Nestorović T., Wuchatsch J.: Methods and possibilities of a virtual design for actively controlled smart systems, Computers and Structures **86** (2008), 240-250

[3] Nestorović T., Marinković D., Chandrashekar G., Marinković Z., Trajkov M.: Implementation of a user defined piezoelectric shell element for analysis of active structures, Finite Elements in Analysis and Design, **52** (2012), 11-22

[4] Nestorović T., Durrani N., Trajkov M.: Experimental model identification and vibration control of a smart cantilever beam using piezoelectric actuators and sensors, Journal of Electroceramics, doi: 10.1007/s10832-012-9736-1, printed online May17th, 2012

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**PL-8. ID-50.**

# WAVE PROPAGATION IN FIBRE REINFORCED COMPOSITE LAMINATES

**D. Milosavljević[1], G. Bogdanović[2]**

[1] Faculty of Engineering
The University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia
e-mail: dmilos@kg.ac.rs
[2] Faculty of Engineering
The University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia
e-mail: gocab@kg.ac.rs

**ABSTRACT.** Composite materials consist of at least two constituents one of which is matrix and another fibre usually lay up through layers mutually bonded to make multilayered composite in the forms of laminates. Fibres carry loads giving strength and matrix bonds fibres together play important role in load transfer to fibres and forms outer shape of composite.

Prediction of dynamic behavior of laminated composites is rapidly assuming considerable importance to industry. Thus, here we deal with such composites, and analysis relates to fibre reinforced materials in which continuous strong and stiff fibres, such as carbon, boron etc., are embedded in a relatively soft matrix, such as resin.

If material is reinforced with one family of fibres it has one privileged direction making material locally transversally isotropic in relation to that direction. Fibre direction may be defined as unit vector field $\boldsymbol{a}$ which may vary from point to point. Trajectories of unit vectors $\boldsymbol{a}$ are defined as fibres in relation to which material is locally transversally isotropic. Since fibre direction depends on position, here we consider coordinate free formulation of constitutive equations [1].

The non-linear dynamic theory of finite elasticity is quite difficult and still today relatively few analytical solutions have been obtained for the full governing equations. These are based on early work of Hadamard and there have been examined cases simple enough to involve only a very limited material response. In the search of more complex motions only partial results seem to be possible [2].

In recent times, researchers interested in continuum mechanics have usually restricted their attention to special classes of response functions such as neo-Hookean or Moony-Rivlin materials. On the other hand, researchers interested in acoustics have concentrated their interest towards the theory of small, but finite amplitude, waves. Possibility to obtain exact solutions of nonlinear dynamic elasticity is important in many fields of applications. These solutions often give opportunity to investigate more complex theories of material behavior where dissipative and dispersive phenomena are taken into account. Here we are going to provide partial survey of some results and methods of approximations made to search governing equations.

Here it is going to be made sort of parallel between nonlinear and linear elasticity, and to derive in a rigorous and general way the nonlinear equations, which describe fibre reinforced composites and

laminates. Constitutive relations of finite elasticity are given for models of materials reinforced by one or two families of fibres. Linearization leads to constitutive relations same as those developed by introduction of strain energy function, which make relatively easy search through wave phenomena.

For given deformation strain energy function depends on both strain $\boldsymbol{\varepsilon}$ and fibre direction $\boldsymbol{a}$. Here is given list of matrix products whose traces make proper orthogonal group basis leading to set of invariants which may be used to form general quadratic form of strain energy function. This may be used to form stress strain relations leading to elasticity tensor for material reinforced by one or two families of fibres.

Dynamic behavior of anisotropic media may be seen the best through its behavior during bulk wave propagation. Bulk waves exist in infinite homogeneous bodies and propagate unbounded without disturbances caused by either boundaries or inter-layers. Such waves may be decomposed into finite plane waves propagating along arbitrary direction $\boldsymbol{n}$ in solid.

Properties of these waves are determined by dependence between propagation direction and constitutive properties of media. Three types of such waves may be distinguished in connection to three displacement vectors, which determine acoustic polarization. Three polarization vectors are mutually orthogonal, but in most cases they are neither perpendicular nor parallel to propagation direction.

The most of dynamical systems are naturally nonlinear and, since it is not easy to find closed solutions of such systems, here we are going to write Reimann-Christoffel equation, leading to three non-homogeneous linear equations which determine displacement amplitudes. This equation represents propagation condition of bulk waves as set of three homogeneous linear equations. Proper values of Reimann-Christoffel equation give phase speed of propagation of plane waves, and proper vectors represent polarization vector. This equation is the most important equation of entire theory of elastic wave propagation in crystals. Since acoustic tensor is symmetric tensor of second order, proper values are real and proper vectors are mutually orthogonal. Reimann-Christoffel equation may be solved analytically only for the simplest cases of material symmetry [3].

For successful display of three dimensional wave surfaces numerical analysis, which contain all propagation directions, phase velocities and polarization vectors, has been performed. The most appropriate approach in this consideration is fibre reinforced material for which proper axes coincide with global coordinate system. That is always used when crystallographic axes are known in advance. Materials used in present analysis are fibre reinforced with one or two families of continuous fibres. Since fibres are much stronger than matrix anisotropic properties are very strong.

After examination of bulk waves we are going to restrict our attention to stress free infinite plate, developing dispersion relations. We are going to use developed dispersion relations to examine laminate structures, and then to consider dynamic behavior of such structures.

**Keywords**: Waves, Composites, Strongly Anisotropic, Slowness surfaces, Acoustic tensor,

**References**

[1]. Spencer, A. J, M.,Constitutive theory for strongly anisotropic solids, in *Continuum theory of the mechanics of fibre reinforced materials*, ed. by A.J.M. Spencer, Springer-Verlag Wien-New York, 1984, pp. 1-33.

[2]. Ogden, R. W., *Non-Linear Elastic Deformations*, John Wiley & Sons, New York, 1988.

[3]. Nayfeh, A.H., *Wave Propagation in Layered anisotropic media*, Elsevir, (1995).

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL- 9. ID-28.**

## SOME PROBLEMS OF DYNAMICS OF A HEAVY RIGID BODY CARRYING A MATERIAL POINT

**A. Markeev**

Ishlinski Institute for Problems in Mechanics of
the Russian Academy of Sciences,
Prospect Vernadskogo, 101, block 1, 119526, Moscow, Russia
e-mail: markeev@ipmnet.ru

A system consisting of an outer rigid body (a shell) and an inner body (a material point) which moves according to a given law along a curve rigidly attached to the shell is considered. The system moves in a uniform gravity field over a fixed absolutely smooth horizontal plane. During its motion, the shell may collide with the plane. The coefficient of restitution at impact is supposed to be arbitrary.

If the shell executes free flight over the plane, the system mass center moves in a parabola or along a vertical straight line.

The paper presents differential equations governing the motion of the shell relative to its center of mass; they describe both stages of free flight over the plane and instances of collisions of the shell with the plane [1].

The shell is found can execute translational motion, if the material point moves according to the special law. The second Lyapunov method was used to investigate stability of this shell motion.

A general solution of the equations governing the rotational motion of the shell was obtained for the case where the shell is dynamically symmetric and the point moves along its symmetry axis according to an arbitrary law.

Two special cases of the system motion are also considered. In the first case the relative motion of the material point is assumed to be fast and the point mass small in comparison with the shell mass, and in the second one the point executes fast motion in a small neighborhood of a given point of the shell.

Using the classical perturbation theory methods approximate systems of differential equations governing the shell rotation are obtained for these cases. Difference between solutions of these systems and the corresponding exact systems of equations is estimated. First integrals of the approximate systems are found, their integrability is proved, and some special solutions are considered.

The paper also presents the results obtained in the problem of existence and stability of periodic motions of a dynamically symmetric shell colliding with a plane [2].

**Keywords**: rigid body, collisions, stability, rotational motion, fast motions, perturbation theory

### References

[1] Markeev A.P., On the Dynamics of a Rigid Body Carrying a Material Point, *Nonlinear Dynamics*, **8** (2012), No 2, pp. 219 – 229. (in Russian)

[2] Markeev A.P., On Periodic Motion of a Rigid Body Carrying a Material Point in the Presence of Impacts with a Horizontal Plane, *Nonlinear Dynamics*, **8** (2012), No 1, pp. 71 – 81. (in Russian)

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-10. ID-69**.

# ONE APPLICATION OF REGULARLY VARYING FUNCTIONS TO FRIEDMANN EQUATIONS

**Žarko Mijajlović**
University of Belgrade – Faculty of Mathematics, Belgrade, Serbia
e-mail: zarkom@matf.bg.ac.rs

**ABSTRACT.** In this paper we analyze the asymptotic solutions of the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

related to the Friedman cosmological equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

which describes the expansion scale factor $a(t)$ of the universe. Here, $p= p(t)$ is the energy pressure in the universe, $\rho= \rho(t)$ is the density of matter in the universe, $k$ is the space curvature, $G$ is the gravitational constant and $c$ is the speed of light. The variable $t$ represents the cosmic time. We are particularly interested in the solutions satisfying the generalized power law $a(t) = t^{\alpha}L(t)$, where $L(t)$ is a regularly varying function in the sense of J. Karamata, see [3]. For this reason we introduced a new parameter $\mu(t) = q(t)(H(t)t)^2$ where $q(t)$ is the deceleration parameter and $H(t)$ is the Hubble parameter. We prove that the acceleration equation has an asymptotical solutions that satisfy the generalized power law if and only if the integral limit

$$\gamma = \lim_{x\to\infty} x\int_x^{\infty}\frac{\mu(t)}{t^2}dt$$

exists and $\gamma < 1/4$. Thus, the values of the constant $\gamma$ determine the asymptotical behavior at the infinity of the solutions of the acceleration equation, i.e. of the expansion scale factor $a(t)$ of the Universe. Our approach presented in the paper covers all results on cosmological parameters for Standard model of the universe, as presented in [1] or in [2]. Our analysis is based on the theory of regularly varying solutions of the linear second order differential equation developed by V. Marić, see [4].

**Keywords**: Friedmann equations, regular variation, cosmology, power law.

**References**

[1] Liddle, A.R., Lyth, L.H., *Cosmological Inflation and Large-Scale Structure*, Cambridge Univ. Press, Cambridge, 2000.
[2] Peacock, J.A., *Cosmological Physics*, Cambridge Univ. Press, Cambridge, 1999.
[3] Bingham, N.H., Goldie, C.M., Teugels, J.L., *Regular variation*, 491, Cambridge Univ. Press, Cambridge (1987)
[4] Marić, V., *Regular Variation and Differential Equations*, 127, Springer, Berlin (2000).

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-11. ID-84.**

# FRACTIONAL DERIVATIVE VISCOELASTIC MODELS IN THE WAVE THEORY OF IMPACT

**Yury A. Rossikhin and Marina V. Shitikova**
Research Center on Dynamics of Solids and Structures
Voronezh State University of Architecture and Civil Engineering
20-letija Oktjabrja Street 84, Voronezh 394006, Russia
Tel./Fax: +7-4732-773992, e-mail: yar@vgasu.vrn.ru, shitikova@vmail.ru

The recently published state-of-the-art article [1] devoted to the analysis of new trends and recent results in the field of fractional calculus application to dynamic problems of structural mechanics has shown that during the last decade fractional calculus entered the mainstream of engineering analysis and has been widely applied to structural dynamics problems both in discrete and continuous equations. Among many engineering problems considered in [1], the problems of dynamic contact interaction play the important role.

In the present paper, different approaches are reviewed for solving the problems dealing with the shock interaction of thin viscoelastic bodies, such as beams, plates and shell, with bodies of finite dimensions [2]. It is emphasized that fractional derivative viscoelastic models of the shock interaction possess some advantages, since they allow one to obtain the solution in the analytical form. Two approaches are discussed for studying the impact response of fractionally damped systems.

The first one is based on the assumption that viscoelastic properties of the target manifest themselves only in the contact domain, while the other part of the target remains elastic one and its behavior is described by the equations of motion which take rotary inertia and shear deformations into account. It is assumed that transient waves generate in the target at the moment of impact, the influence of which on the contact domain is considered using the theory of discontinuities. To determine the desired values behind the transverse shear wave front, one-term ray expansions are used, as well as the equations of motion of the falling mass and the contact region. This approach results in defining the contact force and the local penetration of target by an impactor from the set of linear fractional differential equations.

The second approach is the immediate generalization of the Timoshenko approach, wherein the internal viscoelastic properties of the whole target and Hertz's contact law are taken into account using Volterra correspondence principle. This approach results in the nonlinear functional equation for determining the contact force or the impactor's relative displacement.

The examples of implementing these two approaches are presented, in so doing several procedures are suggested for the analysis of the impact response of fractionally damped systems depending on the different combinations of magnitudes of its mechanical and viscous features.

## References

1. Yu. A. Rossikhin, M.V. Shitikova, Application of fractional calculus for dynamic problems of solid mechanics: Novel trends and recent results. *Applied Mechanics Reviews* 63 (1) (2010) 010801-1-52.
2. Yu. A. Rossikhin, M.V. Shitikova, Transient response of thin bodies subjected to impact: Wave approach. *The Shock and Vibration Digest* 39 (4) (2007) 273-309.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-12. IF-14.**

# HOMOGRAPHICAL SOLUTIONS OF HAMILTONIAN SYSTEMS AND COMPUTER ALGEBRA

**E. Grebenikov[1] , N. Zemtsova[2]**
Institution of Russian Academy of Sciences Dorodnicyn Computing Centre of RAS, Vavilov st. 40, 119333 Moscow, Russia
[1] e-mail: e-greben@yandex.ru
[2] e-mail: zemni@yandex.ru

**ABSTRACT.** It is known that Hamilton differential equations of cosmic dynamics under certain conditions, imposed on geometrical and dynamic parameters of model, have the homographic solutions in Wintner sense [1,2,3,4,5].
The constructive theory of such solutions (that is ¯finding of the exact conditions guaranteeing their existence) have been implemented by us and our colleagues on the basis of application of system of computer algebra Mathematica[5].
In particular we have proved the existence of new classes homographic solutions for models with various number n gravitating bodies (n=4,5,6,7,8,9,10, ets.). We have developed a method of search of equilibrium points for such models and on the basis of the KAM-theory [6,7] have received sufficient conditions of their stability in Lyapunov's sense.

**Keywords**: dynamic systems, differential equations, stationary solutions, stability, computer algebra.

**References**

[3] Wintner A., The analytical foundations of celestial mechanics. MIR, Moscow, (1967), (in Russian).
[4] Grebenikov, E.A., Kozak-Skovorodkin, D., Jakubiak M., The Methods of Computer Algebra in Many-Body Problem. RUDN, Moscow, (2002), (in Russian).
[5] Zemtsova N.I., Stability of the stationary solutions of the differential equations of restricted Newtonian problem with incomplete symmetry, Nonlinear Dynamics and Systems Theory, **3**(1) (2003),pp. 121-130.
[6] Gadomski L., Grebenikov E.A., Prokopenya A.N., Studying the stability of equilibrium solutions in the planar circular restricted four-body problem, *Nonlinear oscillations*, **10**(1), (2007), 66-82
[7] Wolfram S., The Mathematica book. Wolfram Media, Cambridge University Press, (1999).
[8] Arnold V.I., Small denominators and problems of stability of motion in classical and celestial mechanics, Uspekhi Math.Nauk, 18(6), (1963), pp. 91-192, (in Russian).
[9] Markeev A.P., Libration points in celestial mechanics and cosmodynamics, Nauka, Moscow, (1978), (in Russian)

Naučno društvo Srbije — Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-13. ID-81.**

# PHENOMENOLOGICAL MAPPING AND MATHEMATICAL ANALOGY IN NONLINEAR DYNAMICAL SYSTEMS

**K. (Stevanović) Hedrih** [1]

[1] Mathematical Institute SANU, Department of Mechanics,
and Faculty of Mechanical Engineering University of Niš, Serbia,
Priv. address: 18000- Niš, Serbiam ul. Vojvode Tankosića 3/22,
e-mail: khedrih@eunet.rs

**ABSTRACT.** On the basis of this phenomenological mapping and mathematical analogy, we present that analysis for one type of the system nonlinear dynamics is possible to applied for qualitative analysis of nonlinear phenomena appeared in dynamics of other disparate model or nature system nonlinear dynamics. The linearizations as well as nonlinear approximations of nonlinear differential equations around stationary points correspond to equilibrium positions or relative equilibrium positions of mechanical system dynamics with trigger of coupled singularities are obtained. First approximations of a nonlinear differential equation obtained by different methods and around different known analytical solutions were compared and corresponding conclusions are presented. As special examples are used nonlinear differential equations describing nonlinear dynamics of the mechanical system with coupled rotations in damping field

**Keywords**: Nonlinear dynamics, nonlinear phenomenon, trigger of coupled singularities, phenomenological mapping, approximation.

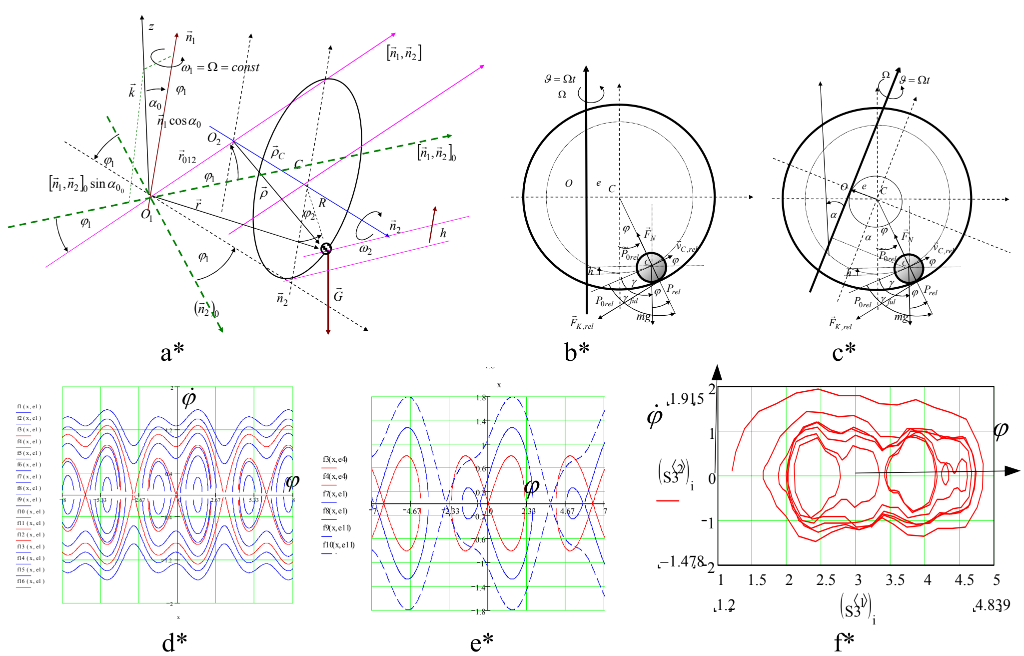



**Figure 1.** Three characteristic models (a*,b*,c*) of nonlinear dynamical systems abstractions of real nonlinear dynamical systems and three visualiyation of nonlinear dynamics in phase plane(d*,e*, f*).

**Some characteristic nonlinear differential equations and approximations around stationary points:** Nonlinear differential equations describing nonlinear dynamics of a heavy mass particle or a heavy disk rolling along rotating circle about axis inclined to the vertical direction with constant angular velocity are in the forms (see Figure 1 . a*, b* and c*, and also References [1-8]):

$$\text{a*}\ \ddot{\varphi}_2 + 2\delta\dot{\varphi} + \Omega^2(\lambda\cos\beta_0 - \cos\varphi_2)\sin\varphi_2 - \Omega^2\frac{r_{012}}{R}\cos\varphi_2 = \Omega^2\lambda\cos\varphi_2\sin\beta_0\sin\Omega t\ ,\ \lambda = \frac{g}{R\Omega^2} \quad (1)$$

$$\text{b*}\ \ddot{\varphi} + \frac{\Omega^2}{\kappa}\langle\lambda - \cos\varphi\rangle\sin\varphi - \varepsilon\frac{\Omega^2}{\kappa}\cos\varphi = 0\ ,\ \text{for}\ \varepsilon = 0\ ,\ \ddot{\varphi} + \frac{\Omega^2}{\kappa}\langle\lambda - \cos\varphi\rangle\sin\varphi = 0 \quad (2)$$

$$\text{c*}\ \ddot{\varphi} + \frac{\Omega^2}{\kappa}\langle\lambda\cos^2\alpha - \cos\varphi\rangle\sin\varphi - \frac{\Omega^2}{\kappa}\left(\varepsilon + \frac{\lambda}{2}\sin 2\alpha\right)\cos\varphi = \frac{\Omega^2}{2\kappa}\lambda\cos(\alpha + \varphi)\sin\alpha\cos\Omega t \quad (3)$$

$$\varepsilon = \frac{e}{(R-r)}\ ,\ \kappa = \left(\frac{\mathbf{i}^2_{C,rel}}{r^2} + 1\right) = \frac{\mathbf{i}^2_{\mathbf{P},rel}}{r^2}\ ,\ \lambda = \frac{g}{(R-r)\Omega^2}\ ,\ \frac{\mathbf{J}_{\mathbf{P},rel}}{M} = \mathbf{i}^2_{\mathbf{P},rel} = \mathbf{i}^2_{C,rel} + r^2\ ,\ \ell_{red} = \left(\frac{\mathbf{i}^2_{C,rel}}{r^2} + 1\right)(R-r)$$

Previous differential equations, also, represent the analogous differential equations of the self rotation heavy rigid body, skew and eccentrically positioned to the axis of self rotation, with coupled rotations about two no intersecting orthogonal axes.

Taking into account possible approximation of nonlinear differential equations (1)-(2)-(3) around stationary points (see trigger of coupled singularities and homoclinic orbit in the form of number "eight" in phase planes in Fig. 1. c*, e( and f*), we obtain series of the approximations and for this these examples, we separate the three following types of linearized approximations as results of linear mapping around stationary states (see Refs. [7]):

$$\ddot{\varphi} + \Omega^2(\lambda - 1)\varphi \approx \Omega^2\lambda ctg\alpha\cos\Omega t\ ;\ \ddot{\varphi} + 2\delta\dot{\varphi} + \omega^2_{0,lin}\varphi = \kappa_2\varphi^2 + \kappa_3\varphi^3\ \text{and}$$

$$\ddot{\varphi}_2 + 2\delta\dot{\varphi} + \varphi_2[\lambda + \gamma\sin\varphi_{s2}\cos\Omega t] + f = h_s\cos\Omega t$$

It is possible to obtain corresponding nonlinear approximations of of nonlinear differential equations (1)-(2)-(3) around stationary points. Then by using known analytical solutions of linearized nonlinear differential equations around stationary point, as the starting solutions, by application Krilov-Bogolyubov-Mitropolyski asymptotic methods and method of variation constants and averaging, different expressions, it is possible to obtain the first approximations of nonlinear differential equation solutions. For nonlinear differential equation: $\ddot{x}_1(t) + 2\delta_1\dot{x}_1(t) + \omega_1^2 x_1(t) = \mp\tilde{\omega}^2_{N1}x_1^3(t)$, (4)

By use two methods starting by known analytical solutions $x_1(t) = R_1(t)e^{-\delta_1 t}\cos(p_1 t + \phi(t))$, $p_1 = \sqrt{\omega_1^2 - \delta_1^2}$ and $x(t) = a(t)\cos[\omega_1 t + \phi(t)]$, and we obtained the first approximations of the solution in the different forms (see Refs. [9]):

$$x_1(t) = R_{01}e^{-\delta_1 t}\cos\left[p_1 t \mp \frac{3}{16\delta_1 p_1}\omega^2_{N1}a_o^2\left(e^{-2\delta_1 t} - 1\right) + \Phi_o\right].\ \text{for}\ \delta_1 \neq 0\ ,\ \varepsilon \neq 0\ ,\ \omega_1^2 > \delta_1^2\ ,\ p_1 = \sqrt{\omega_1^2 - \delta_1^2} \quad (5)$$

$$x_1(t) = a_o e^{-\delta_1 t}\cos\left[\omega_1 t \mp \frac{3}{16\delta_1\omega_1}\omega^2_{N1}a_o^2\left(e^{-2\delta_1 t} - 1\right) + \Phi_o\right].\ \text{for}\ \delta_1 \neq 0\ ,\ \varepsilon \neq 0\ ,\ \omega_1^2 > \delta_1^2 \quad (6)$$

For the case that damping coefficient tends to zero, from both first approximations (5) and (6), we obtain same analytical approximation of the solution for conservative nonlinear system dynamics. For the case that coefficient of the cubic nonlinearity tends to zero, from first approximation (5), we obtain known analytical solution of the linear no conservative system dynamics, but the second obtained approximation (6) give not correct solution. Then we can conclude that, starting different known analytical solutions, for obtaining first approximations are acceptable, but limited by corresponding conditions.

**Acknowledgement.** Parts of this research were supported by the Ministry of Sciences of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 .

## References

[1] Andronov, A.A., Vitt, A.A., Haykin, S.E., (1981), *Teoriya kolebaniy,* Nauka, Moskva., pp. 568.

[2] Hedrih (Stevanović) K., (2005), *Nonlinear Dynamics of a Heavy Material Particle Along Circle which Rotates and Optimal Control*, Chaotic Dynamics and Control of Systems and Processes in Mechanics (Eds: G. Rega, and F. Vestroni), p. 37-45. IUTAM Book, in Series *Solid Mechanics and Its Applications, Editerd by G.M.L. Gladwell, Springer*. 2005, XXVI, 504 p., ISBN: 1-4020-3267-6.

[3] Hedrih (Stevanović) K., (2004), *A Trigger of Coupled Singularities*, MECCANICA, Vol.39, No. 3, 2004., pp. 295-314. , DOI: 10.1023/B:MECC.0000022994.81090.5f,

[4] Hedrih (Stevanović K., (2008), Dynamics of coupled systems, Nonlinear Analysis: Hybrid Systems, Volume 2, Issue 2, June 2008, Pages 310-334.

[5] Hedrih (Stevanović) K R. (2012), Energy and Nonlinear Dynamics of Hybrid Systems, Book Chapter, in *Dynamical Systems and Methods, Edited by Albert Luo, Tenreiro Machado and D Baleanu, , 2012, Part 1, Pages 29-83,* 2012, DOI: 10.1007/978-1-4614-0454-5_2 , ISBN 978-1-4614-0453-8. e-ISBN 978-1-4614-0454-5,

[6] Mitropolyskiy, Yu.A, (1964), Problemi asimptoti-cheskoy teorii nestashionarnih kolebaniy, Nauka Moskva.

[7] Petrović, M., Fenomenološko preslikavanje (Phenomenological mapp), SKA, Beograd, 1933. str. 33.

[8] Rašković D., Teorija oscilacija (Theory of Oscillations), Naučna knjiga, 1952.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**PL-14. ID-85.**

# THE NEW COMPLETE BIFURCATION THEORY OF NONLINEAR DYNAMICAL SYSTEMS AND CHAOS. RARE ATTRACTORS AND APPLICATION TO GLOBAL ANALYSIS

**Mikhail V. Zakrzhevsky**

*Prof.,Dr.habil.sc.ing, Institute of Mechanics,*
*Riga Technical University, email: mzakr@latnet.lv*

A new *complete bifurcation theory of nonlinear dynamical systems (CBT NDS)* and its application, intended for direct global bifurcation analysis of dynamical periodic systems is presented. The bifurcation theory is established for essential nonlinear dynamical periodic systems, described by models of ODE equations or by map-based models of discrete-time equations. Our approach is based on ideas of Poincaré, Andronov and other scientists' results concerning global dynamics, structural stability and bifurcations and chaotic responses of dynamical nonlinear systems and their topological properties.

The main idea of the new CBT is a fact that the NDS in a given parameters and state spaces has finite number (usually not so many) of independent bifurcation groups S(p) with their own complex topology and bifurcations, chaotic behavior, and, in many cases, with rare regular and chaotic attractors (RA). For each point of parameter space it is possible to find all essential fixed points of the periodic orbits (stable and unstable). This periodic skeleton allows to mark out the bifurcation groups and to start global analysis in state and parameter spaces.

The main concepts of the new CBT are: complete bifurcation group (BG); unstable periodic infinitium subgroups (UPI), responsible for chaos; complex protuberances; and periodic skeletons for a system with parameter p. For illustration of the advantages of the new bifurcation theory we use in this presentation several typical nonlinear models: Duffing driven double-well oscillator, a pendulum driven and parametrical excited oscillator (see Figs 1, 2). Besides we consider using the method of complete bifurcation groups for several different models of driven 2DOF systems: a flat system with one mass suspended by nonlinear springs in a plane, two masses chain system with non-unique equilibrium positions, and a simple rotor system with asymmetric suspension.

The last 2DOF systems were investigated for comparing two approaches: traditional analytical approximate methods (harmonic balance methods, average and many scale methods, the nonlinear normal mode's method) and the method of complete bifurcation groups and approaches of the bifurcation theory. In all considered examples we have found that the complete bifurcation theory's methods allow finding important unknown regular or chaotic attractors and/or new bifurcation groups with rare attractors RA. Additional illustration of the bifurcation theory, it is possible to find in the author's and hiss colleague's papers where there is rather complete bibliography on the bifurcation theory and rare attractors (see references).

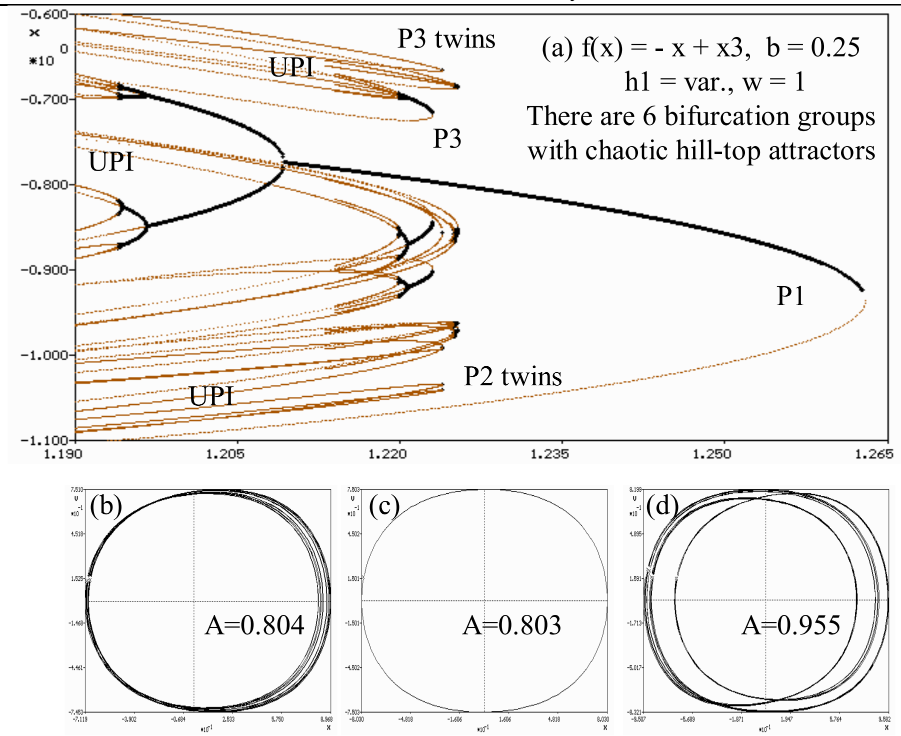



Fig. 1 Driven damped double-well Duffing oscillator. (a) Bifurcation diagrams (black –stable, reddish – unstable) with hilltop (HT) rare periodic and chaotic attractors (ChA) galore; amplitude of excitation h1 = var. (b) asymm. HT ChA, h1 = 1.2193; (c) P1 HT; h = 1.2195; (d) HT ChA-3 rare attractor, h1 = 1.2195.

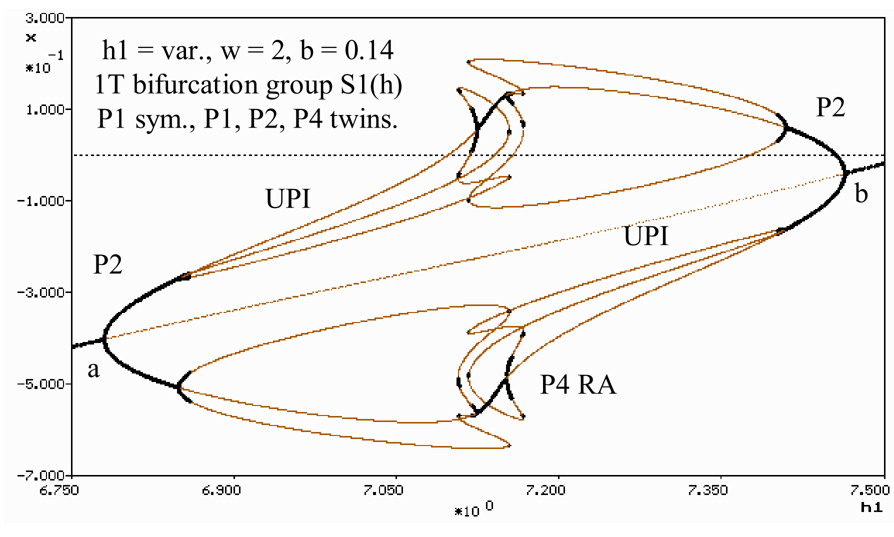



Fig. 2 Driven damped pendulum oscillator. Fragment of 1T bifurcation group (black –stable, reddish – unstable) with complex protuberance (a, b), P1 – P4 stable and unstable orbits, UPI and rare periodic P4 attractors (dark dots in the middle of the figure). Amplitude of excitation h1 = var.

**References:**

[1] M. Zakrzhevsky, New concepts of nonlinear dynamics: complete bifurcation groups…, JVE 10/4, 2008, 421-441.

[2] M. Zakrzhevsky et al, Nonlinear dynamics and chaos. Bifurcation groups and rare attractors, Riga, RTU, 181p, 2012 (in Russian).

**Naučno društvo Srbije** | **Serbian Scientific Society**
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**SL-1. ID-70.**

# DIGITIZATION OF SCIENTIFIC AND CULTURAL HERITAGE IN SERBIA

**Žarko Mijajlović**

University of Belgrade – Faculty of Mathematics, Belgrade, Serbia
e-mail: zarkom@matf.bg.ac.rs

**ABSTRACT.** The aim of this paper is to present the efforts in the area of digitization and digital preservation of scientific and cultural heritage of a group of Serbian scientists. They are from the Faculty of Mathematics of the University of Belgrade, the Mathematical Institute and since recently the Faculty of Natural Sciences of the University in Priština, now situated in Kosovska Mitrovica. Digitized cultural heritage is one of the most important contemporary components of the study of our history and culture. On the other hand, digitized scientific works are the crucial resource and tool for scientific works and fast exchange of scientific information. Also, we understand that every scientific work becomes after some time the part of history and the part of cultural heritage as well. Due to the fast development of computer technologies and Internet, there are many advantages for using digitized works. Besides easy and fast access and exchange of information, the protection of scientific and cultural values is also one of the most important elements.

Some digitization projects started in Serbia already in the middle of the nineties of the previous century. The aim of these early projects was the digitization of national cultural heritage and retro-digitization of mathematical books of old Serbian scientists. The principal participating institutions were Faculty of Mathematics, Belgrade, and Mathematical Institute of the Serbian Academy of Science and Arts, but the other institutions were involved too (Archaeological Institute, Institute of musicology, Institute for monument protection of Serbia, National library, National museum, etc.). The projects were financed by the Ministry of Science of Serbia and Faculty of mathematics. The project *Computer archiving and multimedia presentation of cultural values and national heritage*, see [1], was the most important and comprehensive project in our country in the area of digitization until now. The project consisted of two parts:

- Infrastructure, standards and methodology of design and the architecture of data.
- Design and building of archive databases and program implementation.

In this presentation we describe ***Virtual Library***, one of these sub-projects. It concerns digitization of mathematics-related books, theses, manuscripts, and mathematical journals somehow related to Serbia or our region of South Eastern Europe. The project's goal is to form digital archives, databases and presentations of digitized scientific editions in mathematical sciences (mathematics, mechanics, astronomy) and offer easier on-line access both to old and recent mathematical works. The


Новаꙗ
Сѵрбскаꙗ
Арїѳметика
или
Простое наставленїе къ Хесапу
от
Васіліа Дамяновича.
Въ Мѵткахъ 1767.

**First Serbian mathematical book: Aritmetika, 1767 Vasilije Damjanović**

**Virtual Library.** The overall objective of the *Virtual Library of the Faculty of Mathematics*, University of Belgrade, http://elibrary.matf.bg.ac.rs, is to implement an as much as possible complete collection of retro-digitized books and other digital documents from the past, see [2]. The main part of this project relates to an electronic archive which contains first of all old manuscripts electronic form and

their presentation to the general public.

The project was initially inclined towards mathematics, but since 2009 the books from other areas are also deposited in the Library. The preference is given to the Serbian authors and works that are related to the scientific and cultural region of Southeast Europe. Some of the books in the Library are rare and it is known that only a few copies of them are left in the printed form. Practically they are inaccessible to the general public. We felt it was important to preserve their existence in some way. Not only as a cultural and scientific heritage important for Serbia, but also as part of the World Heritage. We decided to contribute to the preservation of these books and present them to the general public in electronic, digitized form.

The Library has strong support from the Faculty of Mathematics, University of Belgrade, Mathematical Institute of Serbian Academy of Sciences and Arts (SASA), the National Center for Digitization and the Ministry of Science of Serbia.

Virtual Library of the Faculty of Mathematics is the largest Internet oriented database in Serbia of digitized texts with free access. At the time of this writing, the library contains almost 2,000 books. In the Library there are several important collections. For example, the Library contains an important collection consisting of 400 doctoral dissertations in mathematical sciences (most of them are defended at the Faculty of Mathematics). Another important collection consists of rare books from the 18th and 19th century. There are also small collections of digitized books from every republic of former Yugoslavia.

The important part of the Library make the collected works of some leading Serbian scientists from the past: Atanasije Stojković, Bogdan Gavrilović, Milutin Milanković, Đuro Kurepa, Đorđe Stanojević and several others.

Works related to the archive in the Virtual Library are published in the journal NCD Review (SEEDI Communication), issued by the Faculty of Mathematics in Belgrade.

**Keywords**: digitization, old mathematical books, scientific heritage

**References**

[1] Ž. Mijajlović, Z. Ognjanović, *A survey of certain digitization projects in Serbia*. Review of the National Center for Digitization, 4 (Proc. Symp. Digital Preservation of Cultural Heritage, 16–17 Sept. 2003, Borovets, Bulgaria), 52–61, 2004, http://elib.mi.sanu.ac.rs.

[2] Ž. Mijajlović, Z. Ognjanović, A. Pejović, *Digitization of Mathematical Editions in Serbia*, Mathematics in Computer Science, Vol. 3, Issue 3, 251-263, 2010. ISSN 1661-8270.

**Serbian Scientific Society**

**Symposium Nonlinear Dynamics – Milutin Milanković**

**Multidisciplinary and Interdisciplinary Applications**

**(SNDMIA 2012), Belgrade, October 1-5, 2012.**

**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Symposium**

# Nonlinear Dynamics – Milutin Milanković

## Multidisciplinary and Interdisciplinary Applications

## (SNDMIA 2012),

**Belgrade, October 1-5, 2012.**

**(Eigth Serbian Symposium in area of Non-linear Sciences)**

**Editors: Katica R. (Stevanović) HEDRIH**
**Žarko Mijajlović**

# Invited Lectures

**Beograd, October 1-5, 2012.**
**Venue: Mathematical Institute SANU**

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**IL-1. ID-5.**

# GENERALIZED VAN DER POL OSCILLATORS: FROM A LIMIT CYCLE TO THE ENTRAINMENT PHENOMENON

**I. Kovacic**
Department of Mechanics, Faculty of Technical Sciences, University of Novi Sad, 21 000 Novi Sad, Serbia
e-mail: ivanakov@uns.ac.rs

**ABSTRACT.** The classical van der Pol oscillator is one of a key model with a vast number of applications in science and engineering. Its governing equation of motion is given by

$$\ddot{x} + x = \varepsilon\left(1 - x^2\right)\dot{x}, \qquad (1)$$

where $x$ is a generalized coordinate, dots denote differentiation with respect to time $t$ and the coefficient $\varepsilon$ is a positive real number. The nonlinear damping-like force on the right-hand side of eq. (1) dissipates energy for large displacements as the expression in brackets is negative, but it feeds energy for small displacements since this expression is then positive. This behaviour gives rise to self-exciting oscillations. For small values of the coefficient $\varepsilon$, they are characterized by the appearance of a stable limit cycle with the steady-state amplitude $a_s = 2$, which represents its distinctive characteristic. When the classical van der Pol oscillator is harmonically excited, the entrainment phenomenon can occur. It corresponds to the case when this oscillator synchronises with the forcing frequency. Then, the force is said to have entrained the limit cycle oscillations or the limit cycle oscillations are said to have been locked/quenched [1].

This study is concerned with the investigations of the limit cycle and the entrainment phenomenon in the generalized van der Pol oscillators, which is governed by the following equation

$$\ddot{x} + \mathrm{sgn}(x)|x|^{\alpha} = \varepsilon\left(1 - |x|^{\beta}\right)|\dot{x}|^{\gamma}\,\mathrm{sgn}(\dot{x}), \qquad (2)$$

where $\alpha > 0$, $\beta > 0$, $\gamma \geq 0$. Here, the nonlinearity appears both in the damping-like force on the right side, as well as in the restoring force, which is given by the second term on the left-hand side; the sign and absolute value functions are used to assure that these forces have the properties of odd and even functions as in the classical van der Pol oscillator given by eq. (1).

First, the influence of the powers of nonlinearity on the system response in studied for small values of the damping coefficient $\varepsilon$. By applying the averaging method for purely nonlinear systems [2], the amplitude of the limit cycle is determined and its properties analysed with respect to the limit cycle amplitude $a_s$ of the classical van der Pol oscillator.

Then, harmonically excited generalized van der Pol oscillators are investigated from the viewpoint of the occurrence of harmonic entrainment. Locked periodic motion is obtained by adjusting the averaging method. The steady-state amplitude is related to the amplitude of the limit cycle. Effects of the powers of the restoring force and the damping-like force on the occurrence of this phenomenon are determined and those beneficial for potential applications are highlighted.

**Keywords**: van der Pol oscillator, power-form nonlinearity, limit cycle, entrainment

## References

[1] Rand R.H.: *Lecture Notes on Nonlinear Vibrations*, version 52. http://audiophile.tam.cornell.edu/randdocs/nlvibe52.pdf 2005. Accessed 17 March 2012.

[2] Kovacic I., *A* limit cycle and relaxation oscillations in a generalized van der Pol oscillator, *Communication in Non-linear Science and Numerical Simulations*, **16** (2011), pp. 1640–1649.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-2. ID-34.**

# WALL FUNCTION CONCEPT FOR MODELING TURBULENT CHANNEL FLOW

**B. Stankovic[1], M. Sijercic[1], S. Belosevic[1], S. Cantrak[2]**

[1] Institute of Nuclear Sciences "Vinca", Belgrade University,
Laboratory for Thermal Engineering and Energy,
Mihajla Petrovica Alasa 12-14, PO Box 522, 11001 Belgrade
e-mail: branstan@vin.bg.ac.rs
[2] Faculty of Mechanical Engineering, Belgrade University,
Kraljice Marije 16, 11120 Belgrade 35
e-mail: scantrak@mas.bg.ac.rs

**ABSTRACT.** The treatment of wall boundary conditions described by wall functions requires particular attention in turbulence modeling. The single, most common cause of rapid divergence of CFD simulations is the inappropriate specification and implementation of boundary conditions. For the practical problems and computerized engineering calculations regarding both the fluid flow and heat transfer phenomena, the concept of wall functions has the important role. The main objective of the present article is to identify what it is about turbulent flow in the near-wall region that causes problems for turbulence modeling. The near-wall viscosity-affected layer of a turbulent fluid flow poses a number of challenges, from both modeling and numerical viewpoints. The reason is that due to the presence of the solid boundary, the flow behavior and turbulence structure are considerably different from free turbulent flows. In the vicinity of the wall – across the very thin region – there are extremely sharp gradients of mean and turbulent flow variables according to the experimental and DNS data. Over this thin wall-adjacent region turbulence properties change orders of magnitude faster than over the rest of the flow. Also, the turbulent velocity fluctuations are very different in different directions – the turbulence is strongly anisotropic near the wall. Special near-wall treatments – wall functions - are necessary since equations cannot be resolved down to the walls in their original form due to a complex flow structure characterized by rapid changes in the mean and fluctuating velocity components concentrated within a very narrow region in the immediate vicinity of the wall. The concept of wall functions is based on the coarse numerical grid and assumed mean and turbulence properties profiles within this zone. Consequently, the calculation of wall shear stress, average generation and dissipation rates of turbulent kinetic energy and modification of discretized transport equations for wall-parallel momentum and turbulent kinetic energy over near-wall control volumes is carried out. It is of paramount importance that we supply physically realistic, well-posed boundary conditions, otherwise severe difficulties are encountered in obtaining solutions. In addition, the wall function approach is economical, both in computer storage and CPU time, with computations at least an order-of-magnitude faster than with the low-Re approach.

**Keywords**: boundary conditions, wall functions, computational efficiency

**References**

[1] Chieng C C and Launder B E, On the Calculation of Turbulent Heat Transport Downstream from an Abrupt Pipe Expansion, *Numerical Heat Transfer,* **3** (1980), pp. 189-207.

[2] Craft T J, Gant S E, Iacovides H and Launder B E, Development and Application of a New Wall Function for Complex Turbulent Flows, *ECCOMAS CFD CONFERENCE,* Swansea, Wales, UK, 4-7 September 2001

## KONCEPT ZIDNIH FUNKCIJA
## PRI MODELIRANJU TURBULENTNOG STRUJANJA U KANALU

**Apstrakt:** Definisanje graničnih uslova na zidu koji se opisuju zidnim funkcijama predstavlja značajan problem i zahteva posebno razmatranje u sklopu matematičkog modeliranja turbulencije. Najčešći pojedinačni uzrok zbog kojeg program može da ne konvergira ka rešenju ili da konvergira ka pogrešnom rešenju je neodgovarajuća postavka graničnih uslova ili njihova pogrešna ugradnja. Za praktične probleme i kompjutersko-inženjerske proračune, kako kod strujnih tako i kod strujno-termičkih procesa, koncept zidnih funkcija ima važnu ulogu. Glavni cilj ovog rada je da se otkrije šta je to u vezi sa turbulentnim strujanjem u neposrednoj blizini zida što prouzrokuje teškoće, u delu problema koji se odnosi na modeliranje turbulencije. Sloj fluida u kojem je uticaj viskoznosti prisutan postavlja niz pitanja i problema koje je potrebno rešiti i sa aspekta modeliranja i sa numeričkog aspekta. Razlog je u činjenici da su zbog prisustva čvrste granice karakteristike strujanja fluida i turbulentnih struktura znatno drugačije u odnosu na slobodne turbulentne tokove. U samoj blizini zida, u veoma tankom sloju fluida, dolazi do vrlo izraženih promena (velikih gradijenata) osrednjenih i turbulentnih strujnih veličina, prema eksperimentalnim merenjima i DNS podacima. U ovom tankom sloju koji prijanja uz zid, turbulentne veličine se menjaju brže za nekoliko redova veličine u poređenju sa strujanjem van tog regiona. Pored toga, turbulentne fluktuacije se veoma razlikuju u različitim pravcima – turbulentno strujanje je izrazito anizotropno u blizini zida. Poseban tretman ove zone uz zid je neophodan zato što se jednačine koje opisuju strujanje fluida ne mogu rešiti do samog čvrstog zida u svom izvornom obliku – moraju se modifikovati pomoću zidnih funkcija – zbog složene strukture strujanja koju karakterišu nagle promene osrednjenih i turbulentnih veličina na veoma malom prostoru u neposrednoj blizini zida. Koncept zidnih funkcija zasniva se na gruboj numeričkoj mreži i pretpostavljenim profilima osredjenih i turbulentnih veličina unutar ovog regiona, i na osnovu njih se određuju analitički izrazi za tangencijalni napon na zidu, produkciju i disipaciju turbulentne kinetičke energije u blizini zida i vrši modifikacija diskretizovanih transportnih jednačina za količinu kretanja i turbulentnu kinetičku energiju u nizu kontrolnih tačaka do zida. Od velikog je značaja da se postave u fizičkom smislu realni, dobro zasnovani granični uslovi, inače može doći do ozbiljnih teškoća u procesu dobijanja rešenja jednačina. Osim toga, primenom koncepta zidnih funkcija postižu se značajne uštede kako u pogledu kompjuterskog vremena potrebnog za proračun, tako i u pogledu kompjuterske memorije.

**Ključne reči:** *granični uslovi, zidne funkcije, efikasnost proračuna*

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-3. ID-52.**

# EXACT SOLUTIONS TO THE MULTIDIMENSIONAL GENERALIZED NONLINEAR SCHRÖDINGER EQUATION AND THEIR STABILITY

**Milivoj Belić[a], Wei-Ping Zhong[b], Nikola Petrović[c], Najdan Aleksić[c]**

*[a]Texas A&M University at Qatar, 23974 Doha, Qatar*
*e-mail <milivoj.belic@qatar.tamu.edu>,*
*[b]Shunde Polytechnic, Shunde 528300, China*
*[c]Institute of Physics, University of Belgrade, P.O. Box 68, 11001 Belgrade, Serbia*

**ABSTRACT:** The nonlinear Schrödinger equation (NLSE) is one of the most useful generic models in applied mathematics that naturally arises in many fields of physics. Its generalizations, such as the Gross-Pitaevskii and Ginzburg-Landau equations, are gaining widespread use in nonlinear optics [1]. However, stable exact localized solutions to the NLSE are known only in (1+1) dimensions [(1+1)D], for the simple reason that the inverse scattering method, responsible for the existence and stability of 1D solitons, works only in (1+1)D. There are no exact stable solitons in (2+1)D or (3+1)D. Recently, great interest has been generated when it was suggested that the (2+1)D generalized NLSE with varying coefficients may lead to stable 2D solitons. The stabilizing mechanism has been the sign-alternating Kerr nonlinearity in a layered medium [2]. Different methods of soliton management have been introduced. A vigorous search for the stabilized localized solutions of multidimensional NLSE has been launched; however, out of necessity, it has been numerical. We present here analytical periodic traveling and soliton solutions to the generalizations of NLSE in (2+1)D and (3+1)D and discuss their stability [3].

**Keywords:** Solitons.
**PACS:** 05.45.Yv; 42.65.Tg

## References

[1] Y. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, New York, 2003).
[2] B. A. Malomed, *Soliton Management in Periodic Systems* (Springer, New York, 2006).
[3] M. Belic, N. Petrovic, W. P. Zhong, R. H. Xie, and G. Chen, Phys. Rev. Lett. **101**, 123904 (2008); N. Z. Petrovic, N. B. Aleksic, A. Al Bastami, and M. R. Belic, Phys. Rev. E **83**, 036609 (2011).

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-4. ID-53.**

# BIFURCATIONS AND SYNCHRONIZATION IN SYSTEMS OF REALISTIC NEURONAL MODELS

By **Nikola Buric,**
Institute of Physics, University of Belgrade

Small parts of brain cortex may contain thousands of morphologically and functionally similar interconnected neurons. Realistic models of an individual neuron, like Hodgkin-Huxley, FitzHugh-Nagumo (FN) or Hindmarsh-Rose to mention only a few popular examples, are given by few-dimensional nonlinear differential equations. Transport of information between neurons can be phenomenologically described by time-delayed inter-neuronal interaction. It is also well known that neurons *in vivo* function under influences of many sources of noise. Considering all mentioned factors it is clear that a basic, relatively detailed mathematical model of a small part of realistic cortex should involve an extremely large system of nonlinear stochastic delay-differential equations. This is a problem that can not be approached in its full complexity neither by available analytical methods nor by brute-force numerical simulations. Therefore, analyzes of such complex systems must proceed is several steps that gradually increase the complexity of the analyzed system. Natural research program would involve the following steps:

a) Choice of simplest but representative mathematical models of typical single neuron dynamics;
b) Qualitative analytic study of a small number of coupled neurons;
c) Influence of interaction delays and noise on the typical dynamical features of small neuronal networks;
d) More or less severe approximations which could reduce large networks on much smaller ones, amenable to an analytic or efficient numerical treatment, with the same qualitative dynamical properties.

Each step of this research program involves and is based on application of the theory of dynamical systems. In this talk we shall review some of the results obtained within the framework of the stated research program.

In particular we shall:
a) review relevant bifurcations in a single pair of typical neuronal models with delayed coupling[1];
b) illustrate the methods that have been used to study the important phenomenon of synchronization[2,3];
c) Demonstrate some typical phenomena induced by noise in this simplest systems of interest[4,5];
d) Summarize the mean-field approach to construct approximations of systems with large number of neurons[6,7,8] ;
e) Discuss, using the mean-field approximation, an interesting and unexpected phenomena that occurs in large networks of neuronal excitable units[9].

## References

[1] N. Buric, D. Todorovic, Dynamics of FitzHugh-Nagumo excitable systems with delayed coupling, Phys.Rev E **67**, 066222, (2003).
[2] N . Buric, K. Todorovic, N. Vasovic**,** Synchronization of noisy delayed feedback systems with delayed coupling, Phys.Rev E. **75,** 026209 (2007)
[3] N. Buric, K. Todorovic and N. Vasovic**,** Synchronization of bursting neurons with delayed chemical synapses, Phys.Rev. E, **78**, 036211 (2008).
[4] N. Buric, K. Todorovic and N. Vasovic**,** Influence of Noise on Dynamics of Coupled Bursters, PhysRevE. **75** .067204 (2007).
[5] N. Buric, K. Todorovic and N. Vasovic**,** Influence of Noise on Dynamics of Coupled Bursters, PhysRevE. **75** .067204 (2007).

[6] N. Buric, K. Todorovic, D. Rankovic and N. Vasovic**,** Mean field approximation for noisy delay coupled excitable neurons, Physica A, **389** (2010) 3956.

[7] N. Buric, K. Todorovic and N. Vasovic**,** Influence of Interaction Delays on Noise Induced Coherence in Excitable Systems, Physical Review E, **82** 037201 (2010).

[8] I. Franovic, K. Todorovic, N. Vasovic and N. Buric, Stability, bifurcations, and dynamics of global variables of a system of bursting neurons, CHAOS **21**, 033109 (2011).

[9] I. Franovic, K. Todorovic, N. Vasovic and N. Buric, Spontaneous formation of synchronization clusters in homogenous neuronal ensembles induced by noise and interaction delays, Phys.Rev.Lett**, 108,** 094101 (2012) .

Naučno društvo Srbije — Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-5. ID-51.**

# GEOMETRIC RESONANCES IN BOSE-EINSTEIN CONDENSATES WITH TWO-AND THREE-BODY INTERACTIONS

**A. Balaž[1], H. Al Jibbouri[2], I. Vidanović[3], Axel Pelster[4]**

[1] Scientific Computing Laboratory, Institute of Physics Belgrade,
University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia
e-mail: antun.balaz@ipb.ac.rs
[2] Institute for Theoretical Physics,
Free University of Berlin, Arnimallee 14, 14195 Berlin, Germany
e-mail: hamidj@fu-berlin.de
[3] Scientific Computing Laboratory, Institute of Physics Belgrade,
University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia
e-mail: ivana.vidanovic@ipb.ac.rs
[4] Department of Physics and Research Center OPTIMUS,
Technical University of Kaiserslautern, 67663 Kaiserslautern, Germany
e-mail: axel.pelster@physik.uni-kl.de

**ABSTRACT.** The experimental discovery of Bose-Einstein condensation (BEC) in dilute atomic vapors has motivated a broad interest in ultracold atoms and molecules, and paved the way for extensive studies of a wide range of experimental and theoretical topics. In particular, many experiments have focused on investigating collective excitations of harmonically trapped BECs, as they can be measured very accurately and thus provide a reliable method for extracting the respective system parameters of such ultracold quantum gases systems.

The excellent agreement between the measured frequencies and theoretical predictions is one of the first important achievements in the investigation of such systems. At the mean-field level, they can be successfully described by the time-dependent Gross-Piteavskii (GP) equation for the macroscopic wave function of a BEC at zero temperature. Due to the inherent nonlinearity in this equation of motion, a wide variety of interesting phenomena are observed in collective excitations of BECs, including frequency shifts [1], mode coupling, damping, as well as collapse and revival of oscillations.

Here we present the results of a study of the dynamics of the condensate in general and its collective oscillation modes in particular by changing the geometry of the trapping potential. The asymmetry of the confining potential leads to important nonlinear effects, including resonances in the frequencies of collective oscillation modes of the condensate [2]. We describe the BEC dynamics at zero temperature by the nonlinear GP equation for the condensate wavefunction with two- and three-body interactions. Within a variational approach [3][4] the partial differential equation of Gross and Pitaevskii is transformed into a set of ordinary differential equations for the widths of the condensate in an axially-symmetric harmonic trap with both two- and three-body interactions.

We discuss in detail the resulting stability of the condensate. First, we consider the case of an attractive two-body interaction and a vanishing three-body interaction. Then we consider the case when we have attractive two-body and repulsive three-body interactions. We show that a small repulsive three-body interaction is able to extend the stability region of the condensate as it increases the critical number of atoms in the trap.

We study in detail geometric resonances and derive explicit analytic results for the frequency shifts for the case of an axially-symmetric condensate with two- and three-body interactions based on a perturbative expansion and a Poincaré-Lindstedt method [1]. We calculate frequency shift for a quadrupole and a breathing mode, and compare the derived analytical results with the results of numerical simulations. We also compare results of numerical simulations for radial and longitudinal widths of the condensate and the corresponding excitations spectra with the analytical results obtained using the perturbation theory.

Finally, we analyze the resonant mode coupling and generation of second harmonics of the collective modes, which are induced by nonlinear effects. At first we consider a BEC in the initial state corresponding to the stationary ground state with a small perturbation proportional to the eigenvector of the quadrupole mode, which leads to quadrupole mode oscillations. In the linear case, we have small-amplitude oscillations of the condensate size around the equilibrium widths, and we are in the regime of linear stability analysis. However, when the frequencies of collective modes are approached, we obtain a resonant behavior, which is characterized by large amplitude oscillations. In this case it is clear that a linear response analysis does no longer provide a qualitatively good description of the system dynamics.

**Keywords**: Bose-Einstein condensation, nonlinear dynamics, parametric resonance

## References

[1] Vidanović I, Balaž A, Al Jibbouri H, Pelster A, Nonlinear Bose-Einstein-condensate Dynamics Induced by a Harmonic Modulation of the s-wave Scattering Length, *Phys. Rev. A* **84** (2011) 013618.

[2] Dalfovo F, Minniti C, Pitaevskii L, *Phys. Rev. A* **56**, (1997) 4855.

[3] Pérez-García V M, Michinel H, Cirac J I, Lewenstein M, Zoller P, *Phys. Rev. Lett.* **77** (1996) 5320.

[4] Pérez-García V M, Michinel H, Cirac J I, Lewenstein M, Zoller P, *Phys. Rev. A* **56** (1997) 1424.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-6. ID-56.**

# NONLINEAR IONIC PULSES ALONG MICROTUBULES

**Miljko V. Satarić**(1)

1) Faculty of Technical Sciences, University of Novi Sad
Trg D. Obradovića 6, 21000 Novi Sad, Serbia
e-mail bomisat@neobee.net

Microtubules (MTs) are cytoskeletal biopolymers shaped as nanotubes that are essential for cell motility, cell division and intracellular trafficking. Here we investigate their polyelectrolyte character that plays a very important role in ionic transport throughout cellular environment. The model we propose demonstrates an essentially nonlinear behavior of ionic currents which are guided by MTs. These features are primarily due to the dynamics of tubulin's C-terminal tails which are extended out of the surface of the MT cylinder [1,2]. We also demonstrate that the origin of nonlinearity stems from the nonlinear capacitance of each tubulin dimer. This brings about conditions required for the creation and propagation of solitonic ionic waves along the MT axis. We conclude that a MT plays the role of a biological nonlinear transmission line for ionic currents[2]. These currents might be of interest for cell division and perhaps can play some important role even in cognitive processes in nerve cell.

We expect that this kind of localized ionic waves could have the basic role in many vital cellular activities.

First the process of cell division needs the synchronized depolymerisation of MTs. It can be caused by the localized $Ca^{++}$ waves of our model, reaching MT plus ends and triggering the onset of massive detachment of tubulin dimers from MT tips.

If just pure diffusion of $Ca^{++}$ ions through the bulk cytosol is left to rule this depolymerisation, the mistakes in this process would overwhelm the needed coordination.

In the very interesting paper [3] it was explained that in hair bundles (kinocilium) in the inner ear, consisting of the MT doublets, the dynein motor-driven oscillations are controlled by $Ca^{++}$ ions directed from ion channels along MTs.

These ions cause a fraction of dynein motors to detach and thus tune the oscillations of kinocilium as the reaction on a corresponding acoustic signal. This mechanism can also be explained by the localized $Ca^{++}$ waves elaborated here!

## References

[1] M.V. Satarić, D. Ilić, N. Ralević and J.A. Tuszynski
Eur. Biophys. J. 38, 637 (2009)
[2] M.V. Satarić, D. Sekulić and M. Živanov
Journ. Comput. Theor. Nanosci. 7, 2281 (2010)
[3] T. Duke, J. Phys. Condensed Matter 15, S1747 (2003)

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-7. ID-57.**

# SOLITON DYNAMICS IN ONE-DIMENSIONAL WAVEGUIDE ARRAYS WITH SATURABLE, SELF-DEFOCUSING NONLINEARITY

**M. Stepić[1], P. P. Beličev[1], A. Maluckov[1], I. Ilić[1], D. Kip[2], A. Kanshu[2], C.E. Rüter[2], V. Shandarov[3]**

[1] Vinča Institute of Nuclear Sciences,
Mike Petrovića Alasa 12-14, 11000 Belgrade, Serbia
[2] Faculty of Electrical Engineering, Helmut Schmidt University,
22043 Hamburg, Germany
[3] State University of Control Systems and Radioelectronics,
634050 Tomsk, Russia
e-mail: mstepic@vinca.rs

**ABSTRACT.** The last two decades have witnessed a strong interest in periodic optical systems, such as waveguide arrays and photonic crystals [1, 2]. These systems exhibit many attractive features for which no counterpart exists in homogeneous media, including forbidden gaps in their transmission spectra [3], existence of strongly localized nonlinear modes (discrete and gap solitons) and the exciting possibility of controlling diffraction. One-dimensional waveguide arrays consisting of evanescently coupled parallel channels are fabricated in various photorefractive materials, including photovoltaic lithium niobate crystal possessing saturable self-defocusing nonlinearity [4] that is responsible for solitons' stabilization and their enhanced mobility [5]. We analyze both experimentally and theoretically soliton dynamics in two types of photonic lattices: in uniform waveguide arrays with locally introduced coupling defect [6, 7] and in more complex, binary lattices with alternating spacing between channels of constant width [8, 9]. Coupling defect interrupts the lattice periodicity and offer an additional physical mechanism for light confinement, acting as a waveguide in whose vicinity light can be confined, causing the existence of the so-called, defect modes inside the bandgaps. On the other hand, an extra periodicity opens an additional mini-gap where light propagation is again forbidden, thus allowing for extended nonlinear interaction of light with binary lattices and existence of novel, stable types of lattice solitons that may be potentially useful for the development of future all-optical photonic devices.

**Keywords**: nonlinearity, waveguide arrays, solitons

## References

[1] Lederer F, Stegeman GI, Christodoulides DN, Assanto G, Segev M, Silberbeg Y, Discrete solitons in optics , Physics Reports, **463** (2008), pp. 1-126.

[2] Kivshar YuS, Agrawal GP, Optical Solitons - From Fibers to Photonic Crystals. San Diego: Academic Press, (2003).

[3] Mandelik D, Eisenberg HS, Silberberg Y, Morandotti R, Aitchison JS, Band-Gap Structure of Waveguide Arrays and Excitation of Floquet-Bloch Solitons, Physical Review Letters, **90** (2003), pp. 053902.

[4] Chen F, Stepić M, Rüter CE, Runde D, Kip D, Shandarov V, Manela O, Segev M, Discrete diffraction and spatial gap solitons in photovoltaic LiNbO3 waveguide arrays, *Optics Express*, **13** (2005), pp. 4314-4324.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-8. ID-54.**

# ULTRAFAST-LASER FABRICATION OF PHOTONIC COMPONENTS

**J. Petrović[1], V. Mezentsev [2]**

[1] Institute of Nuclear Sciencies Vinca
Mihaila Petrovica Alasa 12-14, 11000 Beograd, Serbia
e-mail: jovanap@vin.bg.ac.rs

[2] Aston University
Aston Institute of Photonics Technologies, Aston Triangle, B4 7ET, UK
e-mail: v.mezentsev@aston.ac.uk

**ABSTRACT.** Driven by the increased demand for optical devices in communications, sensing and quantum optics, photonics technology is becoming one of the key future technologies. The advancements in this technology rely on successful and affordable fabrication of high-speed all-optical devices and of the optical devices compatible with existing electronics. As a consequence, alternatives to the expensive lithographic techniques are intensely sought [1].

In this talk, we will introduce recently developed femtosecond-laser material processing as one of the most promising fabrication techniques in photonics. It is based on the refractive index modification caused by the highly nonlinear laser-material interaction and the subsequent changes in the material structure. A several orders of magnitude difference in the time scales of these processes permits a two-stage modeling of the whole fabrication process [2]. Indeed, the laser-pulse induced multi-photon ionization happens over a few femtoseconds and the heating of the ionized electrons completes during the sub-picosecond laser pulse. Thereby created large temperature and pressure gradients initiate the mechanical processes that can last from a few seconds up to a month and that result in a permanent refractive index change [3].

We will describe a mathematical model of the laser-pulse – electron-plasma system and use it to compare the roles of linear (dispersion, diffraction) and nonlinear (ionization, self-focusing) effects in the interaction dynamics. The experimental evidence of the inscription in different non-linear regimes will be presented. A good agreement between the fingerprints of the calculated plasma distribution and the experimentally obtained refractive index profiles corroborates the model.

We will further give examples of the ultrafast laser processing of glasses, polymers and semiconductors, and will demonstrate the waveguides, fibre gratings and microchannels as end-products ready for applications. Finally, we will describe the experiment in which the nonlinear engineering of the laser frequency has been used to enhance the resolution and efficiency of the inscription process.

The talk will be concluded by a comparison with other available fabrication techniques and the outlook of the future developments.

**Keywords**: femtosecond laser, material processing, photonic devices, nonlinear optics

**References**

[1] Gattass R R, Mazur E, Femtosecond laser micromachining in transparent materials, *Nature Photonics* **2** (2008) 219

[2] Mezentsev V et al. Adaptive Modelling of the Femtosecond Inscription in Silica, *Proceedings. of SPIE* **6107**, (2006) 61070

[3] Allsop T, et al. Spectral characteristics and thermal evolution of long period gratings in photonic crystal fibre fabricated by NIR femtosecond laser using point-by-point inscription, *Journal of Optical Society of America B*, **28** (2011) 2105

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-9. ID-55.**

# SELF-ORGANIZATION IN THE HIDDEN MARKOV MODEL OF WAVELET SIGNAL PROCESSING

**M. Milovanović[1], M. Rajković[2]**

[1] Mathematical Institute of the Serbian Academy of Arts and Sciences, Belgrade
[2] Vinca Institute of Nuclear Sciences, Belgrade, Serbia, e-mail: milanr@vinca.rs

## ABSTRACT.

A model of self-organization in the wavelet signal processing is developed to provide criterion for the choice of the optimal wavelet basis. The method, based on the Hidden Markov model of wavelet coefficients, enables quantification of self-organization and with the optimal basis, performs superior denoising at the same time. Applications for one-dimensional and two-dimensional signals are demonstrated on patterns formed by cellular automata and experimentally recorded signals of neutral and confined plasma turbulence.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-10. ID-36.**

# DIFFERENTIAL EQUATIONS OF MOTION OF MECHANICAL SYSTEMS WITH NONLINEAR NONHOLONOMIC CONSTRAINTS – VARIOUS FORMS AND THEIR EQUIVALENCE

**Zekovic N. Dragomir**
University of Belgrade,
Faculty of Mechanical Engineering
Kraljice Marije 16, 11120 Belgrade, Serbia
e-mail: dzekovic@mas.bg.ac.rs

**Abstract:** In the mechanics of nonholonomic systems there are a number of forms of differential equations of motion that can be translated into the same external form, i.e. it is shown that they are mutually equivalent for nonlinear nonholonomic systems.
In this paper, the above approach is extended to nonholonomic systems with nonlinear constraints. It is shown that various forms of differential equations of motion are a consequence of the manner of introducing nonholonomic constraints into the Lagrange-d'Alembert principle.
The paper also considers a mechanical system subject to the action of nonlinear nonholonomic constraints of the forms:

$$\Phi^\nu\left(q^i,\dot q^i\right)=0 \quad \rightarrow \dot q^\nu=\psi^\nu\left(q^i,\dot q^\alpha\right) \quad (1)$$
[1]

Independent generalized velocities $\dot q^\alpha$ are expressed using specified generalized kinematic parameters $\dot\pi^\alpha$ by the relations:

$$\dot q^\alpha=\theta^\alpha\left(q^i,\dot\pi^\beta\right) \quad (2)$$

Using the constraints (1) the dependent generalized velocities $\dot q^\nu$ are also expressed by means of independent kinematic parameters $\dot\pi^\alpha$ by the relations:

$$\dot q^\nu=\psi^\nu\left(q^i,\dot q^\alpha\right)=\psi^\nu\left[q^i,\theta^\alpha\left(q^i,\dot\pi^\beta\right)\right]=\theta^\nu \quad (3)$$

and the expression for all generalized velocities $\dot q^i$ reads (taking into account (2) and (3):

$$\dot q^i=\theta^i\left(q^j,\dot\pi^\alpha\right) \quad (4)$$

To derive the equations of motion, we used the expression for kinetic energy of the form:

$$T=\frac{1}{2}g_{ij}\dot q^i\dot q^j \quad (5)$$

[1] In this paper, the Einstein summation convention is used. Indices take the following values:
$i,j,k,r,s=1,\dots,n;\quad \alpha,\beta=1,\dots,m;\quad \nu=m+1,\dots,m+l=n$

and the expression for kinetic energy of the form (taking into account (4)):

$$T^* = \frac{1}{2} g_{ij} \theta^i \theta^j \tag{6}$$

Variations of the generalized coordinates $q^i$ are in accordance with the Hertz-Helder principle, i.e.:

$$\delta q^i = \frac{\partial \theta^i}{\partial \dot{\pi}^\alpha} \delta \pi^\alpha \tag{7}$$

After taking into account certain transformations in the procedure of derivation, one obtains the equations of motion of the system in the form as follows:

$$\frac{d}{dt}\frac{\partial T^*}{\partial \dot{\pi}^\alpha} - \frac{\partial \theta^k}{\partial \dot{\pi}^\alpha}\frac{\partial T^*}{\partial q^k} - \frac{\partial T}{\partial \dot{q}^k}\left(\frac{d}{dt}\frac{\partial \theta^k}{\partial \dot{\pi}^\alpha} - \frac{\partial \theta^k}{\partial q^s}\frac{\partial \theta^s}{\partial \dot{\pi}^\alpha}\right) = Q_k \frac{\partial \theta^k}{\partial \dot{\pi}^\alpha} = Q^*_\alpha \tag{8}$$

$$G_{\alpha\beta}\ddot{\pi}^\alpha + g_{ij}\frac{\partial \theta^i}{\partial q^r}\frac{\partial \theta^j}{\partial \dot{\pi}^\beta}\theta^r + g_{ij}\Gamma^i_{sr}\theta^s\theta^r\frac{\partial \theta^j}{\partial \dot{\pi}^\beta} = Q_i\frac{\partial \theta^i}{\partial \dot{\pi}^\beta} = Q^*_\beta \tag{9}$$

and in this case, conditionally speaking, the equations (9) represent a direct form of the equations (8). From the form of equations of motion of nonlinear, nonholonomic systems (8) and (9) there follow the equations of motion of Maggi, Voltere, Appell, Voronec, Chaplygin, Ferrers, Boltzmann, and Hamel, which means that they are mutually equivalent.

Key words: equation of motion, system, nonholonomic, constraint, nonlinear

**Acknowledgements:** This research was supported by the Ministry for Science and Technology of Serbia (ON 174001) through Mathematics Institute of the Serbian Academy of Sciences and Arts.

## REFERENCES

[1] Y.I.Neymark and N.A.Fufaev, Dynamics of Nonholonomic Systems, Nauka, Moskva, 1967, p.519.

[2] V.V.Dobronravov, Basis of Mechanics of Nonholonomic Systems, Vissaja skola, Moskva, 1970, p.270.

[3] V.Covic, Differential Equations and Stability of Motion of Nonholonomic Systems with Application to Dynamic Structures, Doctoral Dissertation, Faculty of Mechanical Engineering, Belgrade, 1976.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**IL-11. ID-3.**

# SOME ASPECTS OF BIRD IMPACT THEORY

**M. Ugrčić**[1]

[1] Economics Institute
Kralja Milana 18, 11000 Belgrade, Serbia
e-mail: ugrcicmarinko@gmail.com

**ABSTRACT.** One of quite possible and very dangerous accident is a bird strike into the aircraft in the flight. This case is characterized by the high speed impact of the bird onto aircraft structure, causing the large dynamic deformations and damage of the elements which may lead to disintegration of the construction. That's why theoretical researching and numerical simulation of the bird strike are very important and should be implemented during design phase of a development and testing processes.

Assuming the variables of target (flat rigid or elastic panel) as constant, the number of impact variables is high. This makes the bird strike analysis complex. The various parameters include bird material and density, impact velocity, bird mass, bird material configuration, bird aspect ratio, material porosity, obliquity of impact and contact properties.

Most of the initial models of bird impact was developed on the basis of the classical impact theory and used force-impulse equation. Unfortunately, these models failed to predict the damage details. Further, the elementary one-dimensional theory of hydrodynamics was used to study bird impact. The hydrodynamic theory assumes that during impact at high velocity of the projectile material tends to behave as a fluid. The mass-momentum-energy conservation analytical equations and simple pressure-density-energy equation of state were used to describe the material behavior [1,2]. More appropriate interpretation of bird impact was carried out by finding out precise equations of state models for the shock phase and steady flow state phase.

Regarding the finite elements methods, actually, three possibilities have been successfully employed to simulate this phenomenon. These are the Lagrangian, Arbitrary Lagrangian-Eulerian (ALE) and Smooth Particle Hydrodynamics (SPH) formulations, which differ mainly in their method of discretization.

The Lagrangian formulation is based on Lagrangian mechanics, and was the earliest approach to be used in bird strike analysis [3,4]. In Lagrangian analyses, the bird is divided into parts and the element mesh is bounded to the material. Birds undergo extreme deformation during a strike, and this can cause problems in Lagrangian meshes as some elements may take negative volumes. Solving dynamic transient nonlinear problems requires a very small and frequently unacceptably small time step.

The ALE formulation [4,5,6,7] allows parts to contain more than one material (e.g. water and air), and uses arbitrary reference coordinates that allow material to flow through the mesh (Eulerian behavior) rather than the nodes moving with the material (Lagrangian). It does not experience a time step problem, but high distortion can still cause negative volume issues.

The SPH method is a more recently developed to discrete parts that uses particles of mass rather than an element mesh to represent the bird [4,8]. The SPH avoids problems caused by mesh distortion, but still requires further development to become reliably stable and consistent.

This paper presents some results of theoretical analysis and numerical simulations of dynamic loading of the bird body and flat thin plate loaded by the bird impact. The bird was modeled as porous water-air material, shaped as a flat or hemi-spherically ended cylinder. Modeling was carried out by the use of the SPH method in ANSYS AUTODYN® code for porous material. In this way, the dependence of sound speed and bulk modulus on porosity in equation of state (EOS) was developed. Lagrangian target model was considered as simple flat rigid or elastic Aluminum alloy plate. The comparative analysis of numerical results for Hugoniot shock theory and SPH method were given. As well, some results of experimental data were included.

**Keywords**: Bird strike, bird modeling, impact simulation, finite element analysis, SPH method

### References

[1] Wilbeck J. S., Rand J. L., The Development of Substitute Bird Model, *ASME Journal of Engineering for Powe*r, Volume **103**, (1981), pp. 725-730.

[2] Lavoie M. A., Gakwaya A., Ensan M. N., Zimcik D. G., Review of Existing Numerical Methods and Validation Procedure Available for Bird Strike Modeling, *International Conference on Computational & Experimental Engineering and Sciences*, (2007), pp. 111-118.

[3] Nizampatman L. S., Models and Methods for Bird Strike Load Predictions, *PhD Dissertation*, Faculty of Graduate School, Wichita State University, USA, (2007)

[4] ANSYS AUTODYN User's Manual *Theory manual*, ANSYS Inc. Southpointe, 275 Technology Drive, Canonsburg, PA, USA, (2010).

[5] Benson, D. J., An efficient, Accurate, Simple ALE Method for Nonlinear Finite Element Programs, *Computer Methods in Applied Mechanics and Engineering*, **72**, (1989), pp. 305-350.

[6] Meo M., Vignjević R., Maengo M., Numerical Simulations of Low-Velocity Impact on an Aircraft Sandwich Panel, *Composite Structures*, **62**, (2003), pp. 353–360.

[7] Ivančević, D., Smojver I., Hybrid Approach in Bird Strike Damage Prediction on Aeronautical Composite Structures. *Composite Structures*, **94**, 1, (2011), pp. 15-23.

[8] Heimbs, S.: Bird Strike Simulations on Composite Aircraft Structures, *2011 SIMULIA Customer Conference*, Barcelona, Spain, May 17-19, (2011), pp. 73-86.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL-12. ID-73**.

# THE USE OF FINITE ELEMENTS METHOD IN VIBRATIONAL PROPERTIES CHARACTERIZATION OF MOUSE EMBRYO IN ICSI

**A. Hedrih[1], M. Ugrčić[2]**

[1] State University in Novi Pazar
Vuka Karadžića bb, 36300 Novi Pazar, Serbia
e-mail: handjelka@hm.co.rs

[2] Economics Institute
Kralja Milana, 18, 11000 Belgrade, Serbia
e-mail: ugrcicmarinko@gmail.com

**ABSTRACT.** To determine the vibration characteristics (natural frequencies and mode shapes) of a mouse embrio during intracytoplasmic sperm injection (ICSI) the modal analysis is used. The spherical mouse embryo 60 µm in diameter is modeled as elastic finite elements biostructure consisting of 6µm thick micromembrane and 38 µm in diameter nucleus. Embryo modeling and modal analysis were based on the use of the finite elements method in the modal analysis system of ANSYS software. The modal analysis was carried out for six basic modes of embryo natural frequencies. The numerical analysis of dependence of embryo own frequencies on the boundary conditions and external loads are presented. The relevant illustrations of the typical variations of the shape, deformation and particle velocities of vibrating embryo are given.

**Keywords**: mouse embryo vibrations, finite elements method, free vibrations, forced vibrations.

## 1. Introduction

ICSI is a common method used in artificial insemination. In ICSI method oocyte/embryo is fixed with vacuum pipette at one site of the cell and on the opposite site is the microinjecting pipette. The whole system is in the petry dish that is full of liquid. The aim of this study was to determine vibration characteristics of this system using finite elements method in the modal analysis system of ANSYS software.

## 2. FEM modeling of mouse embryo

In modal analysis the embryo model was considered as three-dimensional axis-symmetric problem. As basic physical characteristics of mouse embryo we use parameters given in the ref [2,4]. The differential equation of motion in the modal subspace is in the following form:

$$[I]\{\ddot{y}\}+[\Phi]^T[C][\Phi]\{\dot{y}\}+\left(\left[\Lambda^2\right]+[\Phi]^T\left[K_{asym}\right][\Phi]\right)\{y\}=\{0\} \tag{1}$$

where is: $[\Lambda^2]$ - a diagonal matrix containing the first $n$ eigen frequencies $\omega_i$.
Introducing the $2n$-dimensional state variable vector approach, equation (1) can be written in reduced form as follows:

$$[I]\{\dot{z}\}=[D]\{z\} \tag{2}$$

The 2$n$ eigen values of Equation (1) are calculated using the QR algorithm (Press et al. [3]). The inverse iteration method (Wilkinson and Reinsch [5]) is used to calculate the complex modal subspace eigen vectors. In modal analysis the embryo model was considered as three-dimensional axis-symmetric problem. The mouse embryo with basic parts of micro-robotic cell manipulation system described in [4] is simplified according to the model setup. The statistic data related to the number of nodes and elements for each component of the system after medium quality meshing procedure is given. All materials, including biomaterials (biomembrane and nucleus), then medium materials (air and liquid medium MPC) and, finally, mechanical equipments materials (structural steels) are considered as isotropic elasticity features materials.

## 3. Results

The numerical integration of Eq. 2 facilitates the solutions for elements of diagonal matrix [Λ] containing the first $n$ eigen frequencies $\omega_i$. Modal distribution of natural frequencies of the mouse embryo is given in Fig 1. The modal analysis was carried out for six basic modes of embryo natural frequencies. The relevant illustrations of the typical variations of the shape and particle velocities and shape deformation distribution of vibrating embryo in mode 6 are given in Fig 2. and 3.

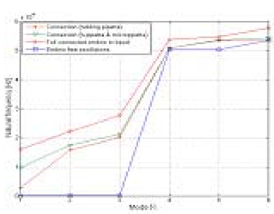


**Figure 1.** Modal distribution of natural frequencies of embryo vs. boundary conditions

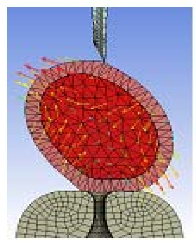


**Figure 2.** Shape and particle velocities distribution in extreme points of embryo vibrations in mode 6

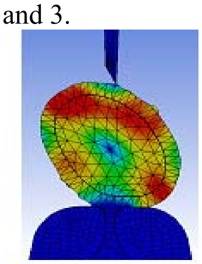


**Figure 3.** Shape and deformations distribution points in extreme points of embryo oscillations ir mode 6.

## 4. Conclusion

Based on the results of numerical analysis given in the paper it is shown that the robust finite elements model of mouse embryo and basic parts of micro-robotic cell manipulation system were correctly created. All necessary contacts and boundary conditions were regularly involved facilitating the modal analysis and simulation of all situations of the embryo vibrations. As well, the determinations of the vibrational characteristics of mouse embryo free oscillations and embryo oscillations affected by boundary conditions were successfully carried out. The work presented in the paper confirms the possibility to use the finite elements method coupled with numerical modal analysis as a powerful tool in the vibrational characterization of bio structures such as the mouse embryo. This could be useful in biomechanical determination of successful ICSI.

*Acknowledgement.* Parts of this research were supported by the Ministry of Education and Science of the Republic of Serbia through the Mathematical Institute SANU Belgrade Grant OI174001" - Dynamics of hybrid systems with complex structures. Mechanics of materials". The authors would like to thank Dr Milan Bojanović, Director of CAD PROFESSIONAL SYS, for their support of the research and facilitate the use of licensed academic software ANSYS Products.

## References


[1] ANSYS AUTODYN *User's Manual Theory manual*, ANSYS Inc. Southpointe, 275 Technology Drive, Canonsburg, PA, USA, (2010).

[2] Liu X., Fernandes R., Jurisicova A., Casper F.R., Sun Y. In situ mechanical characterization of mouse oocytes using a cell holding device. *Lab on a Chip*, 10 (2010), pp. 2154–2161.

[3] Press W.H., et al., Numerical Recipes in C, *The Art of Scientific Computing*, Cambridge University Press, (1993)

[4] Sun Y., Wan K.T., Roberts K.P., Bischof J.C., Nelson B. J., Mechanical Property Characterization of Mouse Zona Pellucida, *IEEE Transactions On Nanobioscience*, Vol. 2, 4, (2003), pp. 279-286.

[5] Wilkinson J.H., Reinsch C., Linear Algebra, *Handbook for Automatic Computation,* Springer-Verlag, New York, NY, Vol. II., (1971), pp. 418–439.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL – 13. ID-15.**

# RAYLEIGH-BENARD CONVECTIVE INSTABILITY IN THE PRESENCE OF TEMPERATURE MODULATION

**M. Jovanović[1], D. Živković[2], J. Nikocijević[3],**

[1,2,3] University of Niš, The Faculty of Mechanical Engineering,
A.Medvedeva 14, 18000 Niš, Serbia, e-mail: jmilos@masfak.ni.ac.rs

**ABSTRACT.** This paper is devoted to the analysis of two-dimensional viscous fluid flow between two parallel plates, where the lower plate is heated and the upper one is cooled .The temperature difference between the plates is gradually increased during the certain time period , and afterwards it is temporarly constant. The temperature distribution on the lower plate and upper plate is not constant in x-direction, there is longitudinal sinusoidal temperature modulation imposed on the mean temperature. The plates and fluid layer between them are inclined at angle $\alpha$ with regard to horizontal plane. Here, isotropy is broken even in the absence of forcing and the anisotropy may compete with external modulation. Even in the absence of an applied uniform temperature gradient, a pure temperature modulation leads to periodic convection patterns.
We investigate the wave number and amplitude influience of this modulation on the subcritical stability and the onset of Rayleigh-Benard convective cells and its instability, by direct numerical simulation of 2D Navier-Stokes and energy equation.

**Keywords**: word, word direct Numerical Simulation of Navier Stokes equation, Bousineque approximation, Rayleigh and Prandtl number, nonlinear stability analysis.

## References

[1] Lancaster P, Free Vibrations of Lightly Damped Systems by Perturbation Methods, *Quarterly Journal of Mechanics and Applied Mathematics*, **13** (1960), pp. 138-155. (Times New Roman, 10pt.)

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**IL- 14. ID-86.**

# APPLICATION OF R-FUNCTIONS THEORY TO STUDY NONLINEAR VIBRATIONS OF LAMINATED SHALLOW SHELLS AND PLATES

**L.V. Kurpa, N. Budnikov, Tatiyana Shmatko**

NTU "KhPI", Kharkov, Ukraine
E-mail: kurpa@kpi.kharkov.ua, l.kurpa@mail.ru

**Abstract.** In the present paper the nonlinear vibrations of laminated plates and shallow shells are investigated. The proposed method of studying is based on the R-functions theory and variational methods (RFM). Formulation of the problem is carried out in classical shell theory (CST) and first-order shear deformation theory (FSDT). The nonlinear system of ordinary differential equations is reduced to system of ordinary differential equations. The proposed method is applied to research nonlinear forced vibrations of two-layered cross-ply plates with complex planform.

**Introduction.** Research of the geometrically nonlinear vibrations of the laminated plates and open shallow shells is one of important issues of nonlinear dynamics. In general case this problem may be only solved by numerical methods. Many researchers are studying this problem [1, 4–6, 10]. Some review of achievements in this field is presented in works [1–3]. The main approach which is applied to solve this problem is finite elements method (FEM) combined with method of harmonic balance, Bubnov-Galerkin, multiscales method and another. In the given paper the algorithm of meshless discretization, based on combination of the classical approaches and modern constructive tools of the R-functions theory [8] is developed. Application of R-functions theory allows studying geometrically nonlinear dynamic response of the laminated shallow shells and plates with complex shape and different boundary conditions.

**Mathematical statement.** A laminated shallow shell of an arbitrary planform with radii of curvature $R_x$, $R_y$ which consists of $M$ layers of the constant thickness $h_i$ is considered. Investigation we will carry out by first-order shear deformation theory (FSDT) and the classical shell theory (CST). According to these theories, it is assumed that the tangent displacements are linear functions of coordinate $z$, and the transverse displacement $w$ is constant through the thickness of the shell. While the CST adopts Kirchhoff's hypothesis, FSDT does not adopt them but in this case it is assumed that the normal to the midsurface remains straight after deformation, but not necessary normal to the middle surface. The nonlinear strain-displacement relations of the laminated shell can be presented as follows:

$$\{F\}=[A]\cdot\{\varepsilon\} \qquad (1)$$

where

$$\{F\}=\{N_{11},N_{22},N_{12},M_{11},M_{22},M_{12},Q_x,Q_y\},\ [A]=\begin{bmatrix}[C] & [K] & 0\\ [K] & [D] & 0\\ 0 & 0 & [S]\end{bmatrix},$$

$$\{\varepsilon\}=\{\varepsilon_{11},\varepsilon_{22},\varepsilon_{12},\chi_{11},\chi_{22},\chi_{12},\varepsilon_{23},\varepsilon_{13}\},\ \varepsilon_{11}=u,_x+\frac{w}{R_x}+\frac{1}{2}w,_x^2,\ \varepsilon_{22}=v,_y+\frac{w}{R_y}+\frac{1}{2}w,_y^2,$$

$$\varepsilon_{12} = u,_y + v,_x + w,_x w,_y , \; \varepsilon_{13} = \delta\left( w,_x + \psi_x - \frac{u}{R_x} \right), \; \varepsilon_{23} = \delta\left( w,_y + \psi_y - \frac{v}{R_y} \right),$$

$$\chi_{11} = \delta\psi_x,_x - (1-\delta)w_{,xx} , \; \chi_{22} = \delta\psi_y,_y - (1-\delta)w,_{yy} , \; \chi_{12} = \delta(\psi_x,_y + \psi_y,_x) - 2(1-\delta)w,_{xy} .$$

Here $u$, $v$ and $w$ are the displacements at the midsurface, $\psi_x$ and $\psi_y$ are the rotations about the $y$- and $x$-axes respectively. Constants $C_{ij}$ and $D_{ij}$ are the stiffness coefficients of the shell, which are defined by the following expressions [5, 6]:

$$\left(C_{ij}, K_{ij}, D_{ij}\right) = \sum_{m=1}^{M} \int_{h_m}^{h_{m+1}} B_{ij}^{(m)}\left(1, z, z^2\right)dz, \quad (i, j = 1,2,6),$$
$$S_{ij} = k_i^2 \sum_{m=1}^{M} \int_{h_m}^{h_{m+1}} B_{ij}^{(m)} dz, \quad (i, j = 4,5) \tag{2}$$

Here $B_{ij}^{(m)}$ are stiffness coefficients of the $m$-th layer, $k_i$, $i = \overline{4,5}$ are shear correction factors. Next, we assume that $k_4 = k_5 = 5/6$, that is. $S_{45} = S_{54}$. Indicator $\delta$ is the tracing constant which takes 1 and 0 for the FSDT and CST respectively.

Differential equations of the motion in operator form are:

$$LU = NL + mU,_{tt} \tag{3}$$

where $U = \{u; v; w; \psi_x; \psi_y\}^T$, $m = \{m_1, m_1, m_1, m_2, m_2\}^T$ $\left( m_1 = \rho h, m_2 = \frac{\rho h^3}{12} \right)$, $L$ is matrix $L = \lfloor L_{ij} \rfloor_{\substack{i=\overline{1,5} \\ j=\overline{1,5}}}$. The elements $L_{ij}$, $i, j = \overline{1,5}$ of the matrix $L$ are linear operators [9]. The components $NL_i$, $i = \overline{1,3}$ of the vector $NL = \{NL_1(w), NL_2(w), NL_3(u, v, w), 0, 0\}^T$ are nonlinear operators:

$$NL_1(w) = N_{11}^{(Nl)},_x + N_{12}^{(Nl)},_y , \; NL_2(w) = N_{12}^{(Nl)},_x + N_{22}^{(Nl)},_x ,$$

$$Nl_3(u, v, w) = \left(N_{11}w,_x + N_{12}w,_y\right),_x + \left(N_{12}w,_x + N_{22}w,_y\right),_y - k_1 N_{11}^{(Nl)} - k_2 N_{22}^{(Nl)} , \; \left( k_1 = \frac{1}{R_x}, k_2 = \frac{1}{R_y} \right).$$

Expressions $N_{11}^{(Nl)}, N_{12}^{(Nl)}, N_{22}^{(Nl)}$ join group of the nonlinear terms in relations for $N_{11}, N_{12}, N_{22}$.

Differential equations (3) are supplemented by corresponding boundary conditions.

**Method of solution.** The first step is studying linear problem in order to find the natural frequencies and eigenfunctions $\{U^{(c)}\} = \{u^{(c)}, v^{(c)}, w^{(c)}, \psi_x^{(c)}, \psi_y^{(c)}\}^T$ satisfying the given boundary conditions. Note that solving linear problem we will not ignore inertia forces. Solution of linear problems has been widely discussed in [9]. Let us note that in generic case this problem may be solved by RFM [8]. Then unknown function are presented as

$$w = \sum_{i=1}^{n} y_i(t) w_i^{(c)}(x, y), \quad \psi_x = \delta \sum_{i=1}^{n} y_i(t) \psi_{xi}^{(c)}(x, y), \quad \psi_y = \delta \sum_{i=1}^{n} y_i(t) \psi_{yi}^{(c)}(x, y)$$
$$u = \sum_{i=1}^{n} y_i(t) u_i^{(c)}(x, y) + \sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j u_{ij} , \; v = \sum_{i=1}^{n} y_i(t) v_i^{(c)}(x, y) + \sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j v_{ij} \tag{4}$$

where $y_k(t)$ are unknown functions in time, $w_i^{(c)}(x, y), u_i^{(c)}(x, y)$, $v_i^{(c)}(x, y)$, $\psi_{xi}^{(c)}(x, y)$, $\psi_{yi}^{(c)}(x, y)$ are components of the $i$-th eigenfunctions of linear vibrations of the shell. Functions $u_{ij}, v_{ij}$ must be solutions of the following system [9]

$$\begin{cases} L_{11}u_{ij} + L_{12}v_{ij} = -Nl_1^{(2)}\left(w_i, w_j\right), \\ L_{21}u_{ij} + L_{22}v_{ij} = -Nl_2^{(2)}\left(w_i, w_j\right). \end{cases} \tag{5}$$

Obtained system is solved by RFM. Substituting the expressions (4) for functions $u, v, w, \psi_x, \psi_y$ in third equation of the system (3) and applying procedure by Bubnov-Galerkin we drive nonlinear system of ordinary differential equations in unknown functions $y_j(t)$, of the following type:

$$y''_j(t) + \alpha_j y_j(t) + \sum_{i=1}^{n}\sum_{k=1}^{n} \beta_{jik} y_i(t) y_k(t) + \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \gamma_{jikl} y_i(t) y_k(t) y_l(t) = \widetilde{F}\,,\ \left(j = \overline{1,n}\right) \qquad (6)$$

Expressions for coefficients $\alpha_j, \beta_{jik}, \gamma_{jikl}$ are found and expressed through double integrals of known functions.

The solving obtained system (6) can be performed by different approximate methods, such as the harmonic balance method (HBM), multiscale, method of Runge-Kutta, Bubnov-Galerkin and others.

The proposed method is applied for studying nonlinear free and forced vibrations of the laminated plates and shallow shells of the different planform and boundary conditions.

**Numerical results.** Below the proposed method is illustrated on examples of two-layered fully clamped cross-ply plates with the complex shape (Fig. 1,2). Geometrical characteristics are: *a*/*b*=1, *a*/*h*=100, *c*=0.2*a*, *d*=0.3*a*, *e*=0.3*a*; mechanical properties are: $E_1/E_2 = 3$, $\boldsymbol{E_1/E_2 = 3}$ $G_{12}/E_2 = 0.5$, $\nu_1 = 0.25$ $\boldsymbol{G_{12}/E_2 = 0.5}$ (Glass/epoxy) [10]. Figure 3 shows the frequency response curves for the nonlinear forced vibrations of the plate under the transverse load $\widetilde{F}(x, y, t) = P\cos(\Omega t)$ $\boldsymbol{\widetilde{F}(x, y, t) = \widetilde{P}\cos\Omega t}$ ($\widetilde{P}$).

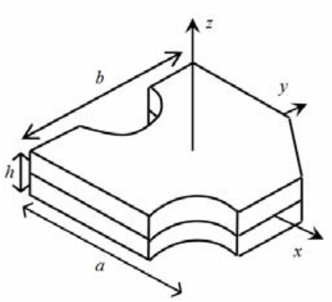


Figure 1

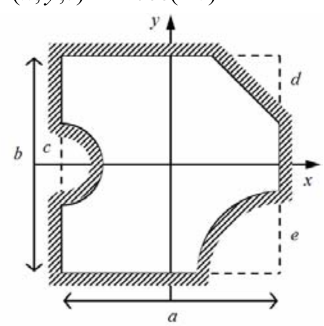


Figure 2

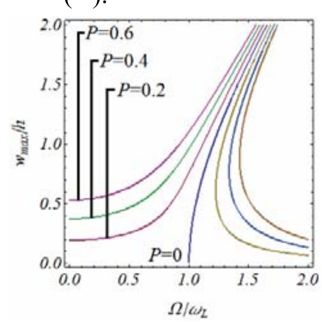



Figure 3

| $w_{max}/h$ | [10] | RFM | $w_{max}/h$ | [10] | RFM |
|---|---|---|---|---|---|
| 0.5 | 1.0562 | 1.0536 | 1.5 | 1.4221 | 1.4113 |
| 1.0 | 1.2076 | 1.2004 | 2.0 | 1.6752 | 1.6623 |

Table 1: A comparison of the frequency ratio for a fully clamped two-layered cross-ply square plate.

In the case when the sizes of cuts approach zero, we obtain a square plate. A comparison of the dependence $\omega_N/\omega_L$ $\boldsymbol{\varpi_{NL}/\varpi_L}$ on the amplitude for two-layered square plate is presented in the Table 1.

**References**

[1] Amabili M. (2008) Nonlinear Vibrations and Stability of Shells and Plates, University of Parma, Italy.

[2] Zhang Y.X., Yang C.H. (2009) Recent developments in finite element analysis for laminated composite plates // Composite Structures 88 147–157

[3] Sharma, Avadesh K.; Mittal, N. D. (2010) Review on Stress and Vibration Analysis of Composite Plates // Journal of Applied Sciences, vol. 10, issue 23, pp. 3156-3166

[4] Qatu S. (2004) Vibration of Laminated Shells and Plates, Elsevier Ltd., Oxford.

[5] Reddy J.N., Arciniega R.A. (2004) Shear Deformation Plate and Shell Theories: From Stavsky to Present. *J. Mechanics of Advanced Materials and Structures* 11:535-582.

[6] Abe A., Kobayashi Y., Yamada G. (2007) Nonlinear dynamic behaviors of clamped laminated shallow shells with one-to-one internal resonance. *Journal of Sound and Vibration* 304:957-968.

[7] Ambartsumian S.A. (1974) The general theory of anisotropic shells. Moscow, Nauka (in Russian).

[8] Rvachev V.L. (1982) The R-Functions theory and its Some Application, Naukova Dumka, Kiev (in Russian).

[9] Kurpa L.V. (2009) Nonlinear free vibrations of multilayer shallow shells with a symmetric structure and with a complicated form of the *plan. Journal of Mathematical Sciences 162, №1: 85-98.*

[10] T. Manoj, M. Ayyappan, K. S. Krishnan, B. Nageswara Rao. (2000) Nonlinear Vibration Analysis of Thin Laminated Rectangular Plates on Elastic Foundations. *ZAMM Z. Angew. Math. Mech.* 80 (3), pp. 183–192.

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

### Multidisciplinary and Interdisciplinary Applications

(SNDMIA 2012), Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Symposium

# Nonlinear Dynamics – Milutin Milanković

## Multidisciplinary and Interdisciplinary Applications

## (SNDMIA 2012),

Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Editors: Katica R. (Stevanović) HEDRIH
Žarko Mijajlović

# Contributed Lectures

Beograd, October 1-5, 2012.
Venue: Mathematical Institute SANU

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-1. ID-1.**

# NONLINEAR DYNAMIC EFFECTS IN CYCLIC MACHINES

**I. Vulfson**

St.-Petersburg State University of Technology and Design. Bolshaya Morskaya,18, 191186, St-Petersburg , Russia.

e-mail: vujo@rambler.ru

**ABSTRACT**. The problem of vibroactivity reducing is one of the most important problems of modern machine dynamics. In this report several developing directions of solution to this problem are discussed.
***The effect arising from the nonlinear geometrical characteristics of mechanisms.*** A distinctive properties of cyclic machines are the motion transformation and programmable displacement of actuators according to nonlinear position function $\Pi(\varphi)$ ( $\varphi$ is the coordinate of an input link). The mechanisms realizing programmed motion ("cyclic" mechanisms) are playing a double role in the vibratory system on the one hand being the source of disturbances for the drive, and on the other hand being critical objected to vibration. In these mechanisms occurs the possibility of asynchronous excitation caused by disruption the dynamic stability condition on the limited time intervals. Using the linearization in the vicinity of program motion after the transformation to the quasinormal coordinates of the original nonlinear system can be described by $H$ differential equations of the form [1–5]:

$$\ddot{\eta}_r + 2n_r(t)\dot{\eta}_r + p_r^2(t)\eta = W_r(t) \qquad (r = \overline{1, H}) .$$

In many mechanisms a change of the "natural" frequency takes place slowly. This does not exclude the possibility of an increase in the vibration amplitude during certain intervals, caused by a local violation of the conditions for dynamic stability. The dynamic stability conditions on any time interval can be written as

$$n_r + 0,5\dot{p}_r / p_r > 0 .$$

In the similar cases the amplitude modulation is seen, when the zone of decrease alternates of the zone of increase. Therefore, contrary to a parametric resonance we do not experience the unlimited increase of amplitudes. Compliance with this condition removes also the possibility of the build-up in the zones of the main parametric resonances.
***The effect arising from the joint action of nonlinear position function and clearances.*** For cyclic mechanisms the clearance effect leads to possibility of vital distortion of kinematical characteristics and increase the drives vibroactivity. Two cases are revealed. In the first case the clearance proves as a nonlinear element to which the opportunity of generating vibratory impact modes is connected. In the second case the reaction to a clearance manifests itself as a impulse in linear systems, and this dynamic effect is equivalent to impact arising from disruption of a continuity of the function $d\Pi / d\varphi$ . Some dynamic criterions that allow forecasting the excitation of vibratory impact regimes are offered.
In the linkages the clearance effect sometime softens due to the conjugate action between the contracting surfaces of hinge. In the given report the concept stated in works of the author in which essentially new model of a clearance was offered develops. The analysis of this model allows defining conditions of stability on the limited time intervals. and critical values of parameters of system at which the excitation close to impact takes place. This effect, named *pseudo-impact*, under certain conditions is transformed to the impact accompanying with disruption of a continuity of contact of a kinematics circuit [2,6].
***Vibrations in the multisection cyclic machines including branched and lattice systems.*** To perform the programmed motion of massive actuating devices of many machines and automatic transfer lines are carried out by long actuators, which are set in motion by multiply duplicated cyclic mechanisms.. In such

cases the drive consists of many sections forming vibratory systems with dynamic structural regularity [2,3,5,7].
The classic theory of regular oscillatory systems is based on the analysis of the chains or lattices consisting of masses and springs. The dynamic models of cyclic machines have more complex internal structure of each repeating module which formed not only a simple connected chain, but also branched and ring structured vibratory systems with nonlinearities and nonstationary dynamic constraints. Therefore, with reference to the given system the problem has specific features and needs separate consideration. In some cases regularity conditions are realized only approximately however quite often even small infringements can lead to spatial localization of vibrations on one or several modules of a drive. The effect of spatial localization of vibrations, which breaks in-phase movement of actuator and raises the vibroactivity of machines, should be investigated at the account of variability of parameters [7].

The peculiarity of excitation in multi section drives of machines with lattice structure has been analyzed where the actuator moves by means of multiply duplicated cyclic mechanisms. By means of such drives it is possible to reduce the clearance joint effect. The conditions wherein the synchronism of actuators movement is broken and grows vibroactivity of the drive are established. Ways of elimination of undesirable dynamic effects are offered.

***Nonlinear dissipation at polyharmonic excitation.*** According to approach based on the idea of the separation of motions the modes can be classed into "base" and "additional". It is taken into account, that usually the engineering determination of nonlinear position dissipative forces is based on the limited initial information, such as energy damping factor $\psi_0$ or logarithmic decrement $\vartheta_0$, which are received experimentally for type objects at monoharmonic vibratory modes
In works [2, 5] the essential decrease of an effective level of dissipation is established at additional movement of system. Physical preconditions for this effect are connected to occurrence of so-called minor loops of the hysteresis located inside a loop, appropriate to vibration with the basic frequency. The total area of minor loops of a hysteresis is proportional to work of forces of the resistance, carried out due to additional movement. Thus the effective area of a loop of the basic movement decreases.
In the analytical form the marked effect results in the following corrective:

$$\psi = \psi_0 \Phi(z)\,;\ \vartheta = \vartheta_0 \Phi(z)\,.$$

Here $\Phi(z) < 1$ is the factor of decrease of the equivalent dissipation characteristics, $z$ is the ratio of speed's amplitudes of the basic and additional movement. Functions $\Phi(z)$ for the widespread forms of loops of a hysteresis are given in [2]. However the functions $\Phi(z)$ depend only slightly on the form of the hysteresis loop. For typical cases the following approximating dependence may be applied:

$$\Phi(z) = z(0,4 + 0,5z)/(1 + 0,5z^2).$$

**Keywords**: cyclic machines, vibrations, dynamic stability, dissipation, clearances.

**References**

[1] Vulfson I.I. On the Vibrations of Systems with Time-depended Parameters.( in Russian). *Prikladnaya matematika i mehanika* , **2**, V.**33**, (1969),pp.331-337

[2] Vulfson. I.I. Vibrations of Machines with Cyclic Mechanisms. (in Russian).Leningrad: Mashinostroenie, 1990.

[3] Vulfson I. Vibroactivity of Branched and Ring Structured Mechanical Drives. New York, London: Hemisphere Publishing Corporation, 1989.

[4] Dresig H., Vul'fson I.I.: Dynamik der mechanismen, Springer, Wien, New York, 1989.

[5] Vulfson I.I. Nonlinear Resonant Oscillations of Drive at the Amplitude-modulation Frequency of High-frequency Excitation. *Journal of Machinery Manufacture and Reliability*. **6,** (2005), pp.13-17.

[6] Vulfson I. I., Preobrazhenskaya M.V. Study of the Oscillatory Modes Raised because Clearances in Cyclic Mechanisms, Connected with Common Actuator. *Journal of Machinery Manufacture and Reliability* **1**, (2008), pp.29-36.

[7] Vulfson I.I. Phase synchronism and spatial localization of cyclic machines' actuators with symmetric dynamic structure. *Journal of machinery manufacture and reliability,* **1**, V**. 40**, (2011), pp.10–15.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-1. ID-1.**

# НЕЛИНЕЙНЫЕ ДИНАМИЧЕСКИЕ ЭФФЕКТЫ В ЦИКЛОВЫХ МАШИНАХ

**И Вульфсон**

Санкт-Петербургский государственный университет технологии и дизайна, Большая Морская, 18, 191186, Санкт-Петербург, Россия
e-mail: vujo@rambler.ru

**АННОТАЦИЯ.** Снижение виброактивности является одной из важнейших проблем современной динамики машин. В докладе рассматриваются некоторые развивающиеся направления этой проблемы.

***Эффекты, обусловленные нелинейностью геометрических характеристик механизмов.*** Отличительная особенностью цикловых машин является преобразование движения и программное перемещение исполнительных органов согласно заданной нелинейной функции положения $\Pi(\varphi)$ ( $\varphi$ – координата входного звена). Цикловые механизмы, реализующие программное движение, одновременно являются источником возбуждения колебаний и объектом виброзащиты. В этих механизмах возможно асинхронное возбуждение, вызванное нарушением условий динамической устойчивости на конечных интервалах времени [1-4]. После линеаризации функций положения в окрестности программного движения и перехода к квазинормальным координатам исходная система дифференциальных уравнений приводится к виду

$$\ddot{\eta}_r + 2n_r(t)\dot{\eta}_r + p_r^2(t)\eta = W_r(t) \qquad (r = \overline{1,H})$$

Условие динамической устойчивости, соблюдение которого исключает возможность возрастании амплитуд возбуждаемых сопровождающих колебаний, может быть записано как

$$n_r + 0,5\dot{p}_r / p_r > 0 .$$

При нарушении этих условий зона нарастания амплитуд чередуется с зоной затухания, однако в отличие от параметрического резонанса амплитудный уровень остаётся ограниченным.

***Эффекты, обусловленные совместным влиянием нелинейной функции положения и зазорами*** Для цикловых механизмов зазоры могут вызвать существенные искажения заданных кинематических характеристик и повышение виброактивности привода. Зазор является нелинейным элементом, который может вызвать виброударные режимы в цикловых механизмах. Однако, при определённых условиях динамический эффект от зазора проявляется как импульс в линейной системе и эквивалентен нарушению непрерывности функции $d\Pi / d\varphi$ . В работе предложены критерии, которые позволяют исключить возможность возникновения виброударных режимов.

В рычажных механизмах иногда удаётся смягчить динамический эффект при пересопряжении в зазорах шарниров. В докладе получила дальнейшее развитие концепция автора, базирующаяся на предложенной динамической модели. Выявлен эффект, названный *псевдоударом,* при котором уровень возбуждаемых колебаний близок к удару даже при отсутствии нарушений непрерывного контакта [2,6].

***Колебания в многосекционных цикловых машинах, включающих разветвлённые и решётчатые системы.*** Для перемещения массивных исполнительных органов во многих машинах и автоматических линиях используются многократно дублированные цикловые механизмы. В таких случаях привод состоит из многих секций, образующих колебательные системы регулярной структуры. Классическая теория регулярных колебательных систем основана на анализе цепочек или решёток, состоящих из масс и пружин. Динамические модели приводов имеют более сложную внутреннюю структуру. При этом в отличие от классических цепочек каждый повторяющийся элемент представляет собой не точечную массу, а разветвлённо-кольцевые колебательные подсистемы. Необходимость отдельного исследования данной проблемы связана со специфическими особенностями цикловых механических систем, среди которых отметим нелинейность и нестационарность динамических связей, возможность нарушения кинематического контакта в зазорах и др. [2,3,5,7].

В некоторых случаях условия регулярности могут быть реализованы лишь приближённо. При этом даже незначительные отклонения (так называемые, «включения») могут привести к пространственной локализации колебаний в одном или нескольких модулях привода. Эффект пространственной локализации колебаний, который нарушает синфазное движение рабочих органов и повышает виброактивность системы, исследован при учёте переменности параметров колебательной системы.

***Нелинейная диссипация при полигармоническом возбуждении.*** Согласно подходу, основанному на разделении движений колебательные режимы могут быть отнесены к основным и дополнительным. Следует принять во внимание, что обычно определение диссипативных сил на инженерном уровне основано на ограниченной информации, таких как коэффициент рассеяния $\psi_0$ или логарифмический декремент $\vartheta_0$, которые получены экспериментально для типовых объектов при моногармонических колебаниях. В работах [2,5] показано существенное уменьшение эффективного уровня диссипации при неодночастотных колебаниях. Физической предпосылкой этого эффекта является появление так называемых частных петель гистерезиса, расположенных внутри петли, соответствующей основному режиму. При этом эффективная площадь основной петли сокращается.

В аналитической форме отмеченный эффект приводит к следующим коррективам:

$$\psi = \psi_0 \Phi(z), \;\; \vartheta = \vartheta_0 \Phi(z).$$

Здесь $\Phi(z) < 1$ – коэффициент уменьшения соответствующей диссипативной характеристики, $z$ – отношение амплитудных значений скоростей при основном и дополнительном движении.

Функции $\Phi(z)$ для типовых форм петель гистерезиса приведены в [2]. Однако эта функция слабо зависит от формы петли, что позволяет воспользоваться следующей аппроксимирующей зависимостью.

$$\Phi(z) = z(0,4 + 0,5z)/(1 + 0,5z^2).$$

**Ключевые слова:** цикловая машина, колебания, динамическая устойчивость, диссипация, зазоры.

**Литература**

[1] Вульфсон И.И. О колебаниях систем с параметрами, зависящими от времени. *Прикладная математика и механика*, **2**, т.33, (1969), с.331-337.

[2] Вульфсон И.И. Колебания машин с механизмами циклового действия. Ленинград: Машиностроение, 1990.

[3] Вульфсон И.И. Виброактивность приводов машин разветвлённой и кольцевой структуры. Ленинград: Машиностроение, 1986.

[4] Dresig H., Vul'fson I.I.: Dynamik der Mechanismen, Springer, Wien, New York, 1989

[5] Vulfson I.I.. Nonlinear Resonant Oscillations of Drive at the Amplitude-modulation Frequency of High-frequency Excitation. *Journal of Machinery Manufacture and Reliability*. **6,** (2005), pp.13-17.

[6] Vulfson I. I., Preobrazhenskaya M.V. Study of the Oscillatory Modes Raised because Clearances in Cyclic Mechanisms, Connected with Common Actuator. *Journal of MachineryManufacture and Reliability* **1**, (2008), pp.29-36.

[7] Вульфсон И.И. Синфазность и пространственная локализация колебаний рабочих органов с симметричной динамической структурой. *Проблемы машиностроения и надёжности машин*. **1**, (2011), с. 12-18.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**Cl-2. ID-9.**

# UNEXPECTED FEATURE OF NEW OSCILLATING HOMOGENEOUS REACTIONS: ALKYNES CARBONYLATION IN Pd COMPLEXES SOLUTIONS

**Sergey N. Gorodsky**

Department of Basic Organic Synthesis
Moscow State University of Fine Chemical Technologies (MITHT)
86 Vernadsky Avenue, 119571 Moscow, Russia
e-mail: Gorodsky@yandex.ru

**ABSTRACT.** The oscillating reactions were discovered in many chemical and biological systems. The most part of oscillatory homogeneous reactions investigated in chemistry are processes of organic substrate oxidation by strong oxidants [1].

For several years in MITHT investigations of oscillating modes of the oxidative carbonylation reactions in homogeneous catalysis by metal complex in alcohol solutions of palladium complexes with various organic substrates were conducted. We have showed that the phenomenon of oscillations can be observed for reactions with participation of different alkynes, and also in different catalytic systems, for example, in system $PdBr_2$ - LiBr, in which the occurrence of chaotic oscillations was demonstrated. The modes of developed oscillations in reactions involving phenyl- and methyl acetylene, 2-methyl-3-butin-2-ol, 2-propin-1-ol in the systems $PdI_2$-KI-CO-$O_2$-$CH_3OH$; $PdBr_2$-LiBr-CO-$O_2$-$CH_3OH$; $PdBr_2$-LiBr-CO-$O_2$-$(CH_3)_2CO$-$H_2O$ have been found [2-5]. Recently interesting modes of oscillations in carbonylation reactions of 1-nonin, 1-decin and 1-dodecin in the system $PdI_2$-KI-CO-$O_2$-$CH_3OH$ were found with a number of features depending on the nature of the substrate.

The study of mechanisms of some other processes in organometallic catalysis allows us to expect occurrence of self-oscillations and other critical phenomena. The searching for such phenomena is important for development of the theory of catalytic reaction mechanisms. The presence of oscillations, besides, is the powerful tool of the mechanistic hypotheses discrimination.

**Keywords**: oscillations, homogeneous reactions, carbonylation.

## References

[1] R. J. Field, M. Burger, Oscillations and Travelling Waves in Chemical Systems, *Wiley, New York*, (1985).
[2] A. V. Malashkevich, L. G. Bruk, O. N. Temkin, *J. Phys. Chem.*, A, **101** (1997) 51, 9825-9827.
[3] S. N Gorodsky, A. N. Zakharov, A. V. Kulik, L. G. Bruk, O. N. Temkin, *Kinetics and Catalysis*, **42** (2001) 2, 285-293.
[4] S. N Gorodsky, E. N. Kalenova, L. G. Bruk, O. N. Temkin, *Russian Chemical Bulletin, Ser. Chem.*, **7** (2003) 1452-1461.
[5] Gorodsky S. N., Bruk, L.G., Istomina, A.E., Kurdiukov, A.V., Temkin, O.N., *Topics in Catalysis*, **2009**, 52, 6-7, 557-562.

**Acknowledgements:** This work was supported by RFBR (grants № 05-03-33151 and № 09-03-01072). Thanks to ACS and RCS for membership.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**Cl-2. ID-9.**

# НЕОЖИДАННОЕ ПОВЕДЕНИЕ НОВЫХ КОЛЕБАТЕЛЬНЫХ ГОМОГЕННЫХ РЕАКЦИЙ: КАРБОНИЛИРОВАНИЕ АЛКИНОВ В РАСТВОРАХ КОМПЛЕКСОВ ПАЛЛАДИЯ

**Сергей Н. Городский**

Кафедра основного органического синтеза, Московский государственный университет тонких химических технологий имени М. В. Ломоносова (МИТХТ)
Россия, Москва, проспект Вернадского, 86
e-mail: Gorodsky@yandex.ru

**РЕФЕРАТ.** Колебательные реакции обнаружены во многих химических и биологических системах. Большая часть обнаруженных в химии колебательных гомогенных реакций являются процессами окисления органических субстратов сильными окислителями [1].
В течение ряда лет в МИТХТ проводились исследования колебательных режимов реакций окислительного карбонилирования различных субстратов в условиях гомогенного катализа комплексами палладия. Нами было показано, что феномен колебаний может наблюдаться в реакциях с участием различных алкинов, а также в различных каталитических системах, например, в системе $PdBr_2$ - LiBr, в которой было продемонстрировано появление хаотических колебаний. Были найдены колебательные режимы в реакциях с участием фенил- и метилацетилена, 2-метил-3-бутин-2-ола, 2-пропин-1-ола, в системах $PdI_2$-KI-CO-$O_2$-$CH_3OH$; $PdBr_2$-LiBr-CO-$O_2$-$CH_3OH$ и $PdBr_2$-LiBr-CO-$O_2$-$(CH_3)_2CO$-$H_2O$ [2-5]. Недавно были найдены интересные режимы колебаний в реакциях карбонилирования 1-нонина, 1-децина и 1-додецина в системе $PdI_2$-KI-CO-$O_2$-$CH_3OH$, отличающиеся друг от друга в зависимости от природы субстрата.
Исследование механизмов некоторых других процессов металлоорганического катализа позволяет нам ожидать появления в них автоколебаний и других критических явлений. Поиск таких феноменов важен для разработки теории механизмов каталитических реакций. Наличие в системе колебаний, кроме того, является мощным инструментом дискриминации гипотез, выдвинутых относительно механизма этой реакции.
**Ключевые слова**: колебания, гомогенные реакции, карбонилирование.

## Литература

[1] Р. Д. Филд, М. Бургер, Колебания и бегущие волны в химических системах, *Вили, Нью-Йорк*, (1985).
[2] А. В. Малашкевич, Л. Г. Брук, О. Н. Темкин, Ж. *Физ. Хим.*, А, **101** (1997) 51, 9825-9827.
[3] С. Н. Городский, А. Н. Захаров, А. В. Кулик, Л. Г. Брук, О. Н. Темкин, Кинетика и катализ, **42** (2001) 2, 285-293.
[4] С. Н. Городский, Е. Н. Каленова, Л. Г. Брук, О. Н. Темкин, Известия Академии наук России, сер. химич., **7** (2003) 1452-1461.
[5] С. Н. Городский, Л. Г. Брук, А. Е. Истомина, А. В. Курдюков, О. Н, Темкин, Вопросы катализа, **2009**, 52, 6-7, 557-562.

**Благодарности:** эта работа поддерживалась РФФИ (гранты № 05-03-33151 и № 09-03-01072). Мои благодарности Американскому и Российскому химическим обществам за членство в этих организациях.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-14 ID-8.**

# CHAOTIC OCCURRENCE OF THE RECORDED EARTHQUAKE MAGNITUDES IN SERBIA

**S. Kostić[1], N. Vasović[2]**

[1] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia, e-mail: srdjan.kostic@rgf.rs
[2] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia, e-mail: kisi@pharmacy.bg.ac.rs

**ABSTRACT.** One of the most challenging tasks in modern seismology concerns upscaling, i.e. resolving evolution law at a large scale, based on the facts obtained at a lower scale. However, in most cases this task is nearly impossible to solve, mostly because of the fact that earthquakes are extremely complex phenomena, so their mechanism cannot be easily reproduced by a simple laboratory or a numerical model. The main reason for this is the lack of a dominating scale: fluctuations of many sizes seem to be important and cannot be neglected, even when interested only in large-scale events [1]. There are two types of features that appear on a small and large scales – scale invariant power law statistics and chaotic behavior. It was already shown by Burridge and Knopoff [2], that their spring-block model follows the Gutenberg-Richter and Omori-Utsu law. In our previous paper, we showed that the parameters *a* and *b* of Gutenberg-Richter law, obtained for this model, are in a range of parameter values which characterize seismicity in Serbia. Moreover, the frequency of aftershocks in this model could be described by Omori-Utsu law, with a satisfying accuracy [3]. In this paper, we demonstrate that the earthquake magnitude frequency per month, recorded between 1964 and 2011 in Serbia, evolved as a chaotic process. Initial data consisted of 763 recorded earthquakes, according to the data of Northern California Earthquake Data Center. For the numerical calculation, we used the open-source program package, developed in [4], with our initial data. The applied algorithm consisted of several steps: firstly, the mutual information method, initially suggested by Fraser and Swinney [5], was used to determine the appropriate value of embedding delay. The calculation was done using the program mutual.exe. Secondly, we applied the technique of Kennel et al. [6] to determine the proper value of embedding dimension, which was done by using the program fnn.exe. Then we applied determinism test, developed by Kantz and Schreiber [7], in order to show that the observed system originates from a deterministic, not a stochastic process. Subsequently, we conducted the stationarity test, suggested by Schreiber [8], so as to ensure that the recorded data originate from system whose parameters are constant during the measurements. Finally, we calculated the maximal Lyapunov number, applying the method proposed by Wolf et al. [9]. Also, even though it was not our primary goal, following Mohammadi and Noorzad [10], we calculated the time period $t$ on the basis of the obtained Lyapunov exponent, during which the prediction of the maximum earthquake magnitude is possible. Such a complex earthquake behavior could be a consequence of heterogeneous geological structures in which fault zones might have fluctuations in strength due to pore pressure variation. Although other more detailed studies and further work will be needed, we believe that our results are important for several reasons. Firstly, they reveal some characteristics of the general physics that is behind the generation of the seismic sequence in terms of a chaotic process. Secondly, we believe that they represent clear example that the earthquakes are predictable phenomena on some occasions. Finally, the strategy applied here can be replicated for other seismically active areas, in order to better understand the chaotic behavior of earthquakes in general.

**Keywords**: earthquake magnitudes, phase space reconstruction, embedding delay, embedding dimension, maximal Lyapunov number

**References**

[2] Sornette, D.V., Pisarenko, V.F. Fractal Plate Tectonics, *Geophysical research Letters*, **30** (2003) pp. 1-4.
[3] Burridge, R. Knopoff, L., Model and theoretical seismicity, *Bulletin of the Seismological Society of America*, **57** (1967), pp. 341-371.
[4] Kostic, S., Vasovic, N. Spring-block model of scaling laws for seismicity in Serbia, *TEHNIKA – mining, geology and metallurgy*, **62-1**(2012), pp. 61-66.
[5] Kodba, S. Perc, M., Marhl, M., Detecting chaos from a time series, *European Journal of Physics*, **26** (2005), pp. 205-215.
[6] Fraser, A., Swinney, H. Independent coordinates for strange attractors from mutual information. *Physical Review A*, **33** (1986), pp. 1134-1140.
[7] Kennel, M., Brown, R., Abarbanel, H. Determining embedding dimension for phase-space reconstruction using a geometrical construction, *Physical Review A*, **45** (1992), pp. 3403-3411.
[8] Kantz, H., Schreiber, T. *Nonlinear time series analysis*, second edition, Cambridge University Press, Cambridge, (2004).
[9] Schreiber, T, Detecting and analyzing nonstationarity in a time series with nonlinear cross-predictions, *Physical Review Letters*, **78** (1997), pp. 843-846.
[10] Wolf, A., Swift, J., Swinney, H., Vastano, J., Determining Lyapunov exponents from a time series, *Physica 16D* (1985), pp. 285-317.
[11] Mohammadi, S., Noorzad, A. Detecting chaos in earthquake time series for seismic hazard prediction, *Proceedings of the 3rd IASME/WSEAS International Conference on Geology and Seismology* ISBN: 978-960-474-058-1 (2009), pp. 17-20.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-15. ID-7a.**

# A SIMPLE MODEL OF EARTHQUAKE NUCLEATION WITH TIME-DELAY

**S. Kostić[1], I. Franović[2], K. Todorović[3], N. Vasović[4]**

[1] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia, e-mail: srdjan.kostic@rgf.rs
[2] University of Belgrade Faculty of Physics, Studentski trg 12, Belgrade, Serbia, e-mail: igor.franovic@gmail.com
[3] University of Belgrade Faculty of Pharmacy, Vojvode Stepe 450, Belgrade, Serbia, e-mail: kisi@pharmacy.bg.ac.rs
[4] University of Belgrade Faculty of Mining and Geology, Djusina 7, Belgrade, Serbia, e-mail: kisi@pharmacy.bg.ac.rs

**ABSTRACT.** The problem of earthquake nucleation is of great interest to seismologists, because of its fundamental importance for the physics of earthquakes as well as because of the immediate practical implications. Understanding the development and initial stages of an earthquake rupture is a major goal of earthquake science. Some researchers suggest that the nucleation process, specifically the size of the nucleation zone, is related to the ultimate size of the resulting earthquake, in that the larger the earthquake, the larger its nucleation zone [1-3]. Others support the view that the size of the nucleation zone is unrelated to the final magnitude of an earthquake [4-6]. However, the influence of the nucleation mechanism on the final impact of earthquake certainly exists, so the modeling of this phenomenon could lead to new insights on the nature of earthquakes. The purpose of this work is to identify the possible deterministic chaotic mechanism by which the earthquakes evolve, in contrast to the fact accepted by some researchers that they represent an example of a random process [7,8]. In this paper we use the Burridge-Knopoff model [9], which is today recognized as a common model for earthquake nucleation mechanism. It consists of one block of a certain rock type, connected through harmonic spring to a moving plate and driven along the rough surface, which causes the whole system to move in a stick-slip fashion. We introduce the effect of time delay in this model with Dieterich-Ruina's rate and state dependent friction law. The time lag describes the memory effect that commonly appears during the stick-slip motion of the block along the rough surface. Standard local bifurcation analysis of delay-differential equations is performed, indicating the dynamical change of the system state from stable equilibrium through oscillatory behavior (first Hopf bifurcation) to torus (second Hopf bifurcation) and eventually to chaos. The results are confirmed by using the software package DDE-BIFTOOL [10,11]. We have obtained exact parametric representation of the Hopf bifurcation curves in the general case of the introduced time-lag. These bifurcation relations provide, for a given value of the parameters, the time-lag that will lead to destabilization of the stationary solution. The corresponding Hopf bifurcation is of the sub-or super-critical type which depends on the sign of the derivatives along the bifurcation curves. These dynamical changes are confirmed by the Fourier power spectra. The obtained results are encouraging, since they strongly support the opinion, besides the Gutenberg-Richter and Omori-Utsu scaling laws, that mechanism of earthquake nucleation could be represented by the spring-block model, which also exhibits chaotic behavior. We believe that this new approach, concerning the involvement of time-delay in the equations for motion of the Burridge-Knopoff model that we used in this work, will help in the future to improve our knowledge of the earthquake physics.

**Keywords**: spring-block model, time-lag, Hopf bifurcation, quasiperiodicity, seismological correlation

**References**

[1] Ohnaka, M, Critical size of the nucleation zone of earthquake rupture interred from immediate foreshock activity, *Journal of Physics of the Earth*, **41** (1993), pp. 45– 56.

[2] Ellsworth, W. L., Beroza, G.C., Seismic evidence for an earthquake nucleation phase, *Science*, **268** (1995), pp. 851– 855.

[3] Dodge, D. A., Beroza, G.C. Ellsworth, W.L., Detailed observations of California foreshock sequences: implications for the earthquake initiation process, *Journal of Geophysical Research*, **101** (1996), pp. 22,371– 22,392.

[4] Abercrombie, R., Mori, J., Local observations of the onset of a large earthquake: 28 June 1992 Landers, California, *Bulletin of the Seismological Society of America*., **84** (1994), pp. 725–734.

[5] Mori, J., H. Kanamori, H., Initial rupture of earthquakes in the 1995 Ridgecrest, California sequence, *Geophysical Research Letters*, **23** (1996), pp. 2340 – 2437.

[6] Kilb, D., Gomberg, J. The initial subevent of the 1994 Northridge, California, earthquake: Is earthquake size predictable? *Journal of seismology*, **3** (1999), pp. 409– 420.

[7] Wang, J., Kuo, C. On the frequency distribution of interoccurrence times of earthquakes. *Journal of seismology*, **2** (1998), pp. 4351-358.

[8] Akkaya, A.D., Yücemen, M.S. Stochastic modeling of earthquake occurrences and estimation of seismic hazard: a random field approach, *Probabilistic Engineering Mechanics*, **17** (2002), pp. 1–13

[9] Burridge, R., Knopoff, L. Model and theoretical seismicity, *Bulletin of the Seismological Society of America*, **57** (1967), pp. 341-371.

[10] Engelborghs, K., Lemaire, V., Belair, J., Roose, D. Numerical bifurcation analysis of delay differential equations arising from physiology modeling, *Journal of Mathematical Biology,* **42 (**2001), pp. 361-385.

[11] Haegeman, B., Engelborghs, K., Roose, D., Pieroux, D. Erneux, T. 2002. Stability and rupture of bifurcation bridges in semiconductor lasers subject to optical feedback, *Physical Review E,* **66** (2002), pp. 046216-1-11.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-15 ID-7.**

# REGRESION AND WEIBULL ANALYSIS OF CROPPING PRACTICES IN CHANGING CLIMATE

**V. Dragičević, Ž. Videnović, B.Kresović, M. Simić, Z. Dumanović, I. Spasojević**

[1] Maize Research Institute "Zemun Polje"
Slobodana Bajića 1, 11185 Zemun Polje, Serbia
e-mail: vdragicevic@mrizp.rs

**ABSTRACT.** Maize is one of the most important crops in the world. Successful production depends primarily on meteorological, but also on other environmental factors. Variations in cropping practices, which include different measures, enable overcoming alterations of environmental factors, especially during periods of extreme weather conditions for plant growth. The aim of this study was to define the most efficient tillage system (no-till, reduced or conventional tillage) and fertilization regime (Ø, 330 or 660 kg $ha^{-1}$ N:P:K) for high maize yield (hybrid ZP 704) under rainfed and irrigation conditions, according to results of long-term maize cropping experiment (1991-2010) in conditions of climate change. Among meteorological factors, we focused on sum of precipitation and growth degree days (GDD - sum of effective temperatures) during maize vegetation.

The observed 20 year period characterizes increasing trend of both, the average temperatures and sum of precipitation, as well as the extreme meteorological conditions for maize growth. In our previous experiments in less extreme weather conditions, we recorded lower differences between irrigated and non-irrigated (rainfed) maize yield [1]. In this research, the positive correlation between grain yield and precipitation was noted only under rainfed cropping, while the increasing level of fertilization and tillage intensity (conventional tillage) had positive effect on grain yield only under irrigation. Polynomial regression analysis ascertained the highest yields at 400 mm of precipitation for all fertilization and tillage regimes, as well as 1550 and 1850 GDD under both, rainfed and irrigation conditions. It was particularly present in treatments with highest inputs (conventional tillage with 660 kg $ha^{-1}$ of N:P:K fertilizers).

Meanwhile, different regimes of fertilization and tillage intensity didn't fit the alterations in grain yield, according to present changes in meteorological conditions, what could give the possibility of grain yield prediction, based on results of long term experiment. Opposite to general inference obtained from regression analysis, Weibull analysis underlined that fertilization with 660 kg $ha^{-1}$ N:P:K could achieve the best results only under the irrigation (7.14 t $ha^{-1}$ of grain yield with reliability of 99%), while the higher inputs were not rational in rainfed conditions. The conventional tillage had its validation under both, rainfed and irrigation conditions. The highest β coefficient values in treatment with 330 kg $ha^{-1}$ N:P:K indicate this fertilization level as the most rational input, especially in combination with conventional tillage, where reliability of achieved grain yield endeavours genetic potential of hybrid under irrigation (9,96 t $ha^{-1}$ of grain yield with reliability of 99%). In rainfed cropping, same fertilization regime diminished differences between reduced and conventional tillage, achieving the maximal yield of 6-7 t $ha^{-1}$. From this point of view, advantage of Weibull analysis reflects through yield prediction, with high reliability [2,3], including variations of meteorological factors, as well as tested cropping practices.

The analysis underlined 330 kg $ha^{-1}$ N:P:K as the optimal fertilization regime, with conventional tillage under irrigation, as well as conventional or reduced tillage in rainfed cropping. The negative environmental factors (alterations of precipitation and temperature) were diminished under irrigation, where high inputs resulted in high yields. But, considering the fact that the rainfed cropping is the most abundant cropping practice in the

world, moderate inputs (330 kg $ha^{-1}$ N:P:K) obtain stability of the maize yield, irrespective of lower yields. It could be concluded that high inputs in fertilizers and tillage energy didn't necessarily resulted in yield increase, particularly under rainfed conditions. On the other hand, high yields attained with high fertilizer and tillage inputs under irrigation, could be reasonable only in seasons with low precipitation (draught).

**Keywords**: maize cropping, fertilization, tillage, Weibull analysis, regression analysis.

**References**

[12] Videnović Ž, Vasić G, Kolčar F**,** Effect of fertilizers and soil tillage on corn yield under dry farming and irrigated conditions**,** *Soil and Tillage Research***, 8 (**1986**)**, pp. 113–118**.**
[13] Kress LW, Miller JE, Impact of ozone on field-corn yield, *Canadian Journal of Botany*, **63** (1985), pp. 2408-2415.
[14] LW Kress, Miller JE, Impact of ozone on grain sorghum yield, *Water, Air, & Soil Pollution*, **25** (1985), pp. 377-390.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-9. ID-74.**

# NONLINEAR OSCILLATIORY FLOWS WITH DIFFERENT SYMMETRY PROPERTIES IN TWO-LAYER SYSTEMS

**Ilya B. Simanovskii**
Department of Mathematics, Technion - Israel
Institute of Technology,\\ 32000 Haifa, Israel
e-mail: cesima@tx.technion.ac.il

Interfacial convection in systems with interfaces has been a subject of an extensive investigation at the past few decades (for a review, see [1], [2]).

There are two basic physical phenomena that produce convective instability in systems with an interface: buoyancy and thermocapillary effect. When heating is from below, the buoyancy instability generates Rayleigh - Bénard convection [3], while the thermocapillary effect is the origin of Marangoni - Bénard convection [4], [1]. Buoyancy (a volume effect) is more important for relatively thick layers, while the thermocapillarity (an interfacial effect) plays the dominant role in the case of thin layers or under microgravity conditions. The simultaneous action of both mechanisms of instability is the most typical.

It is known that the stability problem for the mechanical equilibrium in a system with an interface is not self-adjoint (see, e.g., [1], [5]), thus an oscillatory instability is possible. The mechanism of oscillations, which takes place without interfacial deformations due to the hydrodynamic and thermal interaction between convective flows on both sides of the interface, was found by Gershuni and Zhukhovitsky [6] in the case of transformer oil - formic acid system.

An oscillatory instability of the mechanical equilibrium can be caused by the joint action of buoyancy and thermocapillary effect in a two-layer system heated from below. This phenomenon was first discovered in [7], [1], [8]. Oscillations just above the instability threshold have been observed in experiments of Degen *et. al.* (see [9]).

In the present work, we investigate the nonlinear regimes of convection in the 47v2 silicone oil - water system. The system is bounded from above and from below by two isothermal rigid plates kept at constant different temperatures (heating is from below). It is assumed that the interfacial tension decreases linearly with the increasing of the temperature.

The boundary value problem is integrated in time with some initial conditions for the stream function and the temperature by means of a finite-difference method. Equations and boundary conditions are approximated on a uniform mesh using a second order approximation for the spatial coordinates. The nonlinear equations are solved using an explicit scheme on a rectangular uniform mesh.

It is shown that under the joint action action of buoyancy and thermocapillary effect, the development of oscillatory instability leads to specific types of nonlinear oscillations with different symmetry properties. Transitions between the flows with various spatial structures are studied. New consequence of bifurcations has been found. It is shown that the period of oscillations changes in a non-monotonic way for symmetric and asymmetric oscillations. It is found that in the course of the evolution of asymmetric oscillations, the violation of the symmetry property decreases and the symmetry is restored. With an increase of the Grashof number values, the oscillatory flow becomes unstable and a steady convective flow develops in the system. The region of nonlinear convective oscillations is observed in a finite interval of the Grashof number values bounded from below and from above.

**Keywords**: interface, instabilities, oscillations.

## References

[1] Simanovskii, I. B., & Nepomnyashchy, A. A. 1993 *Convective Instabilities in Systems with Interface*, Gordon and Breach, London.

[2] Nepomnyashchy, A. A., Simanovskii, I. B. & Legros, J. C. 2012 *Interfacial Convection in Multilayer Systems*, Second Edition, Springer, New York.

[3] Gershuni, G. Z. & Zhukhovitsky, E. M. 1976 *Convective stability of incompressible fluid*, Keter, Jerusalem.

[4] Pearson J. R. A. On convection cells induced by surface tension, *J. Fluid Mech.*, **4** (1960) 401.

[5] Renardy Y. Y., Pattern formation for oscillatory bulk-mode competition in a two-layer Bénard problem, *ZAMP* **47** (1996) 567.

[6] Gershuni G. Z., Zhukhovitsky E. M. On monotonic and oscillatoty instability of a two-layer immiscible fluids system heated from below, *Sov. Phys. Dokl.* **27** (1982) 531.

[7] Nepomnyashchy, A. A. & Simanovskii, I. B. Thermocapillary and thermogravitational convection in a two-layer system with a distorted interface, *Fluid Dyn.* **19** (1984) 494.

[8] Juel, A., Burgess, J. M., McCormick, W. D., Swift, J. B. & Swinney, H. L. Surface-tension-driven convection patterns in two liquid layers, *Physica D* **143** (2000) 169.

[9] Degen, M. M., Colovas, P. W., Andereck C. D. Time-dependent patterns in the two-layer Rayleigh-Bénard system, *Phys. Rev. E* **57** (1998) 6647.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-4. ID- 27.**

# STABILITY INVESTIGATION OF A RIGID BODY AND A SYSTEM OF RIGID BODIES UNDER RAPID VIBRATIONS

**O. Kholostova**

Faculty of Applied Mathematics and Physics,
Moscow Aviation Institute (National Research University),
Volokolamskoe Shosse, 4, GSP-3, A-80, 125993, Moscow, Russia
e-mail: kholostova_o@mail.ru

The report presents the results of studying the stability of relative equilibriums and periodic motions of a rigid body and a system of two rigid bodies with an oscillating suspension point. In the first part the motion of a rigid body is considered in assumption that one of the points of the body accomplishes vertical harmonic oscillations of a high frequency and small amplitude. Using an approximate autonomous system of differential equations of motion [1] the existence, bifurcation, and stability of the "lateral" relative equilibriums for which the body mass center and the suspension point do not lie in the same vertical are analyzed. All “lateral” relative equilibriums of the body are found to be unstable in the existence regions both for special and general cases of the body mass geometry [2].

In the second part the investigation of a system consisting of two thin uniform rods is carried out. The system suspension point accomplishes the vertical or horizontal harmonic oscillations. The linear stability of four relative equilibriums of the double pendulum in the vertical is analyzed. In the case of vertical suspension point vibrations of a sufficiently high frequency and a small amplitude, the reversed pendulum and both cases of "folded" pendulum are shown can be stable [3], whereas in the case of horizontal suspension point rapid vibrations only the “hanging” pendulum can be stable.

If two rods of the pendulum are identical, the problem of nonlinear stability of relative equilibriums is solved for the case of rapid horizontal suspension point vibrations and the case of vertical suspension point vibrations of arbitrary amplitude and frequency. In the latter case the stability regions for both variants of "folded" pendulums are shown to exist not only for rapid suspension point vibrations, but also for moderate oscillation frequencies. In the plane of parameters a countable set of linear stability regions is determined for the lower equilibrium position [4].

The problem of existence, bifurcation, and nonlinear stability of the "lateral" high-frequency periodic motions of the pendulum consisting of two identical rods is also solved. In the case of rapid vertical suspension point vibrations all existing "lateral" periodic motions are shown to be unstable [3], and in the case of rapid horizontal vibrations there are both unstable and stable periodic motions.

**Keywords**: rigid body, double pendulum, stability, rapid vibrations, vibration moment, KAM-theory

## References

[15] Markeev A.P., Equations of the Approximate Theory of Motion of a Rigid Body with a Vibrating Suspension Point, *Applied Mathematics and Mechanics*, **75** (2011), No 2, pp. 193-203. (In Russian)
[16] Kholostova O.V., Stability of Relative Equilibriums of a Rigid Body with a Vibrating Suspension Point, *Bulletin of Peoples' Friendship University. Ser. Mathematics. Informatics. Physics*, (2011), No 2, pp. 111-122. (In Russian)
[17] Kholostova O.V., Stability of Relative Equilibriums of a Double Pendulum with Vibrating Suspension Point, *J. of Russian Academy of Sciences. Mechanics of Solids*, (2011), No 4, pp. 18-30. (In Russian)
[18] Kholostova O.V. Motions of a Double Pendulum with a Vibrating Suspension Point, *J. of Russian Academy of Sciences. Mechanics of Solids*, (2009), No 2, pp. 25-40. (In Russian)

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-5. ID- 32.**

# THE NEW FORM OF FORCE FUNCTION OF TWO FINITE BODIES IN TERMS OF MODIFIED DELAUNAY'S AND ANDOYER'S ANGLE VARIABLES

**A. Zlenko**

Department of high matematics of Moscow automobile and road construction state technical university (MADI), Leningradskii avenue 64, Moscow, 125319, Russia,
e – mail : zalaf121@mail.ru

**ABSTRACT.** Knowledge of force function of mutual gravitation of celestial bodies is very important for research of their motion. In work [1] the full force function of two finite bodies is written in the terms of Delaunay's and Andoyer's angle variables and ordinary Kepler's elements. It can be written also in canonical variables of Delaunay and Andoyer. When we are solving any resonant problem or are doing the elimination of short periodic perturbations with the help of averaging the equations of motion we should to find the appropriate terms. This is tiresome work because abovementioned presentation of force function has some disadvantages: 1. for one fixed set of coefficients of angle variables we need seek members in what they are; 2. these members have the functions *cosine* and *sin* with arguments, consisting of angle variables; 3. the multipliers before of *cosine* and *sin* with equal set of arguments are not summarized. This presentation is improved in modified angle variables of Delaunay and Andoyer: 1. one fixed set of coefficients of angle variables is only in one function *cosine* and *sin*; 2. all multipliers, consisting of Kepler's elements, before *cosine* and *sin* are summarized; 3. instead of eight angle variables of Delaunay and Andoyer we have only seven modified angle variables of Delaunay and Andoyer; 4. we can easily to take the terms of needed accuracy.

The new form of force function is more convenient for elimination of the short periodic perturbations and for solving of evolutional and resonant problems in celestial mechanics and astrodynamics.

**Keywords:** gravitation, celestial bodies, force function, Delaunay's and Andoyer's variables.

## REFERENCES

[1] Šidlichovský, M., The elimination of short periodic perturbations in the problem of two finite bodies. *Bull. Astron. Inst. Checosl.,* **32**, no. 3 (1981), pp. 159-167.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-6. ID-35.**

# SOME DYNAMICAL PROBLEMS FOR A PARTICLE TETHERED TO A RIGID BODY

**Alexander V. Rodnikov**[1]

[1] Bauman Moscow State Technical University, 2-nd Baumanskaya 5, Moscow,105005, Russia
e-mail: avrodnikov@yandex.ru

**ABSTRACT.** Some space missions require intermediate bases in their way into deep space. A space station placed near a small planet may be useful for this goal. To exclude running away, one can tether the station to the asteroid surface by a cable. Evidently, this idea produces a set of technical problems because of the asteroid complex spin motion. Nevertheless, restricting to a case of a dynamically symmetric small planet, one can tether the station to poles that are intersection points of the asteroid surface and of the axis of dynamical symmetry. In this way we can formulate a new dynamical problem that is 'To study a particle motion in a gravitational field of a precessing rigid body if the particle is placed on the cable(s) with end(s) fixed in the rigid body pole(s)'. Note that the particle can be tied by one tether with one pole or by two tethers with both poles or can coast along one cable with both ends fixed in the poles. In the last case the cable is called 'the leier'. We claim that if the body gravitational field is invariant with respect to rotations about the axis of dynamical symmetry then some statements are fulfilled for all mentioned methods of tethering. So, the particle on the tether(s) or on the leier can be immovable with respect to axes of precession and of dynamical symmetry only in some points of the plane composed by the mentioned axes and of the plane crossing the body mass center orthogonally to the precession axis. (Note that the libration points of Beletsky's Generalized Restricted Circular Problem of Three Bodies [1] are examples of such equilibria if the rigid body can be replaced with a dumb-bell). Moreover, the considered problems are integrable for zero angle of nutation and for some particular motions if the nutation angle is right. Also we deduce condition of the particle equilibria stability and analize these conditions for the particle tethered by two cables or by the leier. In this paper results from [2,3] are developed.
This work is supported by RFBR (grant #10-01-00406a)

**Keywords**: rigid body, asteroid, space tether system, problem of three bodies, regular precession

## References

[19] Beletsky V.V, Generalized Restricted Circular Three-Body Problem as a Model for Dynamics of Binary Asteroid, *Cosmic Research*, **45** (2007), 6, pp. 408-416
[20] Rodnikov A.V, On a particle motion along the leier fixed in a precessing rigid body, *Rus. J. Nonlin. Dyn.*, **7** (2011), 2, pp. 295–311 (Russian)
[21] Rodnikov A.V, On coplanar equilibria of a space station on the cable fixed in an asteroid, *Rus. J. Nonlin. Dyn.*, **8** (2012), 2, pp. 309–322 (Russian)

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-6. ID-35.**

# НЕКОТОРЫЕ ЗАДАЧИ ДИНАМИКИ МАТЕРИАЛЬНОЙ ТОЧКИ, СОЕДИНЕННОЙ ТРОСОМ С ТВЕРДЫМ ТЕЛОМ

**А. В. Родников**[1]

[1]Московский государственный технический университет им. Н.Э.Баумана, Россия, 105005, Москва, 2-ая Бауманская, 5
e-mail: avrodnikov@yandex.ru

**Тезисы.** Для освоения дальнего космоса могут оказаться полезными базовые космичес-кие станции, помещенные вблизи малых планет. Чтобы сохранить такую станцию в окре-стности астероида, может оказаться удобным “привязать” ее к поверхности малой плане-ты, однако, при этом возникает множество технических проблем, связанных со сложнос-тью движения астероидов вокруг центра масс. Тем не менее, если ограничиться случаем динамически симметричного астероида, можно показать, что станция может быть соединена тросом с полюсами, т.е. с точками пересечения оси динамической симметрии с поверхностью малой планеты. В связи с этим может быть сформулирована новая про-блема динамики, а именно “Изучение движения материальной точки, присоединенной тросом (или тросами) к полюсу (или полюсам) прецессирующего твердого тела в грави-тационном поле этого тела”. При этом материальная точка может быть присоединена одним тросом к одному полюсу, двумя тросами к двум полюсам или одним тросом (который в этой ситуации может быть назван “леером”), оба конца которого помещены в полюсы твердого тела. В последнем случае можно допустить, что материальная точка способна перемещаться вдоль леера. В настоящей работе показывается, что при любом способе соединения станции с полюсами, в случае, когда гравитационное поле астероида инвариантно относительно поворота вокруг оси динамической симметрии, материальная точка может находиться в равновесии по отношению к осям прецессии и динамической симметрии только если она находится или в плоскости, образованной этими осями, или же в плоскости, проходящей через центр масс твердого тела перпендикулярно оси пре-цессии. (Заметим, что точки либрации Обобщенной ограниченной круговой задачи трех тел В.В.Белецкого [1] являются примерами таких равновесий, если твердое тело можно считать гантелью). Кроме того выводятся условия устойчивости описываемых равнове-сий, проводится их анализ в случаях, когда материальная точка закреплена двумя троса-ми или помещена на леер. В работе также того, устанавливается, что рассматриваемая задача интегрируема при нулевом угле нутации и для некоторых частных движений при прямом угле нутации, при этом развиваются результаты, полученные в [2,3].
Работа поддержана РФФИ (грант № 10-01-00406а)

**Ключевые слова**: тяжелое твердое тело, астероид, космическая тросовая система, задача трех тел, регулярная прецессия

## Литература

[22] Белецкий В.В, Обобшенная ограниченная круговая задача трех тел как модель динамики двойных астероидов, *Космические исследования*, **45** (2007), 6, сс. 435-442
[23] Родников А.В, О движении материальной точки вдоль леера, закрепленного на прецессирующем твердом теле, *Нелинейная динамика*, **7** (2011), 2, сс. 295–311
[24] Родников А.В, О компланарных равновесиях космической станции на тросе, закреп-ленном на прецессирующем астероиде, *Нелинейная динамика*, **8**(2012),2, сс. 309–322

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-7. ID-37.**

# THE DEFINITION OF OPTIMAL MASS OF LOADER WITH A STOCHASTIC MODEL OF WORKING PARAMETERS

**I. Ryabicova**

Department of descriptive geometry of Moscow automobile and road construction state technical university (MADI), Leningradskii avenue 64, Moscow, 125319, Russia,
e – mail : irishkarss@mail.ru

**ABSTRACT.** The working cycle of loader includes a number of operations: grip of material, move to the place of unloading and the return (back) to the starting position. Each operation requires from parameters of machine the most effective course of action. The main feature of these machines is the stochastic changing of working parameters. Our goal is to find the optimal mass of loader. Previously considered the formulae for determine the mass of loader with constant parameters. Now we consider some parameters as random values having Gauss distribution law. The random values are following parameters: the specific resistance of material stacked in bucket; the range of displacement of the loader; coefficient that depends of the properties of traction and adherence.

As a result received formulas allows us to set more meaningful values for the optimum mass machines with various operating conditions. The methodology developed, unlike deterministic model, allows us to specify the interval of the most probable values of the mass, which is provided by the loader with the effective development of the soils at wide intervals changes in specific resistance of the material stacked in bucket on varying range of displacement.

**Keywords:** loader, working parameters, random values, Gauss distribution law.

## REFERENCES

[1] Герасимов М.Д., Рябикова И.М. Компьютерное формирование общего вида погрузчика на этапе технического задания: Учеб. Пособие.-Белгород-М.: Изд-во БГДУ, 2011. -84с.

[2] Колмогоров А.Н., Журбенко И.Г., Прохоров А.В. Введение в теорию вероятностей. Москва – Ижевск, Институт компьютерных исследований, 2003. -188 с.

Naučno društvo Srbije Serbian Scientific Society

http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-8. ID-66.**

# ELEMENTS OF MODIFICATIONS AND SENSITIVITY OF DYNAMIC PARAMETERS

**dr Nataša Trišović,**
Faculty of Mechanical Engineering, Belgrade
ntrisovic@mas.bg.ac.rs

**Abstrakt**

Specific calculations of construction enable the closer introduction to its behavior [2]. Elements of the modification (the distribution of membrane and bending stresses [1], deformation energy and kinetic and potential energy of the structural elements) provide a very efficient identification of structural behavior. They define the necessary changes in the structure, which should lead to better structural behavior in service. The problem reduced in mathematical form involves the minimization of functions of the object $F_i(v_j)$ (mass, energy, deformation, voltage level, eigenvalues ...) as a function $v_j$ (coordinates of points, the cross-section, thickness ...) with restrictions $g(v_j)$ (limit voltage, displacement, length, area, volume, frequency ...). Generally, considered functions are non-linear and implicit. Optimization is based on the analysis of the function sensitivity of the object. This paper analyzes structure changes in certain groups of elements depending on the change of kinetic and potential energy. The distribution of kinetic and potential energy at sub-structures for the main form of oscillation can be expressed in percentages, respectively:

**Error! Objects cannot be created from editing field codes.** and **Error! Objects cannot be created from editing field codes.** (1)

Based on the distribution of kinetic and potential energy (expressed in percent), the group of elements suitable for dynamic analysis can be roughly selected. The process of selection will be later in this paper discussed in detail. First will be demonstrated on a simple example the calculation of kinetic and potential energy of the elements, for the first three main types of oscillation.

## The structure changes

Changes in structure may reflect a change in a number of elements in matrix. Depending on the type of the construction or desired changes of elements, they can be modified individually or in groups.

If changes are made to several groups of elements, then the "perturbed" system matrix of stiffness will be equal

$$\left[\Delta K\right]_{system} = \sum_{e=1}^{L}\left[\Delta k\right]_e, \quad \left[\Delta M\right]_{system} = \sum_{e=1}^{L}\left[\Delta m\right]_e \tag{2}$$

Where is $L$ number of modificated elements. Further, any increase in individual matrix elements can be represented as a function of the matrix coefficients of the original system through modifications (or as the sum of the members of which are specifically related to bending, axial tension or twisting, if necessary):

$$\left[\Delta k\right]_e = \alpha_e\left[k\right]_e, \quad \left[\Delta m\right]_e = \beta_e\left[m\right]_e \tag{3}$$

For example, the effect of the plate thickness on the axial stiffness is linear, while the effect on the flexural rigidity of the third order. Usually these relations for the mass change are linear. If necessary, the scope of structural changes can sometimes be expressed and the corresponding inequalities.

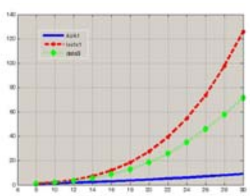


Fig 1 I profile

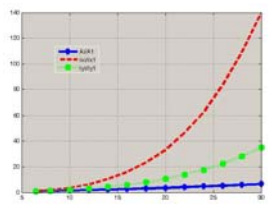


Fig 2 U profile

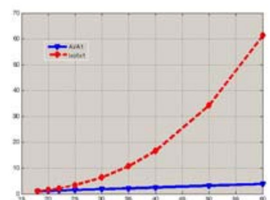


Fig 3
3 box-profile cross-section with thickness δ=2 mm

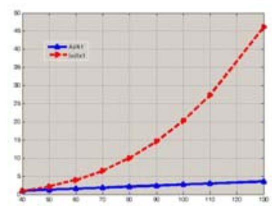


Fig 4
3 box-profile cross-section with thickness δ=3 mm

From the above diagrams may be seen the following conclusions.
Red lines are marked relative axial moments of inertia apply for one of the main central axes of the corresponding cross section of arbitrary cross-section and the corresponding reference, and green for the second axis. Since the bending stiffness of the beam is proportional to the corresponding axial moment of inertia, the correct choice of species profiles can significantly increase the natural frequencies of the system in the phase structure of the original structure. It is evident that the U profile is acceptable for the one direction, while I profile is better for the opposite direction. Full box profiles are more acceptable because they have same properties in the both directions.

**Acknowledgment**

This work has been performed within the projects TR 35011 and ON 74001. These projects are supported by the Republic of Serbia, Ministry of Science and Technological Development, which financial help is gratefully acknowledged.

## Literatura

[1] Trišović, N., Effect of choice of degrees of freedom in modeling the structure of the voltage, Master Thesis, Faculty of Mechanical Engineering, Belgrade, 1995.
[2] Maneski, T., Contribution to the development of design using computer systems supporting structures of machine tools, Ph.D. Thesis, Faculty of Mechanical Engineering, Belgrade, 1992.
[3] Maneski, T., Computer modeling and calculation of structures, Faculty of Mechanical Engineering, Belgrade, 1998.
[4] Maneski, T., Milošević-Mitic, V., blades, D., Settings structural strength, of Faculty of Mechanical Engineering, Belgrade, 2002.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-10. ID-12.**

# ON STABILITY OF SINGULAR TIME DELAY SYSTEMS OVER THE FINITE TIME INTERVAL: CLASSICAL AND LMI APPROACH

**D. Lj. Debeljkovic[1], S. B. Stojanovic[2], G. V. Simeunovic[1], N. J. Dimitrijevic[1]**

[1] Faculty of Mechanical Engineering,
University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35
e-mail: ddebeljkovic@mas.bg.ac.rs
[2]Faculty of Technology,
University of Nis, Bulevar oslobodjenja 124, 16000 Leskovac
e-mail: sb.stojanovic@yahoo.com

**ABSTRACT.**

**A.1 Systems to be considered**. It should be noticed that in some systems we must consider their character of dynamic and static state at the same time. Singular systems (also referred to as degenerate, descriptor, generalized, differential - algebraic systems or semi–state) are those the dynamics of which are governed by a mixture of algebraic and differential equations.

Recently many scholars have paid much attention to singular systems and have obtained many good consequences. The complex nature of singular systems causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control.

They, also, arise naturally as a linear approximation of systems models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large-scale systems, interconnected systems, economics, optimization problems, feedback systems, robotics, biology, etc.

The problem of investigation of time delay systems has been exploited over many years. Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of stability analysis for this class of systems has been one of the main interests for many researchers. In general, the introduction of time delay factors makes the analysis much more complicated.

We must emphasize that there are a lot of systems that have the phenomena of time delay and singular simultaneously, we call such systems as the singular differential systems with time delay.

These systems have many special characters. If we want to describe them more exactly, to design them more accurately and to control them more effectively, we must paid tremendous endeavor to investigate them, but that is obviously very difficult work. In recent references authors had discussed such systems and got some consequences. But in the study of such systems, there are still many problems to be considered. When the general time delay systems are considered, in the existing stability criteria, mainly two ways of approach have been adopted.

Namely, one direction is to contrive the stability condition which does not include the information on the delay, and the other is the method which takes it into account. The former case is often called the delay -

independent criteria and generally provides simple algebraic conditions. In that sense the question of their stability over finite and infinite time interval deserves great attention.

**A.2 Stability concepts**. Practical matters require that we concentrate not only on the system stability (e.g. in the sense of Lyapunov), but also in bounds of system trajectories.

A system could be stable but still completely useless because it possesses undesirable transient performances. Thus, it may be useful to consider the stability of such systems with respect to certain subsets of state-space which are defined a priori in a given problem.

Besides that, it is of particular significance to concern the behavior of dynamical systems only over a finite time interval.

These boundedness properties of system responses, i.e. the solution of system models, are very important from the engineering point of view. Realizing this fact, numerous definitions of the so-called technical and practical stability were introduced. Roughly speaking, these definitions are essentially based on the predefined boundaries for the perturbation of initial conditions and allowable perturbation of system response. In the engineering applications of control systems, this fact becomes very important and sometimes crucial, for the purpose of characterizing in advance, in quantitative manner, possible deviations of system response.

Thus, the analysis of these particular boundedness properties of solutions is an important step, which precedes the design of control signals, when finite time or practical stability control is concern.

It should be noticed that up to now days, there were no results concerning tha problem of non-Lyapunov stability, when the linear continuous singular time delay systems are considered based on LMIs approach.

**A.3 Motivations and contributions.** This contributed lecture and presentation investigate some useful ideas within the Non-Lyapunov stability concept for the class of linear singular continuous time delay systems which can be mathematically described as $E\,\dot{\mathbf{x}}(t) = A_0\mathbf{x}(t) + A_1\mathbf{x}(t-\tau)$ . Moreover we present some sufficient conditions for both practical stability and finite time stability using classical as well as modern approach expressed mainly in form of LMIs.

The geometric theory of consistency leads to the natural class of positive definite quadratic forms on the subspace containing all solutions. This fact makes possible the construction of Lyapunov and Non-Lyapunov stability theory even for the (LCSTDS) in that sense that asymptotic stability is equivalent to the existence of symmetric, positive definite solutions to a weak form of Lyapunov algebraic matrix equation incorporating condition which refer to time delay term.

When we consider a finite time stability concept, sufficient conditions have been derived using the approach based on the Lyapunov-like functions and their properties on the subspace of consistent and admissible initial functions. These functions do not need to have the properties of positivity in the whole state space and negative derivatives along the system trajectories.

When the practical stability has been analyzed the above mentioned approach was combined and supported by the classical Lyapunov technique to guarantee the attractivity property of the system behavior.

Furthermore, part of this result is a geometric counterpart of the algebraic theory in supplemented with appropriate criteria to cover the need for system stability in the presence of actual time delay terms.

Finally, a novel sufficient delay-dependent criterion for the finite time stability, based on LMI`s approach, has been, also, established being less restrictive than those in existing literature.

Moreover all these results are expressed directly throughout systems matrices, quite enough to avoid complex canonical transformations.

Generally, this lecture presents some of the quite new results in the area of the non-Lyapunov stability to the particular class of linear singular continuous time delay systems

**Keywords**: singular systems, finite-time stability, Lyapunov-like functions, LMIs

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-10. ID-12.**

# STABILNOST SINGULARNIH SISTEMA SA ČISTIM VREMENSKIM KAŠNJENJEM NA KONAČNOM VREMENSKOM INTERVALU: KLASIČAN I LMI PRILAZ

**D. Lj. Debeljković[1], S. B. Stojanović[2], G. V. Simeunović[1], N. J. Dimitrijević[1]**

[1]Mašinski fakultet,
Univerzitet u Beogradu, Kraljice Marije 16, 11120 Beograd 35
e-mail: ddebeljkovic@mas.bg.ac.rs
[2]Tehnološki fakultet,
Univerzitet u Nišu, Bulevar oslobođenja 124, 16000 Leskovac
e-mail: sb.stojanovic@yahoo.com

**ABSTRAKT.**

**A.1 Razmatrana klasa sistema.** Dobro je poznato da smo za određene klase sistema praktično, u obavezi, da razmatramo njihove statičke i dinamičke osobine istovremeno. Singularni (poznati još kao: degenerativni, deskriptivni, uopšteni u prostoru stanja, diferencijalno-algebarski, sa polu stanjem, itd.) sistemi su oni sistemi čije je dinamičko ponašanje opisano sistemom (kombinacijom, skupom) algebarskih i diferencijalnih jednačina. Složena priroda singularnih (deskriptivnih) prouzrokuje brojne poteškoće, u njihovom, kako u analitičkom tako i u numeričkom tretmanu a posebno u implementaciji upravljanja. Mnogi naučnici posvetili su dosta pažnje proučavanju ovih sistema i dobili veoma interesantne i zapažene rezultate. Ova sistemi prirodno proistiću iz linaerne aproksimacije modela sistema koji se pojavljuju u mnogim, primenama kao što su: električna kola, dinamika letelica, sistema sa kašnjenjem neutralnog tipa, hemijskim, termičkim i difuzionim procesima, velikim i povezanim sistemima, u ekonomiji, demografiji, optimizacija, sistemima sa povratnom spregom, robotici, biologiji, itd.

Istraživanje dinamičkog ponašanja sistema sa kašnjenjem traje već više decenija. Čisto vremensko kašnjenje je uključeno u brojne tehničke sisteme, kao što su električne, pneumatske i hidraulične mreže (kola), hemijske procese i dugačke prenosne vodove. Prisustvo čisto vremenskog kašnjenja, bez obzira da li je prisutno u upravljanju ili stanju, pa čak i izlazu, može da prouzrokuje neželjene karakteristike prelaznih procesa i dovede do nestabilnost. Samim tim problem njihove stabilnosti bio je od posebnog interesa u istraživanjima mnogih naučnika. U opštem slučaju fenomenom kašnjenja znatno komplikuje dinamičku analizu takvih sistema. Ovi sistemi su opisani običnim diferencijalnim jednačinama sa pomerenim argumentum.

Valja istaći da veliki broj realnih tehničkih sistema istovremeno karakteriše istovremeno prisutno fenomena čisto vremenskog kašnjenja kao i singularnosti u smislu prethodno iznetih činjenica. Ova klasa sistema označena je kao Singularni sistemi sa čistim vremenskim kašnjenjem. Oni imaju veoma veliki broj specifičnih osobina i zaslužuju poseban tretman.

Ovi sistemi poseduju čitav niz specifičnih osobina. Ako želimo da ih opšimo preciznije, da ih projektujemo savršenije a tek kada imamo potrebu da upravljamo njima potrebno je uložiti i veliku količinu energije ali i znanja. Nedavne reference razmatraju ovu klasu sistema i sve više i više spoznaju njihove karakteristike. Imajući u vidu da ih karakteriše čisto vremensko kašnjenje, valja istaći neospornu činjenicu da je pri ispitivanju njihove stabilnosti moguće prići problemu na dva sledeća načina. Prvo, oformiti kriterijume

koji ne uzimaju u obzir iznos čisto vremenskog kašnjenja ili iznaći takve uslove koji taj iznos uzimaju u obzir. U prvom slučaju dobijaju se relativno prosti algebarski izrazi, dok to nije slučaj u drugom prilazu.

**A.2 Koncepti stabilnosti**. Praktična implementacija rezultata nameću potrebu da se koncentrišemo ne samo na stabilnost sistema u smislu Ljapunova već i da razmatramo granice do kojih dosežu trajektorije sistema pri njegovom kretanju. Sistem može da bude stabilan a još uvek potpuno nepodesan pa čak i neprihvatljiv ukoliko se razmatraju pokazatelji njegovog prelaznog procesa. Imajući to u vidu, sasvim je jasno da je poželjno razmatrati stabilnost takvih sistema u odnosu na unapred definisane skupove mogućih (početnih) i dozvoljenih stanja. Mimo toga, od posebnog je interesa razmatrati dinamičko ponašanje sistema na konačnom vremenskom intervalu (a ne na beskonačnom – po Ljapunovu).

Ove osobine granica odziva sistema, tj. rešenja diferencijalnih jednačina koje obrazuju model sistema, veoma su važna sa inženjerske tačke gledišta. Imajući to u vidu, stvorene su brojne definicije tzv. tehničke i praktične stabilnosti . Grubo govoreći te definicije baziraju se na a priorno zadatim granicama do kojih dosežu promene početnih uslova i dozvoljene promene odziva sistema. U inženjerskoj eksploataciji sistema automatskog upravljanja, ova činjenica postaje veoma značajna a kad kad i krucijelna u prilikama kada je neophodno unapred izvršiti karakterizaciju odziva sistema u kvalitativnom smislu, razmatrajući njegove moguće devijacije.

Prema tome, analiza ovih posebnih ograničenja nametnutih odzivu sistema je važan korak koji prethodi izboru upravljačkih signala, u slučajevima kada se razmatra stabilnost na konačnom vremenskom intervalu ili praktična stabilnost.

Valja istači i značajnu činjenicu da sve do današnjih dana problem neljapunovske stabilnosti nije tretian za vremnski neprkidne singularne sisteme sa čistim vremenskim kašnjenjem sa pozicija primene LMI postupaka.

**A.3 Motivacija i doprinosi.** Ova predavanja imaju za cilj da izlože neke značajne ideje koje se javljaju u realizaciji neljapunovske stabilnosti posebne klase vremenski neprekidnih, linearnih singularnih sistema sa čistim vremenskim kašnjenjem, čiji se matematički modeli u prostoru stanja mogu iskazati na sledeći način: $E\,\dot{\mathbf{x}}(t) = A_0\mathbf{x}(t) + A_1\mathbf{x}(t-\tau)$ . Štaviše, prezentovaće se neki novi dovoljni uslovi stabilnosti ove klase sistema, kao za praktičnu tako i za koncept stabilnosti na konačnom vremenskom intervalu a sve sa pozicija klasičnog, odnosnog modernog prilaza oličenog u poznatoj metodi LMI.

Prilaz sa pozicija geometrijskog tretmana savremenih sistema automatskog upravljanja vodi ka prirodnoj klasi pozitivno određenih kvadratnih formi na podprostorima koji sadrže sva rešenja sistema. Ova činjenica omogućava primenu ljapunovske i neljapunovske teorije stabilnosti čak i za vremenski neprekidne, linearne singularne sisteme sa čistim vremenskim kašnjenjem u smislu da je asimptotska stabilnost ekvivalentna postojanju simetrične, pozitivno određenog rešenja slabe Ljapunovljeve algebarske matrične jednačine uključujući i uslov koji se odnosi na član koji unosi kašnjenje.

Kada se razmatra koncept stabilnosti na konačnom vremenskom intervalu, dovoljni uslovi stabilnosti izvedeni su na bazi korišćenja tzv. kvazi-Ljapunovljevih funkcija i njihovih osobina na potprostoru konzistentnih i prihvatljivih početnih funkcija.

Ove funkcije ne moraju da budu pozitivno određene u celome prostoru stanja a ni negativno određene duž trajektorija kretanja sistema..

U slučajevima kada se razmatra praktična stabilnost, prethodno pomenuti prilaz, se kombinuje i podržava klasičnom Drugom metodom Ljapunova kao bi se garantovala osobina atraktivnosti dinamičkog ponašanja sistema.

Štaviše, deo ovih rezultata je geometrijski pandan odgovarajućoj algebarskoj teorije sa dopunskim kriterijumom kako bi se obezbedila stabilnost sistema u prisustvu odgovarajućeg člana koji reprezentuje vremensko kašnjenje.

I konačno, na bazi LMI prilaza, izvedeni su potpuno novi dovoljni uslovi stabilnosti na konačnom vremenskom intervalu zavisni od čisto vremenskog kašnjenja, koji daju manje restriktivne uslove od onih koji egzistiraju u postojećoj literature.

Valja istaći neospornu činjenicu da svi ovde izvedeni rezultat iskazani su preko bazičnih sistemskih matrica, čime je izbegnuto prevođenje polaznog modela u bilo kakve kanoničke forme.

Uopšteno govoreći, ovde ce se predstaviti čitav niz novih rezultata u oblasti istraživanja neljapunovskog koncepta stabilnosti a za jednu posebnu klasu vremenski neprekidnih, linearnih, singularnih sistema sa čistim vremenskim kašnjenjem.

**Keywords**: singularni sistemi, stabilnost na konačnom vremenskom intervalu, linearne matrične nejednakosti

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-11. ID-16.**

# MATHEMATICAL MODEL OF AERIAL ROBOTS AS THE BASIS FOR NEW RESEARCH

**M. Filipovic[1],**

[1] Mihajlo Pupin Institute, The University of Belgrade, Volgina 15, 11060 Belgrade, Serbia, e-mails: mirjana.filipovic@pupin.rs

**ABSTRACT.** The aim of this paper is a detailed analysis *Cable-suspended Parallel Robot – CPR (aerial robot)*, which should enable their strong progress. This would be reflected in the implementation of highly-automated system that would lead the camera precisely in space with minimum participation of human labor. Setting and achieving this goal provides a much wider possibilities for its future use. Such an operation of this system can provide only with application of his high-fidelity mathematical model during the synthesis and analysis, which would further enable the development and application of modern control law.

System for observation of workspace with moving objects is developed to some extent and widely analyzed in the world and in various research areas as well as for different purposes. The cable driven redundant parallel manipulator for a large spherical radio telescope (LSRT), is analyzed and modelled in paper [1].

It is adopted, in this paper that the working space of a camera carrier is in the form of parallelepiped, so that the camera carrier hangs over the ropes connected properly on the four highest points i.e. the four upper angles of the workspace. It is a necessary geometric condition for analysis and synthesis of this system class. See Fig. 1.

A camera carrier moves freely in the space enabling the shooting of the objects from above. It gives a unique feeling to the viewer to observe the event easy from an unusual proximity and to be very close to the action regardless of the size of the observed space.

This paper analyzes the mechanism which involves only the positioning of the camera carrier. Motion of the ropes which carry the camera is controlled. Ropes are unwound (or wound), thus reaching any camera position in space. The gyro sensor, which is installed in the camera carrier, is stabilized towards the horizon. In this paper is assumed that ropes are rigid.

The aim of this paper is to highlight the importance of forming a mathematical model of any CPR, since only that way CPR provides precise guidance in the area. The control of this system is an important component of its operation. In this paper, the control task is defined at the level of motor motion which is the lowest level of control. Task work of CPR can be formulated as much more complex, so for example, the camera needs to recognize and follow the marked object, for example: a ball, a robot or a marked person, a player or co-workers.

Only highly intelligent control systems, based on high fidelity mathematical model of the system, can provide the realization of very complex tasks. The established mathematical model provides an opportunity to modernize CPR significantly and to make its application become much wider. CPR, which is developed in the Mihajlo Pupin Institute and which observes space, is a part of a more complex system presented in [2].

The aim of this paper is to ensure, in the future, accurate and highly automated guidance of the camera in space with minimal involvement of the human factor in the realization of the task for several hours. From the visionary aspect, good mathematical model provides a realistic possibility of further development of CPR, for example in order to participate in the planned match between the robots in the not-too-distant 2050.

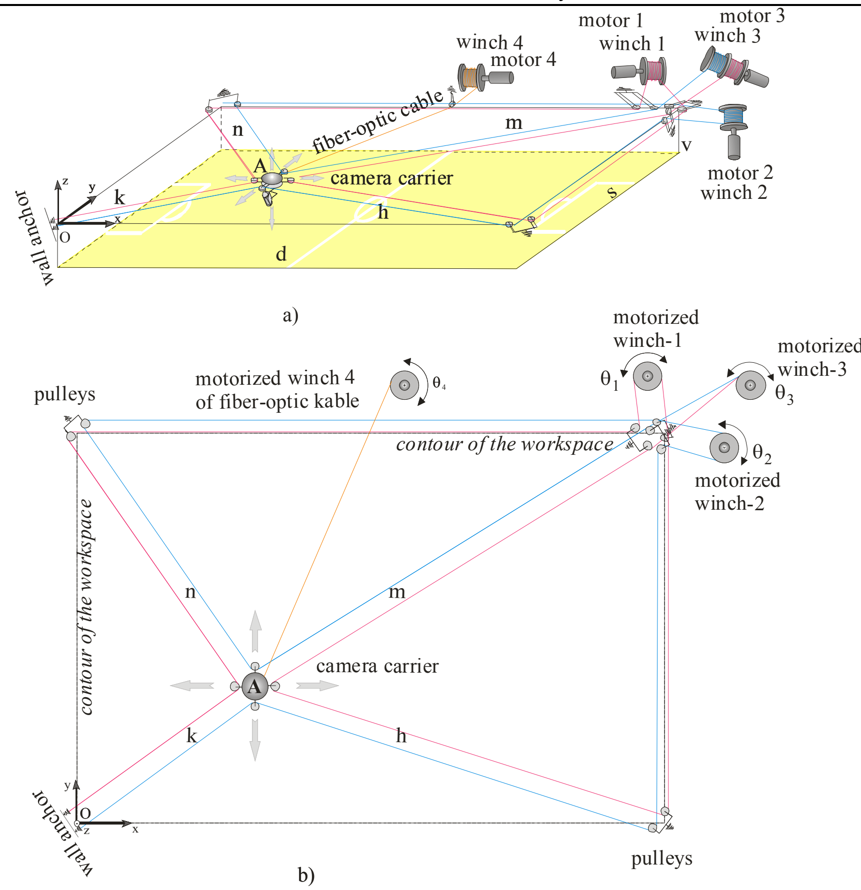



Figure 1. CPR, a) in the space, b) top view.

This camera would not track of people who voluntarily move through the observed space, but humanoid robots, who receive the picture of the observed space and according to that make decisions about their further actions.

Due to large demand and bright prospects of CPR development, the importance of establishing its highly authentic mathematical model is especially emphasized.

Future researches go a step further in implementation of the features of elastic ropes (type of nonlinear dynamic elasticity as defined in [3]-[4] in the mathematical model of CPR.

**Keywords**: aerial robot, observation; workspace; kinematics; dynamics; analysis; synthesis

**References**


[1] B.Y.Duan, “A new design project of the line feed structure for large spherical radio telescope and its nonlinear dynamic analysis,” *Mechatronics*, 9 (1998) pp. 53-64.

[2] A. Rodic, M. Jovanovic, S. Popic, G. Mester, “Scalable Experimental Platform for Research, Development and Testing of Networked Robotic Systems in Informationally Structured Environments,“ *Symposium Series on Computational Intelligence SSCI*, Paris,France, (2011) pp.136-143.

[3] M. Filipović and M. Vukobratović, “Complement of source equation of elastic line,“ Journal *of Intelligent & Robotic Systems, International Journal*, 52(2) (2008) pp. 233-261.

[4] K. (Stevanovic) Hedrih, “Vibration Modes of a axially moving double belt system with creep layer,“ *J VIB CONTROL*,14(10-Sep) (2008) pp. 1333-1347.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-12. ID-11.**

# PRECISE TRAJECTORY TRACKING OF ROBOTIC MECHANISM

**Lj. Kevac[1], M. Filipović[2]**

[1] School of Electrical Engineering
The University of Belgrade, Bulevar Kralja Aleksandra 73, 11000 Belgrade
e-mail: ljubinko.kevac@gmail.com
[2] Mihajlo Pupin Institute,
The University of Belgrade, Volgina 15, 11000 Belgrade
e-mail: mira@robot.imp.bg.ac.rs

**ABSTRACT.** The modeling of a modern robot system as a rigid mechanical system is an unrealistic simplification. Many applications, such as spray painting, plasma cutting and assembly, require good path tracking. Modeling and control of robotic mechanism is very popular and important part of today's Robotics, therefore, there are many papers and books related to this topic. First one who introduced this problem was Spong [1]. In [2] authors present a new way of motor modeling and new form of mathematical model of robotic system in presence of elasticity elements. Depending on a robotic mechanism, it is controlled by one or more motors. In this paper two similar examples of robotic mechanism are analyzed. Both examples are controlled by one motor. First example is completely idealized (everything is rigid) and path tracking is not an issue. Second example is more realistic model of this robotic mechanism. This example has elastic gear, and therefore does not track the motors movement very well. Idea is that the elastic robotic mechanism is managed to follow the motor, and therefore the desired path. Firstly, the model without elasticity is determined and therefore a desired trajectory is chosen. Desired trajectory of the rigid robotic mechanism is used as a referent trajectory with model that has elastic gear. Obtained results have quite a deviation from the referent trajectory. Deviations are present due to the elasticity of the robotic mechanism. This elasticity is usually determined by using know parameters of mechanical object (robotic mechanism), but in this paper it is calculated by subtracting resulted and desired trajectory, which represents one of theoretical ways of calculating mentioned error. With this error, new referent trajectory is determined and it represents previous referent trajectory reduced by the calculated deviation. With this approach, error due the elasticity is used in good purpose and after applying new referent trajectory, the result is very similar to desired trajectory. The new error is present only due the imperfection of PD regulator. Obtained results show that used method of path tracking is very precise and simple which is important in these applications, because of the possible use in more complicated systems.

**Keywords**: robotic mechanism, modeling, control, rigid, elastic, trajectory, path tracking

## References

[1] Spong, W.M. Modeling and control of elastic joint robots, ASME J.of Dynamic Systems¸ Measurement and
Control. 109, 310-319 (1987)

[2] Filipovic M, Potkonjak V and Vukobratovic M (2007) Humanoid robotic system with and without elasticity elements walking on an immobile/mobile platform, *Journal of Intelligent & Robotic Systems, International Journal* 48, pp. 157-186.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-16. ID-10,**

# REGRESION AND WEIBULL ANALYSIS OF CROPPING PRACTICES IN CHANGING CLIMATE

**V. Dragičević, Ž. Videnović, B.Kresović, M. Simić, Z. Dumanović, I. Spasojević**

[1] Maize Research Institute "Zemun Polje"
Slobodana Bajića 1, 11185 Zemun Polje, Serbia
e-mail: vdragicevic@mrizp.rs

**ABSTRACT.** Maize is one of the most important crops in the world. Successful production depends primarily on meteorological, but also on other environmental factors. Variations in cropping practices, which include different measures, enable overcoming alterations of environmental factors, especially during periods of extreme weather conditions for plant growth. The aim of this study was to define the most efficient tillage system (no-till, reduced or conventional tillage) and fertilization regime (Ø, 330 or 660 kg ha$^{-1}$ N:P:K) for high maize yield (hybrid ZP 704) under rainfed and irrigation conditions, according to results of long-term maize cropping experiment (1991-2010) in conditions of climate change. Among meteorological factors, we focused on sum of precipitation and growth degree days (GDD - sum of effective temperatures) during maize vegetation.

The observed 20 year period characterizes increasing trend of both, the average temperatures and sum of precipitation, as well as the extreme meteorological conditions for maize growth. In our previous experiments in less extreme weather conditions, we recorded lower differences between irrigated and non-irrigated (rainfed) maize yield [1]. In this research, the positive correlation between grain yield and precipitation was noted only under rainfed cropping, while the increasing level of fertilization and tillage intensity (conventional tillage) had positive effect on grain yield only under irrigation. Polynomial regression analysis ascertained the highest yields at 400 mm of precipitation for all fertilization and tillage regimes, as well as 1550 and 1850 GDD under both, rainfed and irrigation conditions. It was particularly present in treatments with highest inputs (conventional tillage with 660 kg ha$^{-1}$ of N:P:K fertilizers).

Meanwhile, different regimes of fertilization and tillage intensity didn't fit the alterations in grain yield, according to present changes in meteorological conditions, what could give the possibility of grain yield prediction, based on results of long term experiment. Opposite to general inference obtained from regression analysis, Weibull analysis underlined that fertilization with 660 kg ha$^{-1}$ N:P:K could achieve the best results only under the irrigation (7.14 t ha$^{-1}$ of grain yield with reliability of 99%), while the higher inputs were not rational in rainfed conditions. The conventional tillage had its validation under both, rainfed and irrigation conditions. The highest β coefficient values in treatment with 330 kg ha$^{-1}$ N:P:K indicate this fertilization level as the most rational input, especially in combination with conventional tillage, where reliability of achieved grain yield endeavours genetic potential of hybrid under irrigation (9,96 t ha$^{-1}$ of grain yield with reliability of 99%). In rainfed cropping, same fertilization regime diminished differences between reduced and

conventional tillage, achieving the maximal yield of 6-7 t $ha^{-1}$. From this point of view, advantage of Weibull analysis reflects through yield prediction, with high reliability [2,3], including variations of meteorological factors, as well as tested cropping practices.

The analysis underlined 330 kg $ha^{-1}$ N:P:K as the optimal fertilization regime, with conventional tillage under irrigation, as well as conventional or reduced tillage in rainfed cropping. The negative environmental factors (alterations of precipitation and temperature) were diminished under irrigation, where high inputs resulted in high yields. But, considering the fact that the rainfed cropping is the most abundant cropping practice in the world, moderate inputs (330 kg $ha^{-1}$ N:P:K) obtain stability of the maize yield, irrespective of lower yields. It could be concluded that high inputs in fertilizers and tillage energy didn't necessarily resulted in yield increase, particularly under rainfed conditions. On the other hand, high yields attained with high fertilizer and tillage inputs under irrigation, could be reasonable only in seasons with low precipitation (draught).

**Keywords**: maize cropping, fertilization, tillage, Weibull analysis, regression analysis.

## References

[1] Videnović Ž, Vasić G, Kolčar F**,** Effect of fertilizers and soil tillage on corn yield under dry farming and irrigated conditions**,** *Soil and Tillage Research*, **8 (**1986**)**, pp. 113–118**.**

[2] Kress LW, Miller JE, Impact of ozone on field-corn yield, *Canadian Journal of Botany*, **63** (1985), pp. 2408-2415.

[3] LW Kress, Miller JE, Impact of ozone on grain sorghum yield, *Water, Air, & Soil Pollution*, **25** (1985), pp. 377-390.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL17. ID-20.**

# THE COMPARATIVE ANALYSIS OF NONLINEAR ALGORITHMS FOR TWO TANK SYSTEM CONTROL

**A. Ćosić[1], M. Šušić[2]**
[1] Mihajlo Pupin Institute, Robotics Laboratory, The University of Belgrade, Volgina 15, Belgrade, Serbia, e-mail: aleksandar.cosic@pupin.rs
[2] Mihajlo Pupin Institute, Robotics Laboratory, The University of Belgrade, Volgina 15, Belgrade, Serbia, e-mail: marko.susic@pupin.rs

**ABSTRACT.** Various approaches for the two tank system control are presented in this paper. Considered system consists of two equal tanks, which are connected with a single pipe, on which is installed valve. Also, pipes with valves are placed at the bottom of both reservoirs, which provide liquid outflow. Pump provides inflow to the first tank. Model of the system is highly nonlinear, and stability analysis based on Lyapunov theory of the origin as the unique equilibrium of the system is conducted. Control algorithms based on linearization of original nonlinear model do not provide satisfactory performance, because they are valid only in the close neighborhood of the nominal set point, and they do not ensure stability of the closed loop system during its change. Hence, nonlinear control algorithms are employed, which ensures stability of the closed loop system in wider neighborhood of the nominal set point. Firstly, proportional-integral controller is proposed, because of its simplicity and wide range of utilization in industrial practice, and then nonlinear control algorithms are introduced. Sliding mode control (with and without boundary layer), backstepping procedure and fuzzy logic based controller are nonlinear strategies proposed in this paper. Sliding mode control and backstepping procedure guarantee asymptotic stability of the nominal set point. Fuzzy logic based control is novel control strategy and it is proved to be efficient in control of complex systems. Comparative analysis of the obtained closed loop responses is conducted, and mean square error is adopted as a measure of overall quality. Simulation results are presented as illustration of the proposed control approaches.

**Keywords**: PI control, sliding mode control, backstepping, fuzzy logic

**References**

[1] Vidyasagar M, Nonlinear Systems Analysis, 2nd edition, *SIAM: Society for Industrial and Applied Mathematics,* (2002)
[2] Khalil H, Nonlinear Systems, 1st edition, *MacMillan Publishing Company,* (1992)
[3] Jantzen J, Foundations of Fuzzy Control, 1st edition, *Wiley Interscience*, (2007)
[4] Krstić M, Kanellakopoulos I, Kokotivić P, Nonlinear and Adaptive Control Design, 1st edition, *Wiley Interscience*, (1995)
[5] Pan H, Wong H, Kapila V, de Queiroz M.S, Experimental Validation of a Nonlinear Backstepping Liquid Level Controller for a State Coupled Two Tank System, *Control Engineering Practice, Vol. 13, No. 1,* (2005), pp. 27-40.
[6] Mahieddine Mahmoud S, Chrifi – Alaoui, Van Assche V, Bussy P, Sliding Mode Control of Nonlinear SISO Systems with both Mathed and Unmatched Disturbances, *International Journal of Sciences and Techniques of Automatic Control & Computer Engineering, Vol. 2, No. 1,* (2008), pp. 350-367.
[7] Malmborg J, Eker J, Hybrid Control of a Double Tank System, *6th IEEE Conference on Control Application,* (1997), pp. 133-138.
[8] Suresh M, Srinivasan G.J, Hemamalini R.R, Integrated Fuzzy Logic Based Intelligent Control of Three Tank System, *Serbian Journal of Electrical Engineering, Vol. 6, No.1*, (2009), pp. 1-14.
[9] Wu D, Tan W.W, A Type-2 Fuzzy Logic Controller for the Liquid-level Process, *Proceedings of IEEE Conference on Fuzzy Systems,* (2004), pp. 953-958.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
CL-18. ID-23.

# $H_2$ AND $H_\infty$ BASED OPTIMAL PLACEMENT OF ACTUATORS AND SENSORS FOR ACTIVE VIBRATION CONTROL

**Tamara Nestorović [1], Miroslav Trajkov [2]**

[1, 2] Institute of Computational Engineering, Mechanics of Adaptive Systems
Ruhr-Universität Bochum, Universitätsstr. 150, D-44801 Bochum, Germany
e-mail: [1] tamara.nestorovic@rub.de, [2] miroslav.trajkov@rub.de,

**ABSTRACT.** This paper considers the problem of optimal actuator and sensor placement for active large flexible structures. The proposed placement optimization method is based on balanced reduced models. It overcomes disadvantages arising from demanding numeric procedures related with high order structural models. Optimization procedure relies on $H_2$ and $H_\infty$ norms, as well as on controllability and observability Gramians, related with structural eigenmodes of interest. The optimization procedure was documented by several examples showing a good agreement between the results obtained using different placement indices.

**Keywords**: optimal actuator/sensor placement, $H_2$ and $H_\infty$ norms.

## MODELS AND OBJECTIVE FUNCTIONS USED FOR OPTIMAL PLACEMENT

The finite element (FE) based modeling procedure represents a suitable basis for modeling of piezoelectric structures with large number of degrees of freedom. Applying the FE procedure results in the semi-discrete form of the equations of motion:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}_\mathrm{d}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}\,. \tag{1}$$

where vector **q** contains all degrees of freedom (displacements **u** and electric voltages $\phi$) arranged node-wise:

$$\mathbf{q}^\mathrm{T} = [\mathbf{u}_1^\mathrm{T} \quad \phi_1 \mid \mathbf{u}_2^\mathrm{T} \quad \phi_2 \mid \cdots \mid \mathbf{u}_n^\mathrm{T} \quad \phi_n] \tag{2}$$

Through appropriate transformations, the modal state space representations of each mode can be obtained in the form:

$$\dot{\mathbf{x}}_i = \mathbf{A}_{mi}\mathbf{x}_i + \mathbf{B}_{mi}\mathbf{u}\,, \quad \mathbf{y} = \mathbf{C}_{mi}\mathbf{x}_i \tag{3}$$

The elements of the realization ($\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$, $\mathbf{C}_{mi}$) are used for assessing the optimal actuator/sensor locations based on candidate input/output transfer functions relating corresponding actuators and sensors and applying objective functions in terms of $H_2$ and $H_\infty$ norms of single modes as well as of a whole structure. The $H_\infty$ norm of a single mode can be estimated as:

$$\|G_i\|_\infty \cong \frac{\|\mathbf{B}_{mi}\|_2 \|\mathbf{C}_{mi}\|_2}{2\zeta_i\omega_i}\,, \tag{4}$$

with damping ratio $\zeta_i$ and natural eigenfrequency $\omega_i$, whereas the norm of a structure is approximately determined as the largest of the mode norms.

## OPTIMAL PLACEMENT RESULTS

Placement indices determined based on the $H_\infty$ norm are represented in Fig,1 for a beam clamped on both ends. Left hand side plot represents the placement indices for individually considered eigenmodes 1 to 5. In the right hand side plot the placement indices were calculated based on parallel consideration of the eigenmodes of interest (1 to 5). Locations with largest placement indices indicate the candidates for optimal placement, depending on the number of employed actuators/sensors and on the number of considered modes of interest.

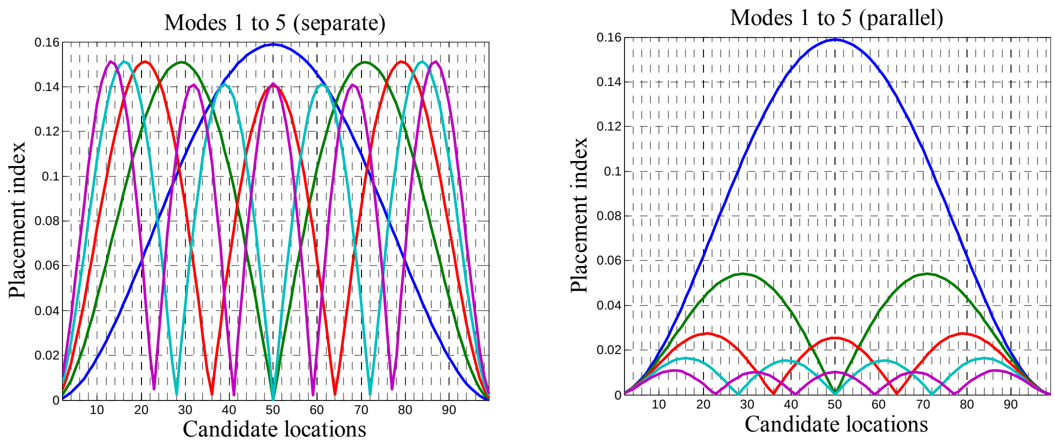



**Fig.1** Placement indices calculated for the beam clamped on both sides based on the $H_\infty$ norm for separate and parallel consideration of the eigenmodes

For a clamped plate the locations with maximal placement indices calculated based on the $H_\infty$ norm for parallel consideration of the five eigenmodes of interest are represented in Fig. 2.

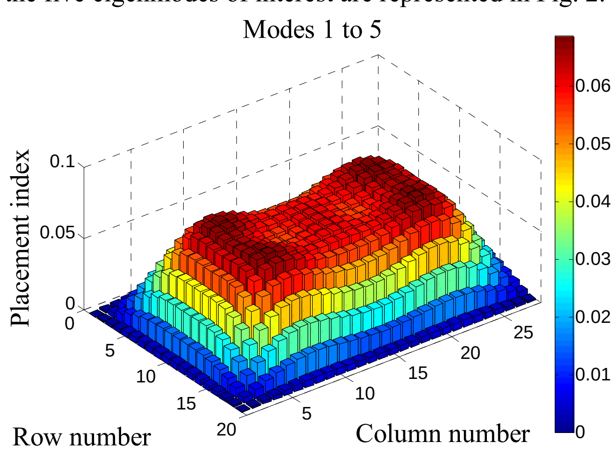



**Fig.2** Placement indices for a clamped plate based on the $H_\infty$ norm (parallel consideration of five eigenmodes)

## ACKNOWLEDGMENT

The authors gratefully acknowledge the funding by the German Research Foundation (DFG), under grant SFB-837.

## References

[1] Nestorović T. Controller *Design for the Vibration Suppression of Smart Structures* Fortschritt-Berichte VDI 8 **1071**, Düsseldorf, VDI Verlag, 2005

[2] Gawronski W. *Advanced Structural Dynamics and Active Control of Structures*, Springer New York, 2004

Naučno društvo Srbije — Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-19. ID-26.**

# OPTIMAL PLACEMENT OF PIEZOELECTRIC ACTUATORS AND SENSORS FOR SMART STRUCTURES USING GENETIC ALGORITHM

**Seyed Mehdi Garmabi [1], Miroslav Trajkov [2] and Tamara Nestorović [3]**

[1, 2, 3] Institute of Computational Engineering, Mechanics of Adaptive Systems
Ruhr-Universität Bochum, Universitätsstr. 150, D-44801 Bochum, Germany
e-mail: [1] seyed.garmabi@rub.de, [2] miroslav.trajkov@rub.de,
[3]tamara.nestorovic@rub.de,

**ABSTRACT.** In recent years a lot of research in vibration control field has been done regarding piezoelectric materials. These materials can be used as actuators and/or sensors depending on particular problem. They are usually applied in the form of piezoelectric patches. With limited number of piezoelectric patches, a problem arises regarding the question, where these patches should optimally be placed on a structure to ease the control of structure. In this paper, $H_\infty$ norm is used as a fitness function and genetic algorithm (GA) and multi objective genetic algorithm are chosen for optimization process. Places of piezoelectric patches are optimized on different structures: cantilever beam, both-end clamped beam and clamped plate.

**Keywords**: optimal actuator/sensor placement, genetic algorithm and $H_\infty$ norm.

## OPTIMAL PLACEMENT USING GENETIC ALGORITHM

The evolution strategies are good choice of design optimization when we have very complicated cases which cannot be optimized with other numeric optimization strategies. The genetic algorithm is one method of the evolution strategies. This method is appropriate for problems with discontinuous solution spaces and multi-objective problems.

One genetic algorithm generally consists of three parts:

1. **Chromosomes** (Individuals): they are result of a problem but they can be optima or not optima. In GA the chromosomes are built of genes. In this case, each of gens represents the place of each piezoelectric patch and chromosome is set of all these places.

$$X = \{x_1, x_2, x_3, \ldots, x_n\} \tag{1}$$

   where vector $X$ is the chromosome and $x_1, x_2, x_3, \ldots, x_n$ are the gens.

2. **Cost or Fitness function:** this is a criterion to evaluate each of chromosomes. In this case, the $H_\infty$ norm is defined as fitness function.

$$F(X) := \|G_i\|_\infty \cong \frac{\|\mathbf{B}_{mi}\|_2 \|\mathbf{C}_{mi}\|_2}{2\zeta_i \omega_i} \tag{2}$$

   where **B**, **C** are respectively input and output matrices of a system in state-space representation. And $\zeta_i$ is damping ratio and $\omega_i$ natural eigenfrequency of the system.

3. **Operators:** they are used to create better new chromosomes from old ones [1, 2].

## OPTIMAL PLACEMENT RESULTS

The mentioned method is applied to find optimal places of piezoelectric patches on different structures. At the end output of the system with optimal placement is compared with not-optimal placement of the piezo-patches with help of Optimal Control method. Some of the obtained results are depicted in the following.

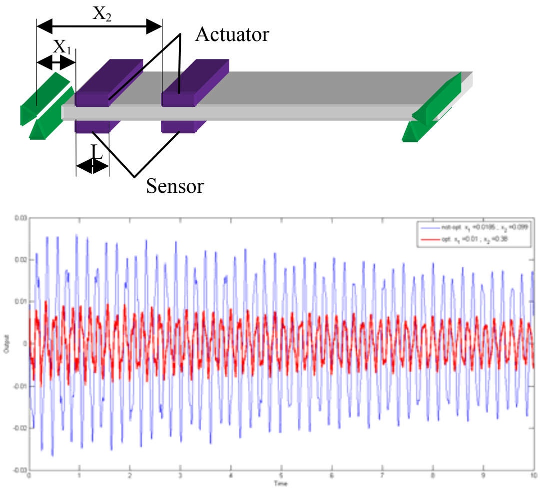



**Fig. 1** The comparison of output of two systems: one with optimal places based on the first mode of the beam and other one with random placement

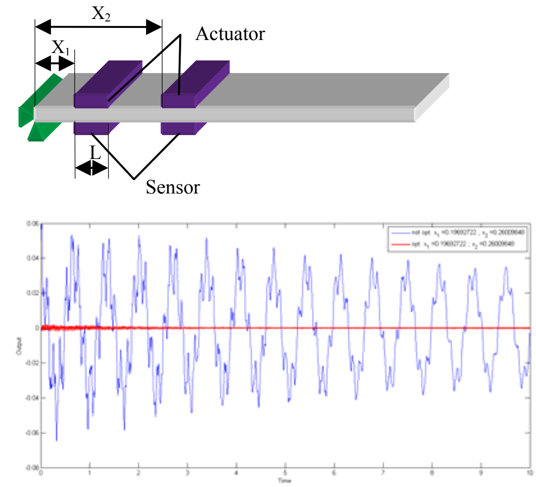



**Fig. 2** Comparison of opt. and not-opt. places of four patches on a cantilever beam regarding first four modes

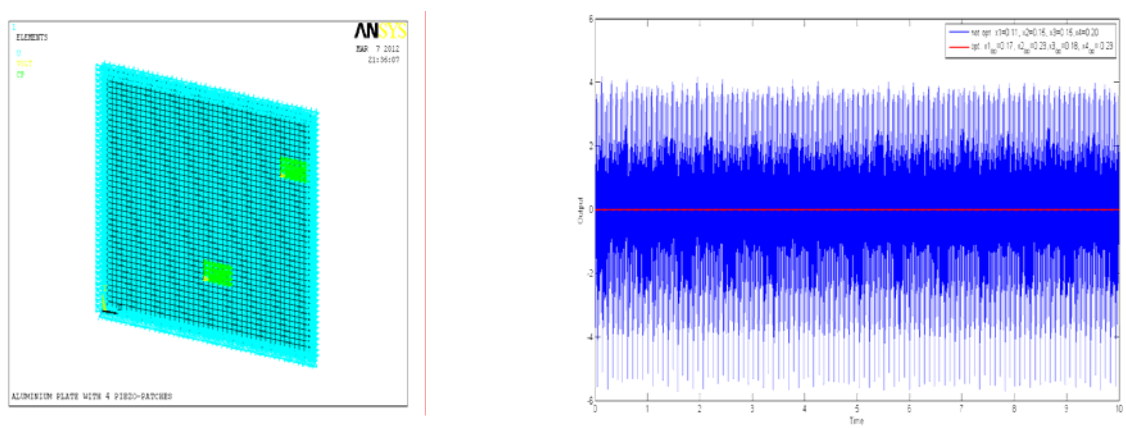



**Fig. 3** The comparison of output of two systems: one with optimal places based on first mode of the plate and other one with random placement


## ACKNOWLEDGMENT

This research has been partly supported by the German Research Foundation (DFG), under grant SFB-837. The authors gratefully acknowledge this support.


### References


[1] Gawronski W. Advanced Structural Dynamics and Active Control of Structures, Springer New York, 2004

[2] Hartmann D. *Desing Optimization,* Ruhr-Universität Bochum, Bochum, 2010

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-20, ID-29.**

# FURTHER RESULTS ON MODELING OF BIOIMPEDANCE OF THE HUMAN SKIN: CALCULUS OF NON-INTEGER ORDER APPROACH

**M. Lazarević[1,2], Z.Vosika,PhD[2] ,G. Lazović [3] , J Simić-Krstić[2], Đ. Koruga[2]**

[1] Department of Mechanics,[2] Module of Biomedical Engineering,
Faculty of Mechanical Engineering,
[2]The University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35
e-mail: mlazarevic@mas.bg.ac.rs
[3] Department of Mathematics, Faculty of Mechanical Engineering,
The University of Belgrade, Kraljice Marije 16, 11120 Belgrade 35

**ABSTRACT.** The fractional integro-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators [1]. Fractional calculus is the essential analytical approach for modeling any kind of complex systems. Further, bioelectro-physical properties of human skin tissue, like most other soft tissues, exhibit electrical behavior [2,3] where human skin consists of several layers with distinct dielectric properties. Today, bioimpedance measurements provide an important method for the noninvasive investigation of tissue structure and properties or for monitoring physiological change (i.e., ‘‘static’’ or ‘‘dynamic’’ human organism properties). Moreover,the complex modulus concept is a powerful and widely used tool for characterizing the electrical behavior of materials in the frequency domain. In this case, according to the proposed concept, bioimpendance moduli can be regarded as complex quantities. In the field of bioimpedance measurements, the Cole (Cole model) impedance model [4] is widely used for characterizing biological tissues because of its simplicity and good fit with measured data, illustrating the behavior of tissue impedance as a function of frequency.

In this study, we apply fractional calculus to modeling of electrical properties of biological systems and derive a new class of models for electrical impedance of human skin. According to literature data, the human skin is usually observed as a relative simple structure, and equivalent electrical model of skin doesn’t include tissue lamination. Such relaxation processes occur because the epidermis is a mosaic in which layers of laminated, inhomogeneous cell structure pile up on top of one another. Frequency-dependent components such as *CPE* (constant phase element), that exists in the single-dispersion Cole model, can be considered composed of an infinite number of lumped components. Recently, authors [5] suggested using the three-layer skin numerical model in the MHz frequency range where each layer of skin is represented by the corresponding Cole–Cole model. So, we proposed the skin structure as a more complex system, consisting of several layers(Fig.1). In relation to our experimental in vivo conditions, structure and complexity of the human skin, we suggested that bio-electrical behavior of the human skin can be described by the series layer Cole model using modified fractional distributed-order based on the Caputo-Weyl fractional derivatives.

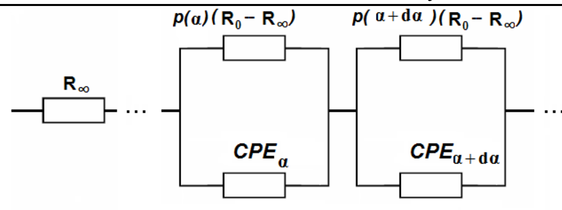



Fig.1 Electrical continuum model of the skin, based on the Cole equation

Our proposed model presents essentially modified single-dispersion Cole model, since it introduces a new parameters $k$ and $\sigma$ in single-dispersion Cole impedance equation. These parameters characterize the width of interval around fractional index α. Comparing our model to well-known Cole models, we conclude that these parameters are important for more accurate describing bioimpedance properties of human skin. Our modified Cole model much better fit to experimentally curve in given frequency range in compare to existing Cole models. The fitting is done using Levenberg-Marquardt nonlinear least squares. In that way, one may conclude that the electrical properties of skin can be modeled using a more discrete Cole impedance element rather than one discrete Cole impedance element.

Last, some our results are related to generalized Cole element as well as constant phase element (*CPE*). These generalizations is described by the novel equation which presented parameter ( $\beta$ ) and corrected four essential parameters ( $R_0, R_\infty, \alpha, \tau_\alpha$ ). Using serial combinations of the primary model elements we may obtain two new models and defined them by the appropriate equations and electrical schemes.

**Keywords**: human skin, calculus of noninteger order (fractional calculus), frequency analysis, bioimpedance, Cole model

Acknowledgement. This work is supported by the Ministry of Education and Science of Republic of Serbia, Project No.41006.

**References**

[1] Podlubny I. , *Fractional Differential Equations*, Academic Press, San Diego, (1999).

[2] Nigmatulin R.R, Ya. E. Ryabov, *Cole-Davidson relaxation as a self-similar relaxation process*, (1997), American institute of Physics [S1063-7834(97)020001-7].

[3] Grimnes S.,O. G. Martinsen, *Bioimpedance and Bioelectricity Basics*, (2008),Elsevier Ltd.

[4] Cole K.S. *Permeability and impermeability of cell membranes for ions*, Cold Spring Harbor Sympos. Quant. Biol. **8**, (1940), pp.110-122.

[5] Huclova S, Baumann D, Talary M S and Frohlich J 2011 Sensitivity and specificity analysis of fringing-field dielectric spectroscopy applied to a multi-layer system modelling the human skin *Phys. Med. Biol.* **56** 7777–793

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-21. ID-2.**

# DYNAMICS OF GEAR-PAIR SYSTEMS WITH PERIODIC VARYING MESH STIFFNESS - SPUR GEARS VS HELICAL GEARS

**I. Atanasovska[1], M. Vukšić Popović[2]**

Institute Kirilo Savić
Vojvode Stepe 51, Belgrade, Serbia
[1] e-mail: iviatanasov@yahoo.com
[2] e-mail: marija.vuksic@iks.rs

**ABSTRACT.** The modelling and calculation of parameters of involute gear pair dynamics is basic requirement for studying of gears stability, [1]. Many authors are interested in the nonlinear dynamics of gears and they have investigated and discussed this problem from wide range of aspects, [2-5]. In this paper, the dynamic model of involute gear pair is used in the comparative study of spur and helical involute gears with their stability as the dominant aspect.

The described dynamic models can be used for the analysis of the gears oscillation parameters, only if all time-varying functions are known. The stiffness and load distribution have been calculated using the developed finite element model, [6]. The special attention was paid to define the procedure for time-varying mesh stiffness calculation. The presented research successfully put together numerical methods for stress and strain calculations and numerical iterative methods for differential equations solving.

The particular gear pair is chosen for numerical calculation and investigation of dynamics of gears. Excellent qualitative superposition of results with results of other authors, [3], led to the conclusion that developed models and procedures are suitable for future research. The obtained results shown as phase portraits confirm that the helical gear pair has more stabile work than the spur gear pair with same main geometry and load. Although this fact has been known in theory of gears, the presented research is important because the developed procedure gives the accurate assessment of the differences between spur and helical gear pairs and can be used for future investigation of optimal gears parameters (e.g. tooth profile, nominal load) with aspect of gear pair stability. Also, the time-varying meshing damping could be incorporate in developed model of gears dynamics, [7].

**Keywords**: gears, stability, time-varying mesh stiffness

## References

[1] Hedrih (Stevanović) K., Knežević R., Structural stability of the planetary reductor nonlinear phase portrait, *Facta Universitatis, Series: Mechanical Engineering*, 1 (7) (2000), pp. 911 – 923.

[2] Dimitrijević D. et al., Dynamic analysis of the stress and strain state of the spur gear pair, *Scientific Technical Review*, LVII 3-4 (2007), pp.20-25.

[3] Amezketa M. et al., Dynamic model of a helical gear pair with backlash and angle-varying mesh stiffness, *Multibody Dynamics, ECCOMAS Thematic Conference Warsaw*, Poland, (2009), 29 June–2 July

[4] Li S. et al., Nonlinear dynamic simulation of coupled lateral-torsional vibrations of a gear transmission system, *IJCSNS International Journal of Computer Science and Network Security*, 6 (8A) (2006), pp.27-35.
[5] Andersson A., Vedmar L., Dynamic model to determine vibrations in involute helical gears, *Journal of Sound and Vibration,* 260 (2003), pp.195–212.
[6] Atanasovska I. et al., Developing of gear FEM model for nonlinear contact analysis, *Proceedings of 1. International Congress of Serbian Society of Mechanics, Kopaonik*, (2007), pp.695-703.
[7] Amabili M., Rivola A., Dynamic analysis of spur gear pairs: steady-state response and stability of the SDOF model with time-varying meshing damping, *Mechanical Systems and Signal Processing*, 11(3) (1997), pp.375-390.

# DINAMIKA ZUPČASTIH PAROVA SA PERIODIČNO PROMENLJIVOM KRUTOŠĆU SPREGE – ZUPČANICI SA PRAVIM ZUPCIMA U POREĐENJU SA ZUPČANICIMA SA KOSIM ZUPCIMA

**I. Atanasovska[1], M. Vukšić Popović[2]**

Institute Kirilo Savić
Vojvode Stepe 51, Belgrade, Serbia
[1] e-mail: iviatanasov@yahoo.com
[2] e-mail: marija.vuksic@iks.rs

**ABSTRACT.** Razvijanje dinamičkog modela evolventnog zupčastog para i proračun njegovih parametara, neophodno je pri izučavanju stabilnosti zupčanika, [1]. Mnogi autori se bave nelinearnom dinamikom zupčanika, tako što istražuju i diskutuju ovu problematiku sa različitih aspekata, [2-5]. U ovom radu, model dinamičkog ponašanja zupčastog para sa evolventnim profilom korišćen je u uporednoj analizi zupčanika sa pravim i kosim zupcima, sa aspekta stabilnosti rada zupčastog prenosnika.
Opisani dinamički modeli mogu se koristiti za analizu oscilacija zupčanika samo kada su poznate sve veličine koje zavise od vremena. Krutost i raspodela opterećenja su zbog toga određivani pomoću posebno razvijenog modela konačnih elemenata, [6]. Posebna pažnja je posvećena definisanju metodologije za određivanje vremenski promenljive krutosti sprege. Prikazana istraživanja su uspešan spoj primene numeričkih metoda za određivanje deformacionog i naponskog stanja i numeričkih iterativnih metoda za rešavanje diferencijalnih jednačina.
Jedan konkretan zupčasti par je izabran za numerički proračun i za izučavanje dinamičkog ponašanja. Odlično kvalitativno poklapanje dobijenih rezultata sa rezultatima drugih autora, [3], potvrđuju primenljivost razvijenih modela u daljim istraživanjima. Dobijeni rezultati prikazani pomoću faznih portreta potvrđuju da zupčasti par sa kosim zupcima ima stabilniji rad od zupčanika sa pravim zupcima, pri istim opterećenjem i osnovnim geometrijskim karakteristikama. Iako je ova činjenica poznata u teoriji zupčanika, prikazano istraživanje je značajno jer je razvijena metodologija koja može da se primeni za preciznu procenu razlike između zupčanika sa pravim i kosim zupcima, kao i za dalja istraživanja optimalnih parametara zupčanika (na pr. profila zubaca, nominalnog opterećenja i sl.) sa aspekta stabilnosti zupčastog para. Takođe, vremenski promenljivo prigušenje može biti ugrađeno u razvijeni model dinamike zupčanika, [7].

**Keywords**: zupčanici, stabilnost, vremenski promenljiva krutost sprege

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-22. ID. 4.**

# FINITE ELEMENT MODELING OF WINDSHIELD- AND WING-BIRD STRIKES

**M. Ugrčić**[1]

[1] Economics Institute
Kralja Milana 18, 11000 Belgrade, Serbia
e-mail: ugrcicmarinko@gmail.com

**ABSTRACT.** Aircraft windshields and wing parts are intensively vulnerable to damage and hence need a certification requirement for a proven level of impact resistance. Bird strike experiments are very expensive and henceforth explicit numerical modeling techniques have grown importance [1-8].

The numerical simulation is carried out using smooth particle hydrodynamics (SPH) method running in the nonlinear explicit finite element code ANSYS AUTODYN® [9].

Three different bird geometries: cylindrical, cylinder with hemispherical ends, and ellipsoidal, are considered for the finite element bird simulation. Majority of the birds responsible for damage in a bird strike are medium sized. Bird mass is fixed at 2 kg and an aspect ratio of 2 is assumed. The airplane velocity can be assumed for the bird. It allows accepting bird velocities 100, 200, 300, 400 and 500 m/s, respectively.

An important step in the bird strike simulation is the development of an "artificial" bird model. The artificial bird models would replace real birds in actual impact simulations. The accurate modeling of an "artificial" bird includes the shape and material modeling of the bird. Regarding selection of the soft "artificial" bird material there are different approaches. The wax, foam, emulsions, rubber and gelatin can be chosen. In this work the porous water (homogenous gas-water mixture) was accepted with porosity 30 and 40% [10].

Depending on the cockpit design two geometries of windshield are chosen. In both cases one-layer windshield geometry is generated as shell part. Equation of state of accepted glass material, Polycarbonate, is considered to be linear with very small plastic part, so that could treat it like brittle material. The thickness of Polycarbonate windshield was considered to be 12 an 24 mm.

The design methodology followed in the development of the typical wing leading-edge is to have skin, rib and baffle plate, which will prevent the bird from impacting the front spar incase of penetration of the skin. A typical wing leading-edge profile of a medium aircraft consisting of nose box skin, baffle and two side ribs is selected for this purpose. Aluminum alloy was used for skin and

baffles and ribs were designed using Kevlar-epoxy composites. The Lagrangian technique was used to model the wing. The thickness of skin was wearied from 2 to 4 mm.

In the paper are presented theoretical analysis and results of numerical simulations of dynamic response of the windshield and wing loaded by the bird impact. The focus is given to the validation of the contact pressure, stress and strain and possible damage of mentioned aircraft parts. Dependence of given parameters on the variation of bird geometry, impact velocity, wall thickness of relevant aircraft parts was discussed. As well, some results of experimental data were given.

**Keywords**: Bird strike, impact simulation, finite element analysis, windshield, wing, SPH method

### References

[8] Wilbeck J. S., Rand J. L., The Development of Substitute Bird Model, *ASME Journal of Engineering for Powe*r, Volume **103**, (1981), pp. 725-730.
[9] Lavoie M. A., Gakwaya A., Ensan M. N., Zimcik D. G., Review of Existing Numerical Methods and Validation Procedure Available for Bird Strike Modeling, *International Conference on Computational & Experimental Engineering and Sciences*, (2007), pp. 111-118.
[10] Nizampatman L. S., Models and Methods for Bird Strike Load Predictions, *PhD Dissertation*, Faculty of Graduate School, Wichita State University, USA, (2007)
[11] Benson, D. J., An efficient, Accurate, Simple ALE Method for Nonlinear Finite Element Programs, *Computer Methods in Applied Mechanics and Engineering*, **72**, (1989), pp. 305-350.
[12] Meo M., Vignjević R., Maengo M., Numerical Simulations of Low-Velocity Impact on an Aircraft Sandwich Panel, *Composite Structures*, **62**, (2003), pp. 353–360.
[13] Ivančević, D., Smojver I., Hybrid Approach in Bird Strike Damage Prediction on Aeronautical Composite Structures. *Composite Structures*, **94**, 1, (2011), pp. 15-23.
[14] Heimbs, S.: Bird Strike Simulations on Composite Aircraft Structures, *2011 SIMULIA Customer Conference*, Barcelona, Spain, May 17-19, (2011), pp. 73-86.
[15] Guida M., Marulo F., Meo M., Riccio M., Russo S., Fiber metal laminate for bird impact conditions. *Numerical and experimental analysis*, Proceedings of ICCST/6, Durban, South Africa, (2007), pp CD-ROM.
[16] ANSYS AUTODYN User's Manual *Theory manual*, ANSYS Inc. Southpointe, 275 Technology Drive, Canonsburg, PA, USA, (2010).
[17] Iannucci L., Bird-strike impact modeling, *Seminar Foreign Object Impact and Energy Absorbing Structure*, London, England, (1998), pp. 11-29

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-23. ID-19.**

# THE POTENTIAL ROLE OF BULK WATER IN BRAY–LIEBHAFSKY OSCILLATORY REACTION

**D. Stanisavljev, M. Milenković**

Faculty of physical chemistry, The University of Belgrade,
Studentski trg 12-16, 11000 Belgrade, Serbia
e-mail: dragisa@ffh.bg.ac.rs

**ABSTRACT.** Bray–Liebhafsky (BL) reaction[1] is one of the oldest oscillatory reactions known for more than 100 years. Bray himself supposed that reaction proceeds through two periodically dominating pathways (branches):

$$2IO_3^- + 2H^+ + 5H_2O_2 \rightarrow I_2 + 5O_2 + 6H_2O \qquad \Delta G^0 = -464.02\,\text{kJ/mol} \qquad (1)$$

$$I_2 + 5H_2O_2 \rightarrow 2IO_3^- + 2H^+ + 4H_2O \qquad \Delta G^o = -482.6\ \text{kJ/mol} \qquad (2)$$

The first one represents reduction of iodate by hydrogen peroxide to iodine, whereas the second one stands for backward reduction of iodine to iodate. Both pathways are complex processes involving a number of insufficiently known elementary steps. Despite that initial chemical composition consists of only hydrogen peroxide, potassium iodate and a suitable strong acid (sulphuric, or perchloric) dissolved in water, the detailed mechanism of its peculiar dynamics is still unknown. The mechanism of switching the whole reaction between two branches is of considerable interest since similar mechanisms in living organisms may be the source of signaling for various metabolic processes.[2]

One of the greatest difficulties in understanding the mechanism of Bray-Liebhafsky reaction (and other similar systems as well) is the lack of fast, sensitive and selective experimental techniques for monitoring the short lived and reactive intermediates in branches (1) and (2). In such situation, one way to theoretically simulate oscillatory dynamics is basically chemical-mathematical in which unknown rate constants of the assumed system of nonlinear differential kinetic equations are fitted to obtain oscillatory numerical solutions.[3] Generally, modeling chemical oscillations is not unique and depends of the assumed components and reaction stoichiometry.[4;5] Closer understanding the realistic reaction mechanism may be further facilitated by understanding the periodic flow of energy through branches (1) and (2). This approach can be introduced by considering (1) and (2) from the thermodynamic and kinetic point of view. Both branches are thermodynamically favorable since they are accompanied by decrease of free energy ($\Delta G^0<0$). Despite of this, process (2) is difficult to proceed[6-8] so that acidic solution of hydrogen peroxide and iodine may stay for ours without appreciable reaction. It can be understood by the kinetic theory of chemical reactions assuming high activation energies of some of its constituting steps. The existence of highly reactive (rich in energy) free radicals in one of the reaction subsystems *i.e.* iodide-peroxide acidic solutions is experimentally confirmed.[9] Despite of this, under the BL reaction conditions, this branch periodically becomes a dominant process. Understanding how energy may predominantly flow

"through" the process with higher energy barrier may further help in approaching the real mechanism. The problem can be rationalized if a non-equilibrium energy distribution may appear with the selective enhancement of process (2). Supporting this, short temperature jumps during oscillations are experimentally detected.[10] The existence of a nonequilibrium energy distribution is not possible to detect directly but it should be accompanied by participating bulk water which is present in high excess and due to large heat capacity absorbs (and exchanges with the environment) most of the energy released in chemical reactions. The indications of changes in bulk water during oscillatory reactions are presented in Figure 1 showing chemical oscillations during BL reaction monitored potentiometrically by Pt electrode (Fig 1.a) where arrows indicate interval in which $^1$H-NMR spectra are recorded. Accompanying changes of $^1$H-NMR spectra during oscillations 1, 4 and 5 are shown in (Fig 1.b)[11].

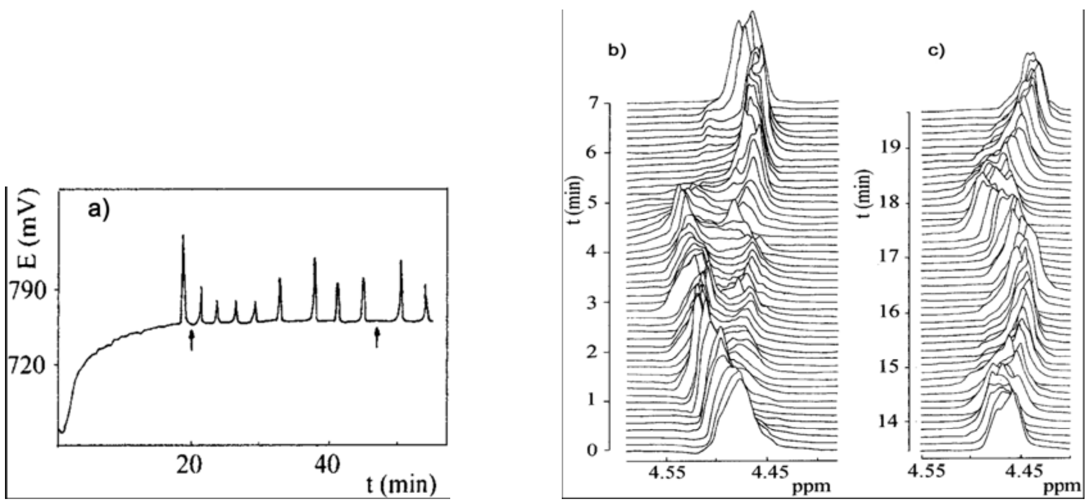



**Fig. 1.** BL-Potentiogram a) and H-NMR spectra during oscillations 1,4 and 5
(Reprinted with permission from[11], Copyright 1998)

**Keywords**: chemical oscillators, Bray-Liebhafsky reaction, Energy distribution

## References

[1] Bray, W. C. *J.Am.Chem.Soc.* **43**, (1921), pp. 1262-1267.
[2] Epstein, I. R.; Pojman, J. A. *An Introduction to Nonlinear Chemical Dynamics*, Oxford, University Press: New York, 1998.
[3] Clarke, B. L. *Advances in Chemical Physics*, Wiley and Sons: New York, 1980.
[4] Schmitz, G. *J.Chim.Phys.*, **84**, (1987), pp. 957-966.
[5] Kolar-Anić, Lj.; Schmitz, G. *J.Chem.Soc.Faraday Trans.* **88**, (1992), pp. 2343-2349.
[6] Liebhafsky, H. A. *J.Am.Chem.Soc.* **54,** (1932), pp.1792-1806.
[7] Schmitz, G. *Phys.Chem.Chem.Phys.* **3**, (2001), pp. 4741-4746.
[8] Furow, S. *J.Phys.Chem*, **91**, (1987), pp. 2129.
[9] Stanisavljev, D.; Milenković, M.; Mojović, M.; Popović-Bijelić, A. *J.Phys.Chem.* **115,** (2011), pp.7955-7958.
[10] Matsuzaki, I.; Woodson, J. H.; Liebhafsky, H. A. *Bull.Chem.Soc.Jap.* **43**, (1970), pp. 3317-3317.
[11] Stanisavljev, D.; Begović, N.; Žujović, Z.; Vučelić, D.; Bačić, G. *J.Phys.Chem.* **102,** (1998), pp. 6883-6886.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-24. ID-31.**

# APPLICATION OF NON-LINEAR FREQUENCY RESPONSE METHOD FOR INVESTIGATION OF PERIODICALLY OPERATED CHEMICAL REACTORS

**D. Nikolic Paunic[1] , M. Petkovska[2]**

[1] Institute of Chemistry, Technology and Metallurgy,
The University of Belgrade, Njegoseva 12, 11000 Belgrade
e-mail: daliborkan@ihtm.bg.ac.rs
[2] Faculty of Technology and Metallurgy,
Department of Chemical Engineering ,
The University of Belgrade, Karnegijeva 4, 11000 Belgrade
e-mail: menka@tmf.bg.ac.rs

**ABSTRACT.** Process intensification aims to increase the efficiency of processes in the chemical industry by development of new concepts of processing methods and equipments. One way to achieve better performance is to operate the process in a periodic way. Periodic operation is achieved by periodic modulation or periodic forcing of one or more inputs into the system around a chosen steady-state point. For nonlinear systems the average value of the output when the input is periodically modulated is different from the steady-state value. For some cases, the periodic process can be superior to the optimal steady-state operation [2], while for some other it gives worse results. Most chemical processes are nonlinear in nature, and consequently they are candidates for possible improvements (increased conversion, selectivity, yield, production rates) through periodic operation.

Evaluating whether a periodic operation of a chemical processes would be beneficial and selecting the optimal operational conditions that maximize the process performance is often carried out by experimental and/or numerical studies. However, the experimental approach, as well as numerical simulations, can be rather time consuming and costly. It is therefore of economic importance to carry out theoretical studies alternatively for assessing the effect of periodic operation of a chemical processes and then selecting the optimal forcing inputs.

We will present a new, fast and easy method for evaluation of system performance in periodic operation, called the nonlinear frequency response method (NFRM). Nonlinear frequency response is a quasi-stationary response of a nonlinear system to a periodic (sinusoidal or co-sinusoidal) input change, around a steady-state. One of the most convenient tools for treating nonlinear frequency responses is the concept of higher order frequency response functions (FRFs), which is based on Volterra series and generalized Fourier transform. This concept is very convenient for analysing weakly nonlinear systems [1].

Frequency response of a weakly nonlinear system, in addition to the basic harmonic, contains a non-periodic (DC) term, and an indefinite sequence of higher harmonics. A nonlinear model with

polynomial nonlinearity ($G$), can be replaced by an indefinite sequence of FRFs of different orders ($G_1(\omega_1), G_2(\omega_1, \omega_2), G_3(\omega_1, \omega_2, \omega_3), \ldots$), which are directly related to the DC component and different harmonics of the response. It is important to notice that the DC component is responsible for the average performance of the periodic process. On the other hand, the DC component has a dominant term which is proportional to the asymmetrical second order FRF $G_2(\omega, -\omega)$, so it can be roughly estimated based on this function. The sign of the asymmetrical second order FRF $G_2(\omega, -\omega)$, defines the sign of the DC component and, consequently gives an answer to the question whether the periodic operation is favourable or not.

The NFRM can be extrapolated to nonlinear systems with multiple modulated inputs. To define the model with multiple modulated inputs, it is necessary to define several sets of FRFs which correlate each output to each input and cross-FRFs which correlate each output to several inputs.

In our previous work [3, 4], the NFRM was used to analyze the periodic performance of three standard reactor types: continuous stirred tank reactor, a plug flow tubular reactor (PFTR), and a dispersive flow tubular reactor (DFTR) with periodic changes of the input concentration. The cases of homogeneous [3] and heterogeneous [4] *n*-th order reaction takes place under isothermal conditions.

This method is also applied for analysis of periodic reactor operation for: 1) isothermal CSTR in which simple *n*-th order homogeneous reaction takes place, when inlet concentration and flow are modulated simultaneously and separately, 2) non-isothermal CSTR for simple *n*-th order homogeneous reaction when inlet concentration and temperature of inlet stream are modulated separately and simultaneously .

It has been estimated that conversion in chemical reactors can be significantly improved by periodic operation, for certain values of the reaction order ($n$) and for some modulated inputs, forcing parameters (periods, amplitudes, phase shift between synchronized inputs). The results obtained by nonlinear frequency response method were compared with the results of numerical solutions, and good agreement was obtained.

**Keywords**: nonlinear frequency response method, frequency response functions, periodic operation, chemical reactor.

## References

[1] Weiner D.D., Spina J.F., *Sinusoidal analysis and modelling of weakly nonlinear systems*, Van Nostrand Reinhold, New York, 1980.

[2] Silveston P.L, *Composition Modulation of Catalytic Reactors*, Gordon and Breach Science Publishers, Amsterdam (1998).

[3] Markovic A, Seidel-Morgenstern A, Petkovska M, Evaluation of the potential of periodically operated reactors based on the second order frequency response function, *Chemical Engineering Research and Design,* 86 (2008), pp. 682-691.

[4] Petkovska M, Nikolic D, Markovic A, Seidel-Morgenstern A, Fast evaluation of periodic operation of a heterogeneous reactor based on nonlinear frequency response analysis, *Chemical Engineering Science* 65 (2010), pp. 3632-3637.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-25. ID-38.**

# APPLICATION OF NONLINEAR FREQUENCY RESPONSE METHOD FOR INVESTIGATION OF GAS ADSORPTION

**D. Brzić, M. Petkovska**

Faculty of Technology and Metallurgy, University of Belgrade, Karnegijeva 4, 11000 Belgrade. e-mail: danica@tmf.bg.ac.rs, menka@tmf.bg.ac.rs

**ABSTRACT.** One of the widespread methods for investigation of the dynamics of adsorption processes in porous solids is frequency response (FR) method. By using small input amplitudes of the sinusoidal perturbed input, the obtained response contains only the first harmonic which means that only the first (linear) frequency response function (FRF) in the frequency domain can be defined. By comparing the pattern of the experimental FRF with the theoretical one derived for an assumed model, the parameters of the dynamic model can be obtained. However, taking into account that adsorption systems are nonlinear, application of the previously described linear tool is not completely justified.

Application of a thorough nonlinear analysis of adsorption processes, however, requires complex mathematical tools, as well as sophisticated experimental systems. Significant effort in this field has been made by Petkovska and Do [1] who developed the nonlinear frequency response (NFR) method which uses the concept of higher order FRFs, based on Volterra series and generalized Fourier transform. Their theoretical studies show a great potential of the NFR method for investigation of equilibrium and kinetics of gas adsorption systems [1,2]. Application of the NFR method is based on the representation of the nonlinear model as a set of higher order FRFs. It has been shown that the shape of the second order FRF enables unambiguous identification of the dynamic mechanism [3] and determination of the nonlinear model parameters [2].

In this work the experimental procedure for obtaining the first and the second order FRFs, for a batch adsorber, is presented. The adsorption system $CO_2$ on commercial zeolite 5A was used for the experiments. The volume of the adsorber was used as the modulated input and the pressure in the adsorber as the measured output. The input amplitudes of 3 and 7 % were used and the experiments were performed in the range of frequencies from 0.0005 to 1 Hz. The volume and the pressure were continuously measured and these measured signals were transferred into the frequency domain by fast Fourier transform and used for estimation of the first and second order FRFs. Based on the estimated second order FRF, a bimodal mechanism have been identified and the corresponding time constants have been determined. The quality of the obtained experimental results has been analyzed and improvements in experimental measurements have been proposed.

**Keywords**: Nonlinear frequency response, Gas adsorption, Frequency response functions

## References

[1] Petkovska, M., Do, D. D., Nonlinear frequency response of adsorption systems: Isothermal batch and continuous flow adsorbers. *Chemical Engineering Science* **53** (1998), 3081-3097.

[2] Petkovska, M., Nonlinear frequency response method for investigation of equilibria and kinetics in adsorption systems. In: Spasic, A.M., Hsu, J.P.(Eds.), Adsorption Systems in Finely Dispersed Particles: Micro Nano and Atto-Engineering. CRC Taylor & Francis, Boca Raton, (2005), pp. 283–327.

[3] Petkovska, M., Do, D., Use of higher-order frequency response functions for identification of nonlinear adsorption kinetics: Single mechanisms under isothermal conditions. *Nonlinear Dynamics* **21** (2000), 353-376.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-26. ID-40.**

# NUMERICAL EVIDENCE OF COMPLEX NONLINEAR PHENOMENA OF THE BELOUSOV–ZHABOTINSKY OSCILLATORY REACTION UNDER BATCH CONDITIONS

**S. M. Blagojević[1], S. N. Blagojević[3] and S. Anić[2]**

[1]Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, Belgrade, Serbia
e-mail: slavica.blagojevic@pharmacy.bg.ac.rs
[2]IChTM – Department of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade
[3]Institute of General and Physical chemistry, Studentski trg 12, Belgrade, Sebia.

**ABSTRACT.** An oscillatory evolution of the concentrations of the reaction species is one of the most striking examples of the nonlinear dynamic phenomena. The most common example is an oscillatory Belousov–Zhabotinsky (BZ) reaction, which is the oxidation of malonic acid (MA), by bromate ions in acidic solution, catalyzed by $Ce^{3+}$ or $Ce^{4+}$ metal ions. Under various conditions in a closed and open reactor, this reaction exibits sustained oscillations, multiple steady states, multiply periodic oscillations, mixed–mode oscillations, chaos and spatially propagating waves. With the aim to explain the experimentally obtained dynamic states of reaction, the numerous models of the mechanism of the BZ reaction and their modifications were proposed [1–3].

The numerical calculations of the dynamic behavior of the BZ reaction under batch condition were carried out by the known stoichiometric model where we included the new species and new reactions that did not exist in the previously known GF model. [2,4]

The numerical simulations of the BZ reaction were carried out under batch conditions at 30 °C and with the constant values of the species (initial) concentrations (in mol $dm^{-3}$): $[H_2SO_4]_0 = 1.00$, $[MA]_0 = 1.60 \times 10^{-2}$, $[KBrO_3]_0 = 6.20 \times 10^{-2}$, $[KBr]_0 = 1.50 \times 10^{-5}$ $[Ce_2(SO_4)_3]_0 = 2.50 \times 10^{-3}$. As a mathematical tool was used MATLAB program package.

Changes in the dynamic structure of the BZ system are presented by the temporal evolution of the logarithm of the concentration of bromide. We have obviously obtained rich nonlinear temporal phenomena such as high– and low–amplitude regular oscillations, aperiodic oscillations, mixed–mode oscillations and low–amplitude sinusoidal oscillations (Figure1), which are typical for open reaction system (Continuously Stirred Tank Reactor (CSTR)).

**Keywords**: nonlinear dynamics, oscillatory reactions, Belousov–Zhabotinsky reaction.

**References**

[1] T. Turanyi, L. Györgyi, R. Field, J. Phys. Chem., **94**, 1990, 7162 – 7170.
[2] L. Györgyi, S. L. Rempe, R. J. Field, *J. Phys. Chem*., **95**, 1991, 3159 – 316.
[3] S. Blagojević, S. Anić, Ž. Čupić, N. Pejić, Lj. Kolar–Anić, *Phys. Chem. Chem. Phys*. **10**(44), (2008), 6658 – 6664.
[4] K. Pelle, M. Wittmann, K. Lovrics, Z. Noszticzius, M. L. Turco Liveri, R. Lombardo, J. Phys. Chem. A, **108**, 2004, 5377 – 5385.

**Acknowledgements** The present investigations were supported by the Ministry of Education and Sciences of the Republic of Serbia, Grants no. 172015 and no. 45001.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-27. ID-41.**

# BIFURCATION ANALYSIS OF THE OSCILLATORY REGION OF A HYPOTHALAMIC-PITUITARY (HPA) AXIS MODEL

**S. Maćešić[1], V. M. Marković[1], A. Ivanović-Šašić[2], Ž. Čupić[2] and Lj. Kolar-Anić[1]**

[1]*Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia*
e-mail: macesic.stevan@gmail.com
[2]*Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia*
e-mail: zcupic@nanosys.ihtm.bg.ac.rs

**ABSTRACT.**

The hypothalamic-pituitary-adrenal (HPA) axis represents a neuroendocrine system involved in maintaining homeostasis in mammalian organisms under physiological conditions and stress [1]. Cortisol, the HPA axis principal hormone in humans, exhibits complex dynamic behavior with two characteristic frequencies: ultradian oscillations, with a period of 20-120 min superimposed on circadian oscillations, with a period of about 24 h. For maintaining homeostasis, it is essential that the dynamics of the HPA axis operate in the adequate oscillatory regime.

The oscillatory dynamics of the HPA axis can be emulated with high reliability by using stoichiometric models of its activity [2-5]. In these models, differential equations describing temporal evolution of the concentrations of the species as dynamical variables are based on the mass-action kinetics according to the proposed mechanism between the species included in the model. In that regard, we utilize in this paper a five-dimensional stoichiometric model of the HPA axis function and investigate transitions between different unstable stationary states that arise for different values of the model parameters. The model comprises of five internal species that are of key significance for dynamic regulation of the HPA axis: corticotropin-releasing hormone (CRH), adrenocorticotropin (ACTH), cortisol (CORT) and aldosterone (ALDO), the chief hormones in the HPA axis, and cholesterol, the principal precursor of cortisol and aldosterone. Each of the species (apart from CRH) possesses an oscillatory region, which makes it possible to study and predict their ultradian dynamics under different parameter conditions. Due to its physiological and medical significance, we specifically pay our attention to and carry out detailed investigations of the oscillatory region of cortisol.

To analyze the repertoire of the cortisol's dynamics within its oscillatory region, we conduct a bifurcation analysis by systematically varying one of the model's parameters (the rate constant of cholesterol elimination). By increasing this bifurcation parameter, we distinguish within cortisol's oscillatory evolution two periodic regimes: one with only small-amplitude (for lower values of the bifurcation parameter) and the other one with only large-amplitude oscillations (for higher values of the bifurcation parameter). Furthermore, in the area of transition between small- and large-amplitude

oscillations the system is in a chaotic dynamical state, with sequences of irregular number of repetitions of small- and large-amplitude oscillations. With the increase of the bifurcation parameter, we also determine that this chaotic regime evolves from the state with high number of small-amplitude and a few large-amplitude oscillations to the state where the situation is opposite.

This and similar studies encompassing the investigation of transitions between periodic and aperiodic dynamical states within oscillatory regions of the systems far away from equilibrium might contribute to broader understanding of self-organizing dynamical properties underlying these systems. In the context of the HPA axis, detailed examinations of the dynamical states which this system can display under different physiological conditions may be of potential benefit to medical treatments of diseases associated with dysregulation of the HPA dynamics, such as major depression, bipolar and posttraumatic stress disorder, as well as to designing more efficacious procedures for administration of drugs affecting HPA axis functioning.

**Keywords**: bifurcation analysis, oscillations, chaos, HPA axis

**Acknowledgement** – This investigation was supported by the Ministry of Education and Science of Republic of Serbia, under the projects No. 172015 and 45001.

## References

[5] Miller WL and Chrousos GP, The Adrenal Cortex, in Felig P, Frohman L (eds) *Endocrinology & Metabolism*, McGraw-Hill, New York, 1960, pp. 387-524.

[6] Jelić S, Čupić Ž and Kolar-Anić Lj, Mathematical modelling of the hypothalamic-pituitary-adrenal system activity, *Mathematical Bioscience,* **197** (2005), pp. 173-187.

[7] Marković VM, Čupić Ž, Ivanović A and Kolar-Anić Lj, The Stability of the Extended Model of Hypothalamic-Pituitary-Adrenal (HPA) Axis examined by Stoichiometric Network Analysis (SNA), *Russian Journal of Physical Chemistry A*, **85** (2011), pp. 2327-2335.

[8] Marković VM, Čupić Ž, Vukojević V and Kolar-Anić Lj, Predictive Modeling of the Hypothalamic-Pituitary-Adrenal (HPA) Axis Response to Acute and Chronic Stress, *Endocrine Journal*, **58** (2011), pp. 889-904.

**[9]** Maćešić S, Čupić Ž and Kolar-Anić Lj, Model of the nonlinear reaction system with autocatalysis and autoinhibition: Stability of dynamic states, Hemijska industrija (2012), doi:10.2298/HEMIND120210034M.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-28. ID-42.**

# A NEW STRUCTURE OF CHAOS IN THE BRAY LIEBHAFSKY OSCILLATORY REACTION

**Ana Ivanović-Šašić[1], Vladimir Marković[2], Željko Čupić[1], Ljiljana Kolar-Anić[2], Slobodan Anić[1,]**

[1]*Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia*
e-mail: ana.ivanovic@nanosys.ihtm.bg.ac.rs
[2]*Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia*
e-mail:

**ABSTRACT.** The deterministic chaos in chemical reaction systems is a phenomenon known for a long time. However, in the first discovered homogeneous oscillatory reaction, Bray–Liebhafsky (BL) reaction [i,ii], this phenomenon has been discovered and more comprehensively described only recently. With increasing the flow rate $j_0$, in both experimental and numerical simulation settings of the BL reaction in continuously fed well stirred tank reactor (CSTR), transitions could be observed between stable steady states and various simple, complex and chaotic oscillations. Moreover, in numerical simulations, additional chaotic transitions are observed between each two mixed-mode oscillatory periodic states.

In particular, when the control parameter, flow rate $j_0$, is increased, between each successive mixed-mode oscillatory dynamic states, always the same transition scenario has been found. Novel chaotic states occurring in these transitions have been characterized using several quantitative and qualitative methods for dynamic systems analysis: from time series, corresponding attractors and Poincare´ maps, to power spectra and the largest Lyapunov exponent method [iii-v]. By using the above-mentioned methods complementary, we have been able to make distinction between chaotic states themselves, since the methods give the possibility for detection of dynamic ‘‘structures’’ in the chaotic regions. To better classify these novel chaotic states, we have adopted the three types of observed chaos as ‘‘period-doubling’’ chaos, ‘‘unstructured’’ chaos and ‘‘mixed-mode structured’’ chaos, and characterized their dynamic structures. ‘‘Period-doubling’’ chaos considers regular doubling in the number of both small and large oscillations with parameter changes, until the chaos with infinite number of periods appears. This type of chaos can be identified only by some analytical techniques. At the end of such period-doubling route to chaos, we identify the ‘‘unstructured’’chaos, a type of chaotic dynamics with the irregular repeating of the sequence of oscillations and, thus, without a recognizable ‘‘structure’’, additionally characterized by a form of wide bands of trajectories in the phase space. ‘‘Mixed-mode structured’’ chaos is denoted by irregular patterns of miscellaneous types of oscillation sequences comprising different number of

small and large oscillations. This type of chaos is most obvious and can be detected from the time series alone.

Here, we continue to describe the fine structure of the observed chaos in BL reaction under CSTR conditions arising between different periodic oscillatory sequences, and their transformations inside one chaotic window. Special emphasis is put to symmetric and asymmetric evolution of chaos observed in corresponding strange attractors and ensuing Poincaré maps. We find the observed scenario of evolution of dynamic states is a universal feature throughout the whole mixed-mode region, as well as throughout other mixed-mode regions obtained under different initial conditions. Resembling dynamic sequences with corresponding chaos between each two successive periodic states can be expected to be found in almost all other models of complex reaction systems exhibiting mixed-mode oscillatory behavior.

**Acknowledgment.** The present investigations were supported by the Ministry of Sciences of the Republic of Serbia, (Grants no. 172015 and no. 45001).

**Keywords**: chaos, chaotic "structures", mixed-mode oscillations, Bray-Liebhafsky (BL) reaction, period-doubling

## References

[1] Bray W. C, A Periodic Reaction in Homogenous and its Relation to Catalysis, *Journal of the American Chemical Society,* **43** (1921), pp.1262-1267.
[1] Bray W. C. and Liebhafsky H. A., Reaction Involving Hydrogen Peroxide, Iodine and Iodate Ion, I. Introduction, *Journal of the American Chemical Society,* **53** (1931), pp. 38-44.
[1] Hilborn R. C., Chaos and Nonlinear Dynamics, Oxford University Press, Oxford, 2nd edn, 2000.
[1] Scott S. K., Chemical Chaos, Clarendon Press, Oxford, 1997.
[1] Wolf A., Swift J. B, Swinney H. L, and Vastano J. A., Determining Lyapunov Exponents From a Time Series, *Physica D*, **16** (1985), pp. 285-317

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-29. ID-43.**

# QUALITATIVE AND QUANTITATIVE ANALYSIS OF THE CHAOTIC SEQUENCE IN THE BRAY-LIEBHAFSY REACTION

**Ana Ivanović-Šašić[1], Marija Janković[2], Stevan Blagojević[3] and Nataša Pejić[4]**

[1]*Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia,* e-mail: ana.ivanovic@nanosys.ihtm.bg.ac.rs
[2]*University of Belgrade, Institute Vinča, Radiation and Environmental Protection Department, P.O. Box 522, 11001 Belgrade, Serbia*
[3]*Departmant of Electrochemistry, Institute of General and Physical Chemistry, P.O. Box 551, 11000 Belgrade, Serbia*
[4]*Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, 11000 Belgrade, Serbia*

**ABSTRACT.** Bray-Liebhafsky (BL) reaction [1,2], the first discovered homogenous oscillatory chemical reaction, in which hydrogen peroxide catalytically decomposes into oxygen and water in the presence of hydrogen and iodate ions, exemplifies a highly nonlinear process that comprises a complex oscillatory evolution of numerous iodine intermediates involved in this reaction. Oscillatory phenomena in BL system have been acknowledged theoretically, in simulation studies of proposed models as well as experimentally, both in closed - batch reactor and open - continuously fed well stirred tank reactor (CSTR). [3] These results have confirmed not only periodic, but also chaotic dynamic behavior in the BL system under CSTR conditions [4].

Initial information on the type of dynamics for a given control parameter is obtained from time-series [5]. Although in time series regular and chaotic dynamics can often be clearly distinguished, in the case of close dynamical states, this method alone cannot be applied for their conclusive identification, indicating the need for further examinations using additional methods. Based on the appearance of the attractor in the phase space, chaotic and periodic behaviors can be further distinguished one from another. In the case of chaotic dynamics, attractor trajectory is open and never crosses itself, forming strange attractor which fills a part of phase space to which it is confined to, and gives the appearance of a band. As in the case of attractor, Poincaré maps [5] can also provide additional information about chaos in the analyzed dynamic state. In the Poincaré maps, made of a selected Poincaré section, in the case of chaotic dynamics, cross points continuously fill in a part of the map. As a complimentary, quantitative indicator for the type of the investigated oscillatory dynamics, chaos can be confirmed by a positive value, while periodicity by a negative value of the largest Lyapunov exponent [5].

In the present paper, we analyzed unusual oscillatory behavior of the Bray–Liebhafsky oscillatory reaction realized in CSTR reactor, by experimentally varying the flow rate $j_0$ as the control parameter. For flow rate $j_0$ in the range between $4.57 \times 10^{-3}$ min$^{-1}$ and $7.43 \times 10^{-3}$min$^{-1}$, between two stable steady states, we obtained various simple and chaotic oscillations, which were characterized by employing complementarily the above-mentioned qualitative and quantitative

methods for dynamic systems analysis: time series analysis, attractor analysis, Poincaré maps and maximal Lyapunov exponent method (calculated using the Wolf algorithm [6]). After finding the Lyapunov exponents, it was concluded that higher values of the positive Lyapunov exponent indicated more complex structure of detected chaotic dynamical states.

Comprehensive application of the abovementioned methods proved to be an efficient methodological approach to obtain reliable enough results for proper characterization and specification of chaotic dynamical states observed not only in numerical simulations but in experimental studies as well.

**Acknowledgements**. The present investigations were supported by the Ministry of Sciences of the Republic of Serbia, (Grants no. 172015, no. III43009 and no. 45001)

**Keywords**: chaos, oscillations, Lyapunov exponent, attractor, Bray-Liebhafsky (BL) reaction

## References

[10] Bray W. C, A Periodic Reaction in Homogenous and its Relation to Catalysis, *Journal of the American Chemical Society,* **43** (1921), pp.1262-1267.
[11] Bray W. C. and Liebhafsky H. A., Reaction Involving Hydrogen Peroxide, Iodine and Iodate Ion, I. Introduction, *Journal of the American Chemical Society,* **53** (1931), pp. 38-44
[12] Kolar-Anić Lj, Čupić Ž, Vukojević V and Anić S, Dinamika nelinearnih procesa, Fakultet za fizičku hemiju, Univerzitet u Beogradu, Beograd, (2011), pp. 227-302.
[13] Ivanović A. Z, Čupić Ž. D, Janković M. M, Kolar-Anić Lj. Z and Anić S. R, The chaotic sequences in the Bray-Liebhafsky reaction in an open reactor, *Physical Chemistry Chemical Physics*, **10** (2008), pp. 5848-5858.
[14] Hilborn R. C., Chaos and Nonlinear Dynamics, *Oxford University Press*, Oxford, 2nd edn, 2000.
[15] Wolf A., Swift J. B, Swinney H. L, and Vastano J. A., Determining Lyapunov Exponents From a Time Series, *Physica D*, **16** (1985), pp. 285-317

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-30. ID-44.**

# BRAY-LIEBHAFSKY REACTION. DYNAMIC STATES WHEN TEMPERATURE IS THE CONTROL PARAMETER

**J. Maksimovć[1], M. Milenković[1], N. Pejić[2], D. Stanisavljev[1], S. Anić[3]**

[1] Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12, Belgrade
[2] Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, Belgrade
[3] IChTM – Department of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade

**ABSTRACT.** The Bray-Liebhafsky (BL) [1,2] reaction is the decomposition of hydrogen peroxide into the water and oxygen in the presence of iodate and hydrogen ions:

$$2H_2O_2 \xrightarrow{IO_3^-, H^+} 2H_2O + O_2 \qquad \text{(D)}$$

The dynamic behavior of the BL reaction is examined in the CSTR (Continuously fed well Stirred Tank Reactor) when temperature is a control parameter. Under the following experimental conditions: $[KIO_3]_0 = 5.9\cdot10^{-2}$ M, $[H_2SO_4]_0 = 5.5\cdot10^{-2}$ M, $[H_2O_2]_0 = 4.0\cdot10^{-2}$ M; stirring speed: 900 rpm; the specific flow rate: $j_0 = 0.007$ min$^{-1}$ and the temperature varied from 41 $^{o}$C to 58 $^{o}$C. The evolution of the BL reaction was followed potentiometrically, by means of a Pt electrode connected to the Ag/AgCl reference electrode.

Time series obtained under the given experimental conditions are illustrated in Fig. 1a. For temperature in the range from 41.0 $^{o}$C to 44.5 $^{o}$C, stable stationary states are found. Simple periodic oscillations exist for temperature in the range from 44.8 $^{o}$C to 58 $^{o}$C. The corresponding bifurcation diagram is presented in Fig. 1b. When we are approaching the bifurcation point, the period of the large-amplitude oscillations remains constant, while the amplitude decreases. The bifurcation point (T = 44.6 $^{o}$C) is found by linear extrapolation of a plot of the square of the large-amplitude oscillations observed close to the bifurcation point versus the temperature (Fig. 2). The bifurcation occurs at the same value of the temperature, when approached from either side, i.e. hysteresis was not observed.

In this paper the experimental evidences for the onset and termination of oscillatory behavior via the supercritical Andronov–Hopf bifurcation [3] are presented.

**Keywords**: Bray-Liebhafsky reaction, dynamic states, bifurcation diagram, bifurcation point

## References

[1] W.C. Bray, *J.Am.Chem.Soc*., **43** (1921), 1262.
[2] W.C. Bray, H.A. Liebhafsky, *J.Am.Chem.Soc*., **53 (**1931), 38.
[3] P. Gray, S. Scott, *Chemical Oscillations and Instabilities: Nonlinear Chemical Kinetics,* Oxford Univ., Oxford, 1990.

# Figures

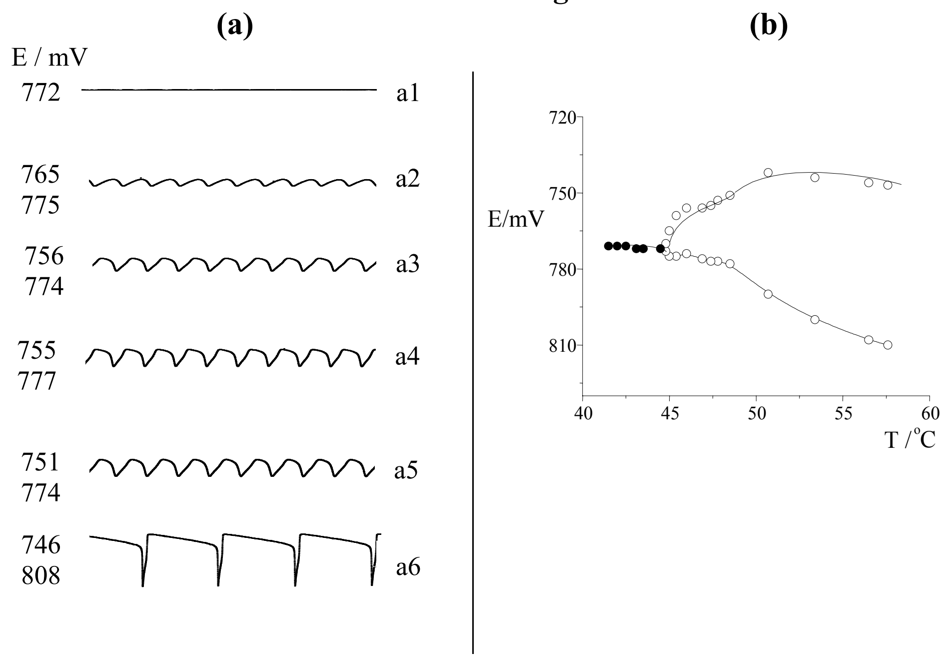



**Fig. 1**. **a)** Time series obtained under the given experimental conditions. a1) 44.5 °C, a2) 45.0 °C, a3) 46.0 °C, a4) 47.4 °C, a5) 48.5 °C, a6) 56.5 °C. **b)** Bifurcation diagram show transition from the stable stationary state (solid circles) to the large-amplitude oscillations (open circles) denoting minimal and maximal potential in an oscillation

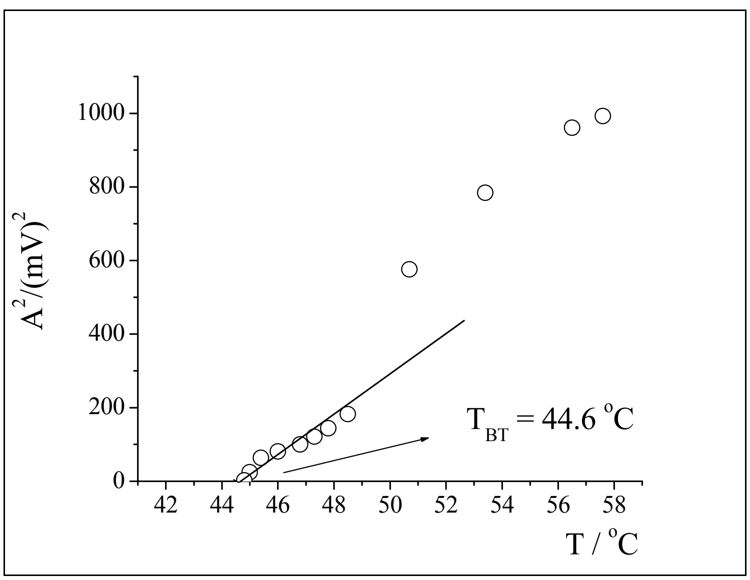



**Fig. 2.** Plot of the square of the large-amplitude oscillations as a function of the tempera

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-31. ID-45.**

# HYPOTHALAMIC-PITUITARY-ADRENAL (HPA) AXIS AS NONLINEAR SYSTEM WITH FEEDBACK

**Lj. Kolar-Anić[1], Ž. Čupić[2], S. Jelić[3], V. Marković[1], S. Maćešić[1], V. Vukojević[4]**

[1] Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, Belgrade, Serbia
e-mail: lkolar@ffh.bg.ac.rs
[2] Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Center of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade, Serbia
e-mail: zcupic@nanosys.ihtm.bg.ac.rs
[3] Department of Theoretical Physics and Physics of Condensed Matter 020/2, Vinča Institute of Nuclear Sciences, University of Belgrade, P. O. Box 522, 11001 Belgrade, Serbia
[4] Department of Clinical Neuroscience, Karolinska Institutet, CMM L8:01, 17176 Stockholm, Sweden

**ABSTRACT.** Oscillatory dynamics appears to be essential for self-organization and self-regulation in living systems. Oscillations with different periodicity, from millisecond to annual range, are implemented at all levels of organization of living organisms [1]. The hypothalamic-pituitary-adrenal (HPA) axis, a neuroendocrine system involved in maintaining homeostasis in mammalian organisms under physiological conditions and stress [2] is no exception to that rule. Cortisol, the HPA axis principal hormone in humans, exhibits complex dynamic behavior with two characteristic frequencies: ultradian oscillations, with a period of 20-120 min [2] superimposed on circadian oscillations, with a period of about 24 h.

The ultradian HPA axis dynamics was described in our previous publication [3] by four-dimensional stoichiometric model that comprises, as dynamic variables, CRH (corticotropin-releasing hormone), ACTH (adrenocorticotropic hormone), ALDO (aldosterone) and CORT (cortisol). In this model there is a reaction which describes the positive feedback actions of cortisol as well as the one that exemplifies cortisol negative feedback. Thus, ultradian self-regulation in the model is achieved through the experimentally established positive and negative feedback effects of cortisol on the HPA system *via* glucocorticoid and mineralocorticoid receptors [4]. However, these receptors are not included directly, but rather introduced implicitly through corresponding autocatalytic and autoinhibitory reactions. These two reactions represent the core of the model's instability.

The ultradian dynamics of the HPA system is described by the following set of ordinary differential equations, obtained from the reactions appearing in the above mentioned stoichiometric model, through the law of mass action:

$$\frac{d[\mathrm{CRH}]}{dt} = \mathrm{k}_0 - \mathrm{k}_1[\mathrm{CRH}] \quad (1)$$

$$\frac{d[\mathrm{ACTH}]}{dt} = \mathrm{k}_1[\mathrm{CRH}] - (\mathrm{k}_2 + \mathrm{k}_3 + \mathrm{k}_6)[\mathrm{ACTH}] - \mathrm{k}_4[\mathrm{ACTH}][\mathrm{CORT}]^2 \quad (2)$$

$$\frac{d[\mathrm{ALDO}]}{dt} = \mathrm{k_m} + \mathrm{k_3}[\mathrm{ACTH}] - \mathrm{k_5}[\mathrm{ALDO}][\mathrm{CORT}]^2 \qquad (3)$$

$$\frac{d[\mathrm{CORT}]}{dt} = \mathrm{k_2}[\mathrm{ACTH}] + \mathrm{k_4}[\mathrm{ACTH}][\mathrm{CORT}]^2 - \mathrm{k_5}[\mathrm{ALDO}][\mathrm{CORT}]^2 - \mathrm{k_7}[\mathrm{CORT}]. \qquad (4)$$

Dynamic variables $[\mathrm{CRH}]$, $[\mathrm{ACTH}]$, $[\mathrm{ALDO}]$ and $[\mathrm{CORT}]$ stand for the concentration of CRH, ACTH, aldosterone and cortisol, respectively.

The stability of the main steady state and selection of the parameters necessary for numerical simulations was examined by the Stoichiometric Network Analysis (SNA), the impressive method that allows the examination of the system of nonlinear differential equations with more than three variables. With this method even the five-dimensional stoichiometric model of the HPA axis with cholesterol as fifth included species was successfully investigated.

To account for the circadian rhythm governed CRH production, an extrinsic periodic function (*D*) has been introduced:

$$D = \mathrm{d_1} - 0.079145093\cdot\mathrm{d_2} + \{0.064\cdot\sin(2\pi\, t / 1440) + 0.12\cdot\mathrm{abs}[\sin(\pi\, t / 1440)]\}\cdot\mathrm{d_2}. \qquad (5)$$

Parameters $d_1$ and $d_2$ in last equation decouple the mean daily CRH level in the hypothalamic-pituitary portal vessels (governed by $d_1$) from the amplitude of the circadian CRH oscillation (governed by $d_2$). This function emulates well the asymmetry of the 24 h rhythm in humans, with the nocturnal phase lasting 8 hours. It affects the inflow rate of CRH into the system, transforming $k_0$ in Eq. (1) into $k_D(t) = k_0 \cdot D$. The multiplier *D* couples the rate constant of CRH production ($k_0$) to extrinsic circadian regulation. Consequently, CRH evolution changes from monotonic to oscillatory, and the dynamics of ultradian oscillations becomes more complex.

Stress and a number of illnesses are associated with short- or long-term perturbations of the HPA dynamics [2, 5], changing the amplitude and/or frequency of HPA hormones discharge and their mean levels. Hence, by mathematical modeling, numerical simulations and dynamical systems theory approaches we are able to investigate more comprehensively the different diseases and suggest refinements to more efficient drug therapies [6].

**Keywords**: hypothalamic-pituitary-adrenal (HPA) axis, ultradian and circadian oscillations, Stoichiometric Network Analysis (SNA), autocatalysis and autoinhibition, nonlinear dynamics

**References**

[1] Goldbetter A, *Biochemical Oscillations and Cellular Rhythms*, Cambridge University Press, Cambridge, 1996.

[2] Miller WL and Chrousos GP, The Adrenal Cortex, in Felig P, Frohman L (eds) *Endocrinology & Metabolism*, McGraw-Hill, New York, 1960, pp. 387-524.

[3] Jelić S, Čupić Ž and Kolar-Anić Lj, Mathematical modelling of the hypothalamic-pituitary-adrenal system activity, *Mathematical Bioscience,* **197** (2005), pp. 173-187.

[4] Makino S, Hashimoto K and Gold PW, Multiple feedback mechanisms activating corticotropin-releasing hormone system in the brain during stress, *Pharmacology Biochemistry and Behavior,* **73** (2002), pp. 147-158.

[5] VanItallie TB, Stress: a risk factor for serious illness, *Metabolism*, **51** (2002), pp. 40-45.

[6] Marković VM, Čupić Ž, Vukojević V and Kolar-Anić Lj, Predictive Modeling of the Hypothalamic-Pituitary-Adrenal (HPA) Axis Response to Acute and Chronic Stress, *Endocrine Journal*, **58** (2011), pp. 889-904.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-32. ID-46.**

# STOHASTIC ANALYSIS OF MONOLAYER GAS ADSORPTION: THE USE OF BI-VARIATE AND MONOVARIATE PROBABILITY GENERATING FUNCTION

**O. Jakšić[1], Z. Jakšić[1], D. Ranđelović[1], Lj. Kolar-Anić[2]**

[1] Institute of Chemistry, Technology and Metallurgy,
University of Belgrade, Njegoševa 12, 11000 Belgrade, Serbia
e-mail: olga@nanosys.ihtm.bg.ac.rs
[2] Faculty of Physical Chemistry,
University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia
e-mail: lkolar@ffh.bg.ac.rs

**ABSTRACT**

A stochastic theory of the reversible second order reaction A + B ↔ C is formulated and its theoretical aspect is brought in connection with the physical parameters of the system on an example of monolayer adsorption. Sorption rates are formulated for the case of ideal gas adsorption on solid surfaces.

The stochastic analysis, which is based on the use of sequences for solving (partial) differential equations, is performed in two different ways. For both of them the calculus is outlined in detail. The sequence which is used in a procedure of stochastic analysis, the probability generating function (PGF), is defined in two different ways: as a bi-variate (variables being the number of free gas molecules and the number of adsorbed gas molecules) and as a mono-variate (the number of adsorbed gas molecules being the only variable). The respective results, obtained by their implementation, are investigated.

A Rigorous solution for the sequence of PGF in a closed form (summed) can not be found in general. PGF involved in the differential equation that describes the process of adsorption and desorption on a solid surface involves solving a nonlinear second order partial differential equation, whether bi-variate PGF or mono-variate PGF is used.

However, in some situations it is not necessary to find an exact expression for the summed PGF in order to obtain its derivatives and with the use of its derivatives to obtain the moments of observed stochastic variable. Here, the first moment is of practical interest for obtaining the stochastic rate equation for the number of adsorbed molecules, the second moment is of practical interest for obtaining the variance and relative fluctuations of the number of adsorbed molecules. The alternate method for obtaining the moments of the number of adsorbed molecules without the use of the exact expression of probability generating function is also proposed.

The solution for the stochastic rate equation obtained by the alternate method is straight forward in the case when bi-variate PGF is used, but in the other case when mono-variate PGF is used that method only helps lower the order of the starting partial differential equation from second to first. The method is proposed for obtaining the stochastic rate equation in that case.
The respective stochastic rate equations obtained in these two different ways, are compared to the deterministic solution obtained by solving the appropriate Riccati quadratic differential equation.

These two different approaches imply using different mathematical methods [1], and give different results. As shown in [2], the stochastic rate equation may be the same as the deterministic equation, but there are conditions when it is not. The obtained results are analyzed from that point of view, since the use of a bi-variate PGF leads to the stochastic rate equation that corresponds to the deterministic equation and the use of mono-variate PGF leads not.

The final conclusion is that the main reason that helps us choose the appropriate approach in dealing with the stochastic phenomena in adsorption-desorption processes, approved from the practical point of view, is the ratio between the overall number of molecules in the system and the number of adsorption centers (*i.e*. the number of binding sites).

**Keywords**: adsorption-desorption processes, stochastic analysis, PGF, probability generating function

## References

[1] Debnath L. (2005) *Nonlinear partial differential equations for scientists and engineers*, Birkhäuser, Boston.

[2] Jachimowski C. J., McQuarrie D. A. and Russell M. E. (1964) A Stochastic Approach to Enzyme-Substrate Reactions, *Biochemistry*, **3**, 1732-1736.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-33. ID-47.**

# ANALYSIS OF REAL SAMPLES BY PERTURBATION OF NON-EQUILIBRIUM STATIONARY STATES IN AN OSCILLATING REACTION

**N. Pejić[1], S. Anić[3], J. Maksimović[2], N. Sarap[3]**

*[1]Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, 11000 Belgrade, Serbia, e-mail:nata@pharmacy.bg.ac.rs*
*[2]Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, P.O. Box 137, 11001 Belgrade, Serbia*
*[3]IHTM, University of Belgrade – Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia*
*[4]Institute of Nuclear Science Vinča, Radiation and Environmental Protection Department, University of Belgrade, Mihajla Petrovića Alasa 12-14, P.O. Box 522, 11000 Belgrade, Serbia*

**ABSTRACT.**
The application of the pulse perturbation technique in analysis of real samples based on an oscillatory reaction is reported. In particularly, the reaction involving the catalytic decomposition of $H_2O_2$ in the presence of both $H^+$ and $IO_3^-$ ions (the Bray-Liebhafsky (BL) oscillatory reaction) [1], was used for convenient determination of several substances in both pure forms and complex samples [2]. The addition of small amounts of analyte examined to the BL oscillatory reaction as matrix alters some of its properties; this effect can be used for analytical protocol to determine different substances which are of interest [3]. Here, we developed the method based on the Pulse Perturbation of the Oscillatory reaction system being in a Stable Steady State (PPOSSS) in the vicinity of the bifurcation point in the BL matrix, which is realized in a continuously stirred tank reactor, for the determination of piroxicam (PX) in real samples (pharmaceuticals).

Generally, the minimal necessary steps in PPOSSS procedure are: selection of a matrix that will be perturbed with analyte (in our case, BL oscillatory reaction), bifurcation analysis, i.e. determination of suitable dynamic state that is, the stable steady state near bifurcation point that will be perturbed, and perturbation analysis.

For bifurcation analysis, by variation of the some control parameter we obtain various dynamic states that are used for construction of the bifurcation diagrams, which define all stationary states of the matrix. After this, the stable steady state near the bifurcation point can be selected; for example, when the inflow concentration of $H_2SO_4$ was the control parameter [4] the following dynamic state is selected: T = 64.0 °C, $j_o = 2.95\times10^{-2}$ min$^{-1}$, $[KIO_3]_0 = 5.9\times10^{-2}$ mol L$^{-1}$, $[H_2O_2]_0 = 1.5\times10^{-1}$ mol L$^{-1}$ and $[H_2SO_4]_0 = 8.55\times10^{-2}$ mol L$^{-1}$, since the bifurcation point was found for $[H_2SO_4]_0 = 8.70\times10^{-2}$ mol L$^{-1}$. Temporal evolution of the system was recorded by means of a Pt electrode and double junction Ag/AgCl electrode.

Matrix response behavior is investigated by injecting PX into the matrix, when BL reaction is operated under conditions where stable non-equilibrium stationary states endured (Fig. 1., Zones II). Thus, before perturbation, the system is in a stable stationary state while its corresponding potential denoted as $E_s$ is constant (Fig. 1, Zone II). When a trace amount of PX was injected into BL matrix, an initial overshoot-decay response was observed (Fig. 1, perturbation injection zone III). The response to each PX perturbation was evaluated using maximal potential displacement, $\Delta E_m = E_p - E_s$, where $E_p$ is the maximal potential value

attained after the perturbation was performed and $E_s$ is the potential corresponding to the stable stationary state before the perturbation is performed (Fig. 1.). In the PX concentration range between $6.8 \times 10^{-5}$ mol $L^{-1}$ and $1.9 \times 10^{-3}$ mol $L^{-1}$, the regression equation of the standard series calibration curve is $\Delta E_m = -5.7 - 1.5 \times 10^4\ c_{PX}$ (r = 0.997).

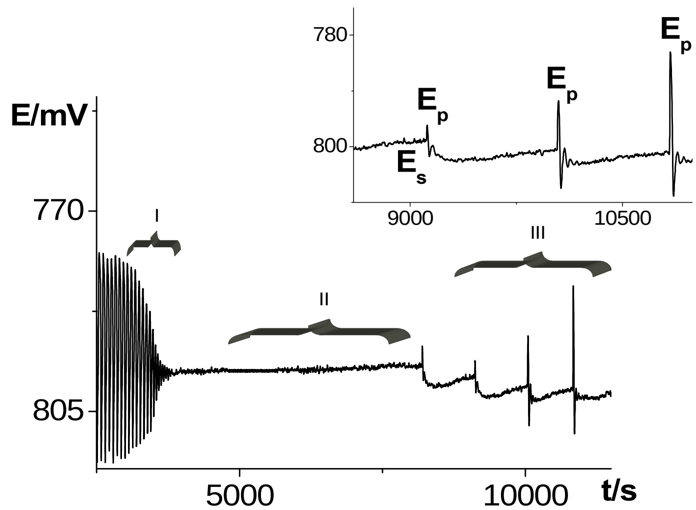



Fig. 1. The transition oscillations (I), and stable stationary state (II) together with the typical potentiometric responses of the BL matrix, obtained after its perturbation (III) with different concentrations of PX (from left to right): $9.7 \times 10^{-5}$ mol $L^{-1}$, $2.9 \times 10^{-4}$ mol $L^{-1}$ and $7.7 \times 10^{-4}$ mol $L^{-1}$.

Proposed method was applied to the determination of PX in real samples (tablets and injections (Pfizer, Greek)). The obtained RCV ≤ 104.7 % indicating that the developed method is free from any interference, and provides accurate results; it is a useful method for quantitative analysis of PX in pharmaceutical formulations.

**Conclusion** We presented here the potential of an analytical protocol to measure PX in pure and pharmaceutical dosage forms by using pulse perturbation technique. It is a useful analytical tool for analyzing samples of interest by very simple, modular equipment that can be assembled from parts available in any analytical laboratory.

**Acknowledgements –** The present investigations were supported by the Ministry of Education and Sciences of the Republic of Serbia, Grants no. 172015.

**References**

[1] W.C. Bray, J. Am. Chem. Soc., **43 (**1921), pp. 1262−1267
[2] N. Pejić, Hem. Ind. **63** (2009), pp. 455–466 (in Serbian).
[3] V. Vukojević, N. Pejić, D. Stanisavljev, S. Anić, Lj. Kolar−Anić, Analyst, **124** (1999), pp. 147−152
[4] N. Pejić, M. Vujković, J. Maksimović, A. Ivanović, S. Anić, Ž. Čupić, Lj. Kolar-Anić, *Russ. J. Phys. Chem. A,* **85** (2011), pp. 66–72.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-34. ID-48.**

# EMULSIONS AND DOUBLE EMULSIONS AS PARTICULAR EXAMPLES OF MEMRISTIVE SYSTEMS

**A.M. Spasic**

*Institute for Technology of Nuclear and Other Mineral Raw Materials, Dept. of Chem.Engng., 86 F. d'Esperey St., P.O. Box 390, 11000 Belgrade, Serbia e-mail: a.spasic@itnms.ac.rs*

**ABSTRACT.**

Since the events at the interfaces of finely dispersed systems have to be considered at the molecular, atomic, and/or entities level it is inevitable to introduce the electron transfer phenomenon beside the classical heat, mass, and momentum transfer phenomena commonly used in chemical engineering. Here discussed representative finely dispersed systems are selected emulsions and double emulsions.

Three possible mathematical formalisms have been derived and discussed related to the proposed physical formalism, that is, to the developed theory of electroviscoelasticity. The first is stretching tensor model, where the normal and tangential forces are considered, only in mathematical formalism, regardless to their origin, mechanical and/or electrical. The second is classical integer order van der Pol derivative model. Finally, the third model comprise an effort to generalize the previous van der Pol differential equations, both, linearized and nonlinear; where the ordinary time derivatives and integrals are replaced by the corresponding fractional-order time derivatives and integrals of order $p<2$ ($p = n - \delta$, $n = 1, 2$, $\delta << 1$). In order to justify and corroborate more general approach the obtained calculated results were compared to those experimentally measured using the representative liquid-liquid system [1-7].

Based on the definition of a "missing" basic element *memristor* for electrical circuit analysis, and taking into account four fundamental circuit variables, the electric current $i$, the electromotive force $v$, the charge $q$, and the magnetic flux $\varphi$ the set of fundamental functional relations is presented. Hence, the memristor as a current-controlled device is defined and the memristance as a generalized resistance is introduced for easier and deeper understanding of the nonlinear phenomena that occur at developed interfaces of selected emulsions and double emulsions. In 1976 Chua and Kang generalized the memristor concept to a much broader class of nonlinear dynamical systems, named memristive systems; this, almost, forgotten story come back in 2008 by Strukov et al. [8].

Further on, when an incidental uniform physical field, for example electromagnetic, is applied on the system emulsion/droplet or double emulsion/droplet-film-structure, causing the motions of both electrons and ions, than the structure of memristance may be discussed. The memristance consists of two resistances in series, the low resistance $R_{int}$ at the interface and close to the interface layers, and the much higher resistance $R_{bul}$ in the bulk layers.

Now, according to the presented electro hydrodynamic approach, emulsions and double emulsions will be considered as the composite system “droplet + film” or “droplet-film structure + droplet homophase (S + E)”. The history of this system consists of the initial/formation, intermediate/transition, and final/rigid states. 1. Every stationary state (initial, intermediate, and final) is characterized by interaction in the composite system that is of the same kind – being able to give rise to the occurrence of decoherence with the cluster arrangements as the “pointer basis states”. 2. The nonstationary state is characterized by the change in the character of interaction in the composite system. The net effect takes the following “phases” each having its own characteristic time: a) under the action of an external or incidental physical field a formation of the droplet-film structure occurs, producing the nonstationary state; b) the transition or relaxation of the electroviscoelastic droplet-film structure into the rigid one, establishing the new final stationary state, this transition or relaxation process may be considered as a kind of a memory storage process, therefore, the system either electroviscoelastic droplet or droplet-film structure submerged into the other immiscible liquid phase, could be considered as the particular example of memristive systems; c) decoherence process, the final stationary state.

The implications and applications of presented research may be useful, for example in a solution of the entrainment problems in solvent extraction, that is, in a breaking of emulsions or double emulsions, than for a deeper elucidation of adhesive processes, rupture processes, and coalescence. Further on, these concepts and approaches could help better understanding of the complex phenomena related to: colloid and interface science, chemical and biological sensors, electro-analytical methods, biology or biomedicine (in particular hematology, genetics, electroneurophysiology), classical limit of quantum mechanics, ionics, spintronics, fractional-quantum Hall effect-fluids, decoherence sensitivity, quantum computation, entities-quantum particles entanglement [1-11].

**Keywords:** Electrohydrodynamics, Electroviscoelasticity, Nanorheology, Emulsions. Double emulsions, Memory storage processes

**Acknowledgement** – This investigation was supported by the Ministry of Education and Science of Republic of Serbia, under the projects, former 142034, and at present as the two continuation projects, No. 46010 and 34009.

## References

[1] Spasic A.M, Jovanovic J.M, Jovanovic M, Modeling Selected Emulsions and Double Emulsions as Memristive Systems, *Advances in Colloid and Interface Science,* **174** (2012), pp. 31-49.
[2] Hsu J-P, Spasic A.M, *Interfacial Electroviscoelasticity and Electrophoresis*, CRC Press-Taylor & Francis, Boca Raton London New York, 2010.
[3] Spasic A.M, Lazarevic M.P, A New Approach to the Phenomena at the Interfaces of Finely Dispersed Systems, *Journal of Colloid and Interface Science*, **316** (2007), pp. 984-995.
[4] Spasic A.M, Jokanovic V, Krstic D.N, A Theory of Electroviscoelasticity: A New Approach for Quantifying the Behavior of Liquid-Liquid Interfaces under Applied Fields, *Journal of Colloid and Interface Science,* **186** (1997), pp. 434-446.
[5] Spasic A.M, Djokovic N.N, Babic M.D, Marinko M.M, Jovanovic G.N, Performance of Demulsions: Entrainment Problems in Solvent Extraction, *Chemical .Engineering .Science*, **52** (1997), pp. 657-675.
[6] Spasic A.M, Mechanism of the Secondary Liquid-Liquid Droplet-Film Rupture on Inclined Plate, *Chemical Engineering Science,* **47** (1992), pp. 3949-3957.
[7] Spasic A.M in. (ed) Delgado A.V, *Interfacial Electrokinetics & Electrophoresis,* Marcel Dekker, New York, 2002, pp. 837-868.
[8] Strukov D.B, Snider G.S, Stewart D.R, Williams R.S, The Missing Memristor Found, *Nature* **453/1** (2008), pp. 80-83.
[9] Dugic M, *Decoherence in a classical limit of quantum mechanics,* Notebooks of Physical Sciences, SFIN XVII (2), Institute of Physics, Belgrade,:2004.
[10] Dugic M, Rakovic D, Plavsic M, A in (eds) Spasic A.M, Hsu J-P, *Finely Dispersed Particles: Micro-, Nano-, and Atto-Engineering,* CRC Press-Taylor & Francis, Boca Raton London New York, 2006, pp. 317-331.
[11] Spasic A.M, Electrohydrodynamics of Developed Liquid/Liquid Interfaces: Fractional Order Time Delay Systems, *Russian Journal of Physical Chemistry A*, **85** (2009), pp. 1563-1570.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-35. ID-75.**

# TYPES OF BIFURCATION POINTS IN BRAY-LIEBHAFSKY OSCILLATORY REACTION

**B. Stanković[1], Ž. Čupić[2] and Lj. Kolar-Anić[1]**

[1]Faculty of Physical Chemistry, University of Belgrade, Studentski trg 12-16, 11000 Belgrade, Serbia
e-mail: branislav20@yahoo.com
[2]Institute of Chemistry, Technology and Metallurgy, University of Belgrade, Department of Catalysis and Chemical Engineering, Njegoševa 12, 11000 Belgrade, Serbia
e-mail: zcupic@nanosys.ihtm.bg.ac.rs

**ABSTRACT.**

Bray-Liebhafsky reaction, the hydrogen peroxide decomposition into the water and oxygen in the presence of hydrogen and iodate ions, is the first discovered oscillatory chemical reaction [1]. This, apparently simple oscillatory reaction, represents a complex homogeneous catalytic oscillatory process, which involves numerous iodine intermediates such as $I_2$, $I^-$, HIO, $HIO_2$ and $I_2O$ and their concentrations are changing oscillatory [2].

Here we analyze time series obtained by numerically simulating the model of the Bray-Liebhafsky oscillatory reaction system in open reactor [3], with the aim to construct bifurcation diagram in parameter space of flow rate and inflow hydrogen peroxide concentration.

To identify types of bifurcation points in which system transforms from stable to unstable state and from unstable to stable state, it is necessary to know correlation between amplitude of oscillations and flow rate and correlation between period of oscillations and flow rate near the bifurcation point.

Analyzing these correlations we have found that system transforms from stable to unstable state through supercritical Andronov-Hopf bifurcation for almost all hydrogen peroxide concentrations in inflow. In narrow interval of hydrogen peroxide concentrations in bottom of bifurcation diagram, where x-axis represents flow rate and y-axis represents inflow hydrogen peroxide concentration, system transforms from stable to unstable state through subcritical Andronov-Hopf bifurcation. System transforms from unstable to stable state through supercritical Andronov-Hopf bifurcation for most hydrogen peroxide concentrations in inflow, but below some concentration it transforms through saddle loop bifurcation.

We have also found that system transforms from unstable to stable state through supercritical Andronov-Hopf bifurcation for all hydrogen peroxide concentrations in inflow, for which mixed-mode dynamics, sequences of large and small-amplitude oscillations, are obtained.

Cognition of types of bifurcation points in which system transforms from stable to unstable state and from unstable to stable state is significant for understanding global changes in system dynamics.

**Keywords**: Bray-Liebhafsky reaction, supercritical Andronov-Hopf bifurcation, subcritical Andronov-Hopf bifurcation, saddle loop bifurcation, mixed-mode dynamics

**Acknowledgement –** This investigation was supported by the Ministry of Education and Science of Republic of Serbia, under the projects No. 172015 and 45001.

## References

[1] Bray WC, Periodic reaction in homogenous solution and its relation to catslysis, *J. Am. Chem. Soc*, **43** (1921), pp. 1262-1261

[2] Furrow SD, Chemical oscillators based on iodate ion and hydrogen peroxide, in Field RJ, Burger M (eds) *Oscillations and Traveling Waves in Chemical Systems*, J. Wiley, New York, 1985, pp. 171-192

[3] Kolar-Anić Lj, Čupić Ž, Schmitz G, Anić S, Improvement of the stoichiometric network analysis for determination of instability conditions of complex nonlinear reaction systems, *Chem. Eng. Sci*, **65** (2010), pp 3718

Naučno društvo Srbije — Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-49. ID-61.**

# COMPARATIVE ANALYSIS OF OPTIMAL SIZE WITH THE RESULTS EJECTORS EXPERIMENTAL EJECTORS TESTED IN DISTRICT HETING

**Šefik M.Bajmak**

University of Pristine, Faculty of Technical Sciences
email: bajmak@yahoo.com

ABSTRACT. Ejectors are used to lower water temperatures in district heating systems at the level of temperature systems $90/70^0C$. Main advantages of ejectors, a mixing device, the simplicity, safety and which has no moving parts. The ejectors work is necessary to have a connection to the consumer a significant difference between the effortless flows and return temperature of the thermal network, the account which gets increased water velocity at the exit of the ejector nozzle, required to create the effect of injection (mixing). Therefore, the flow of water into the local system for heating changes in direct proportion to the flow of water from the thermal network through ejector nozzle. A serious flaw with ejector scheme is the absence of an independent water circulation in the local heating plant. When interrupting the flow of water from the thermal network ejector nozzle, for example, the exclusion of the network due to thermal breakdown, interruption of the circulation of water in the heating plant, which can lead to freezing of water in service. In the computation of pressure drop in the heating system to 15,000 Pa is usually applied ejector, and for several pressures using centrifugal pumps.

Starting from the energy balance for the horizontal position of the ejector has that:

$$E_{ej} = E_{u1} + E_{u2} - E_{gu} = P_{t1}\dot{V}_1 + P_{t2}\dot{V}_2 - E_{gu} = P_{t4}\dot{V}_3 \tag{1}$$

We consider incompressible fluid, where $P_1 = P_2 = P_{12}$, using the conditions $\dot{V}_3 = \dot{V}_1 + \dot{V}_2$ and after ordering the equations (1) we have that:

$$\Delta P_r = P_{t1} - P_{t2} = \frac{1}{2}\left( \frac{W_{1C}^2 \cdot \rho_1}{\varphi_1^2} - \frac{W_{2C}^2 \cdot \rho_2}{\varphi_4^2} \right) \tag{2}$$

$$\Delta P_t = P_{t4} - P_{t2} = \frac{1}{2} m_1^2 \left[ \frac{2\varphi_2}{f_3} - \left( \frac{1}{\rho_1 \cdot f_1} + \frac{U^2}{\rho_2 \cdot f_2} \right) - \frac{U^2}{\varphi_4^2 f_2^2} \frac{\overline{\rho} + U}{1+U} \frac{\rho_3}{\rho_2} + \left(2 - \varphi_2^3\right) \frac{1+U}{\rho_3 f_3^2} \right] \tag{3}$$

Starting from expression (2) and (3) we can form a relationship (4), which represents the **characteristic equation with central ejector nozzle**:

$$\frac{\Delta P_t}{\Delta P_r} = \frac{\varphi_1^2 f_1}{f_2}\left[2\varphi_2 + \left(2\varphi_2 - \frac{1}{\varphi_4^2}\right)\frac{U^2}{\bar{\rho}}\frac{f_1}{f_2} - \left(2-\varphi_3^2\right)\frac{(1+U)(\rho+U)}{\bar{\rho}}\frac{f_1}{f_3}\right]$$
$$= \frac{\varphi_1^2 d_1^2}{d_3^2}\left[2\varphi_2 + \left(2\varphi_2 - \frac{1}{\varphi_4^2}\right)\frac{U^2}{\bar{\rho}}\frac{(d_1/d_3)^2}{1-(d_1/d_3)^2} - \left(2-\varphi_2^3\right)\frac{(1+U)(\rho+U)}{\bar{\rho}}\frac{d_1^2}{d_3^2}\right] \quad (4)$$

The equation of the characteristics and features of ejector nozzles, are derived under the condition that the ejector works with single-phase incompressible fluid. Primary and secondary fluid on its way through the ejector passes through zones of different pressures.

In the system of centralized supply of heat to power stations in operation with hot water pressure in some parts of the apparatus may be lower than the saturation vapor pressure at the temperature of water streaming. Such places, where the cavitations occurs, the output of most parts of the ejector nozzle and inlet parts of the mixing chamber.

Therefore, when calculating cavitations regime, usually gets speed vapor cavitations in the section which is equal to the local speed of sound. Cavitation begins by evaporation of one or both fluids when the pressure in the ejector suction chamber falls below the vapor partial pressure of the fluid. When cavitations occurs, coefficient of efficiency (KKD) ejectors rapidly decreases, and the secondary fluid flow is reducing, which may lead to termination of the ejector.

Characteristics of ejectors operating with water as described by equation (4) can be presented in the following form:

$$\frac{\Delta P_t}{\Delta P_r} = 2\frac{f_1}{f_2}\left[A + B\frac{f_1}{f}U^2 - C\frac{f_1}{f_3}(1+U)^2\right] \quad (5)$$

Where A, B and C coefficients.

Results tested ejectors are presented in tabular and graphic depending on the injection coefficient U. A comparative analysis of the obtained optimal size $\frac{\Delta P_t}{\Delta P_s}$, $\eta$ with ejectors tested data we defined speed coefficients $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ for more types of ejectors for a variety of related $f_1/f_3$. In this way, theoretical equations for the characteristics of ejectors are closer to the actual situation.

**Key words**: Ejector, optimal size, experiment, district heating

## References


[1] Берман , Д.Л.: Характеристики водяаныџ струйных насосов (элеваторов) и их расчет, Теплоэнергетика, 1935, Но.8, стр 32-35

[2] Соколв, Е.Я., Зингер, Н.М.: Струиные апараты, Энергия, стр.288.,Москва , 1970.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-49. ID-61.**

# KOMPARATIVNA ANALIZA OPTIMALNIH VELIČINA EJEKTORA SA REZULTATIMA EKSPERIMENTALNOG ISPITANOG EJEKTORA U SISTEMU DALJINSKOG GREJANJA

**Šefik M.Bajmak**
Univerzitet u Prištini, Fakultet tehničkih nauka
bajmak@yahoo.com

**ABSTRACT.** Ejektori služe za snižavanje temperature vode u sistemu daljinskog grejanja na nivou temperature sistema $90/70^0C$. Osnovne prednosti ejektora , kao uređaja za mešanje, su jednostavnost, sigurnost u radu i što nema pokretnih delova. Za rad ejektora neophodno je imati na priključku potrošaća značajnu razliku napora između polaznog i povratnog voda toplotne mreže , na račun koje se dobija povećana brzina vode na izlazu iz mlaznice ejektora, neophodna za stvaranje efekta injektiranja (mešanja). Zbog toga se, protok vode u lokalnom sistemu za grejanje menja upravo proporcionalno sa protokom vode iz toplotne mreže kroz mlaznicu ejektora. Ozbiljne nedostatke šeme sa ejektorom je odsutvo nezavisne cirkulacione vode u lokalnom postrojenju za grejanje. Pri prekidu dotoka vode iz toplotne mreže u mlaznici ejektora, na primer, pri isključenju toplotne mreže zbog kvara , prekida se cirkulacija vode u postrojenju za grejanje , što može dovesti do smrzavanja vode u eksploataciji. Pri računskom padu pritiska u sistemu grejanja do 15.000 Pa obično se primenjuje ejektor , a za više pritiske se koriste centrifugalne pumpe.

Polazeći od bilansa energije za horizontalni položaj ejektora imamo da je :

$$E_{ej} = E_{u1} + E_{u2} - E_{gu} = P_{t1}\dot{V}_1 + P_{t2}\dot{V}_2 - E_{gu} = P_{t4}\dot{V}_3$$

(1)

Razmatramo nestišljivi fluid, gde je $P_1 = P_2 = P_{12}$ ; korišćenjem uslova da je $\dot{V}_3 = \dot{V}_1 + \dot{V}_2$ i posle sređivanja jednačine (1) imamao da je :

$$\Delta P_r = P_{t1} - P_{t2} = \frac{1}{2}\left(\frac{W_{1C}^2 \cdot \rho_1}{\varphi_1^2} - \frac{W_{2C}^2 \cdot \rho_2}{\varphi_4^2}\right) \tag{2}$$

$$\Delta P_t = P_{t4} - P_{t2} = \frac{1}{2} m_1^2 \left[\frac{2\varphi_2}{f_3} - \left(\frac{1}{\rho_1 \cdot f_1} + \frac{U^2}{\rho_2 \cdot f_2}\right) - \frac{U^2}{\varphi_4^2 f_2^2}\frac{\overline{\rho} + U}{1+U}\frac{\rho_3}{\rho_2} + \left(2 - \varphi_2^3\right)\frac{1+U}{\rho_3 f_3^2}\right] \tag{3}$$

Polazeći od izraza (2) i (3) možemo formirati odnos (4) koji predstavlja **jednačinu karakteristike ejektora sa centralnom mlaznicom:**

$$\frac{\Delta P_t}{\Delta P_r}=\frac{\varphi_1^2 f_1}{f_2}\left[2\varphi_2+\left(2\varphi_2-\frac{1}{\varphi_4^2}\right)\frac{U^2}{\bar{\rho}}\frac{f_1}{f_2}-\left(2-\varphi_3^2\right)\frac{(1+U)(\rho+U)}{\bar{\rho}}\frac{f_1}{f_3}\right]$$
$$=\frac{\varphi_1^2 d_1^2}{d_3^2}\left[2\varphi_2+\left(2\varphi_2-\frac{1}{\varphi_4^2}\right)\frac{U^2}{\bar{\rho}}\frac{(d_1/d_3)^2}{1-(d_1/d_3)^2}-\left(2-\varphi_2^3\right)\frac{(1+U)(\rho+U)}{\bar{\rho}}\frac{d_1^2}{d_3^2}\right] \quad (4)$$

Jednačina karakteristike ejektora i karakteristike mlaznice , su izvedene pod uslovom da ejektor radi sa jednofaznom nestišljivom tečnošću. Primarna i sekundarna tečnost na svom putu kroz ejektor prolazi kroz zone različitih pritisaka. U sistemu centralizovanog snabdevanja toplotnom energijom pri radu strujnih pumpi sa vrelom vodom, pritisak u nekim delovima aparata može biti niži od pritiska zasićenja pare na temperaturi vode koja prostrujava (struji) . Takva mesta, na kojima se javlja kavitacija, su na ejektoru uglavnom izlazni delovi mlaznice i ulazni delovi komore mešanja. Prema tome, pri proračunu kavitacionog režima , obično se dobija brzina parne faze u kavitacionom preseku koja je jednaka lokalnoj brzini zvuka. Kavitacija počinje isparavanjem jedne ili obe tečnosti, kada pritisak u usisnoj komori ejektora padne ispod parcijalnog pritiska napona pare tih tečnosti. Kada dođe do kavitacije , koeficijent korisnog dejstva (KKD) ejektora naglo opada , a protok sekundarne tečnosti se smanjujue , pa može doći do prestanka rada ejektora.

Karakteristika ejektora koji radi sa vodom opisanom jednačinom (4) može se predstaviti u sledečem obliku

$$\frac{\Delta P_t}{\Delta P_r}=2\frac{f_1}{f_2}\left[A+B\frac{f_1}{f}U^2-C\frac{f_1}{f_3}(1+U)^2\right] \quad (5)$$

Pri čemu su A, B i C koeficijenti.

Rezultati ispitanog ejektora su prikazani tabelarno i grafički u zavisnosti od koeficijenta injekcije U.

Komparativnom analizom dobijenih optimalnih veličina $\frac{\Delta P_t}{\Delta P_s}$, $\eta$ sa podacima ispitanog ejektora definisali smo brzinske koeficijente $\varphi_1,\varphi_2,\varphi_3,\varphi_4$ za više tipova ejektora i za različite odnose $f_1/f_3$ . Na taj način , teoretsku jednačinu za karakteristiku ejektora približili smo je faktičkom stanju.

**Ključne reći**: ejektor, optimalne veličine , eksperiment, daljinsko grejanje

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-50. ID-68.**

# NUMERICAL EVIDENCE OF COMPLEX NONLINEAR PHENOMENA OF THE BRAY–LIEBHAFSKY OSCILLATORY REACTION UNDER CSTR CONDITIONS

**S. N. Blagojević[1], S. M. Blagojević[2], Ž. Čupić[3]**

[1]Institute of General and Physical chemistry,
Studentski trg 12, Belgrade. e-mail: stevan.blagojevic@gmail.com
[2]Faculty of Pharmacy, University of Belgrade, Vojvode Stepe 450, Belgrade,Serbia.
[3]IChTM –Department of Catalysis and Chemical Engineering, Njegoševa 12, Belgrade

**ABSTRACT.** By numerically simulated Bray–Liebhafsky (BL) reaction in a continuously fed well stirred tank reactor (CSTR) we discussed the complex time evolutions. It was found that attractors and shape of 1D maps can be one of the criteria for mixed–mode dynamics with complex sequence.

The BL reaction is the decomposition of hydrogen peroxide into the water and oxygen in the presence of iodate and hydrogen ions [1]:

$$2H_2O_2 \xrightarrow{H^+,\ IO_3^-} O_2 + 2H_2O$$

The overall reaction can be understood as the sum of two processes: reduction of iodate to iodine and oxidation of iodine to iodate by hydrogen peroxide. Oscillatory dynamics of the BL reaction under CSTR conditions can be simulated with the model consisting of eight reactions proposed by Lj. Kolar–Anić *et al.* [2]. We analyzed time series obtained by numerically simulating the proposed model of BL reaction [2], under isothermal CSTR conditions for the flow rate $j_0 = 4.829250 \times 10^{-3}$ min$^{-1}$. Numerical simulations are based on an idealized deterministic model, since they can generate data with high numerical precision of $10^{-16}$. All calculations were performed using MATLAB program package. The differential equations derived from the model were integrated using the ode15s solver. Relative and absolute error tolerance values of $3 \times 10^{-14}$ and $1 \times 10^{-20}$ were used in all simulations. Initial values of the concentrations were: $[IO_3^-]_0 = 4.74 \times 10^{-2}$ mol dm$^{-3}$, $[H^+]_0 = 9.58 \times 10^{-2}$ mol dm$^{-3}$, and $[H_2O_2]_0 = 1.55 \times 10^{-1}$ mol dm$^{-3}$.

Attractor analysis and the Poincaré or 1D maps made of selected Poincaré section, are qualitative methods capable of providing information regarding period–doubling and/or chaos in the analyzed dynamic state [3]. The complex time series of iodide ions have shown that in examined time interval (8000 minute) mixed–mode oscillations with four large and one small–amplitude oscillations were found (Figure 1). In the case of chaotic dynamics, an infinite trajectory (strange attractor) emerging from periodic cycle fills in the part of phase space by spirally cycling in the vicinity of the periodic state $4^1$. (Figure 2). Different shapes of Poincaré 1D maps are found at sections were chosen hyperplane cuts large cycles only (Figure 3 (a)), small cycles only (Figure 3 (b)) or large and small cycles both (Figure 3 (c)).

**Keywords**: nonlinear dynamics, oscillatory reactions, Bray–Liebhafsky reaction.

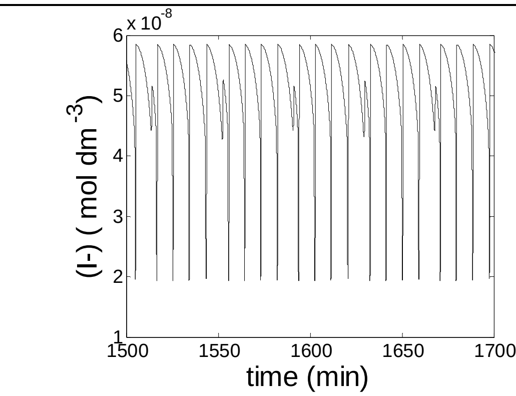



**Figure 1.** The time sequences of evolution of the BL system from the simulations, flow rate $j_0 = 4.829250 \times 10^{-3}$ min$^{-1}$.

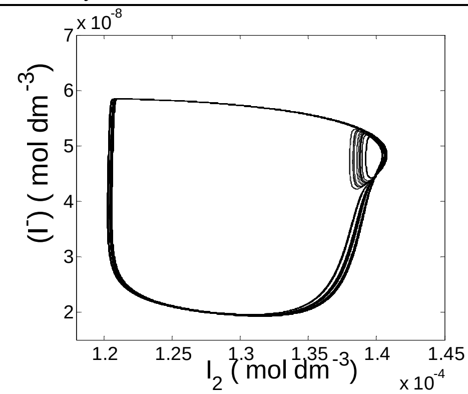
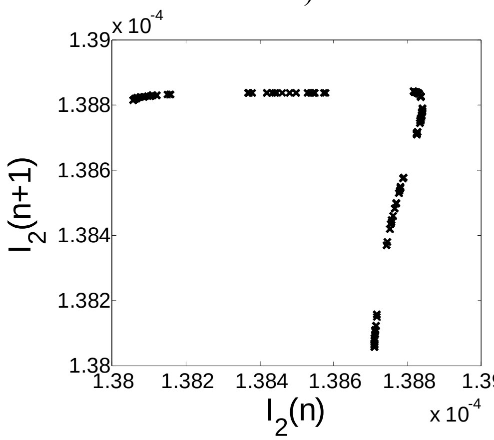
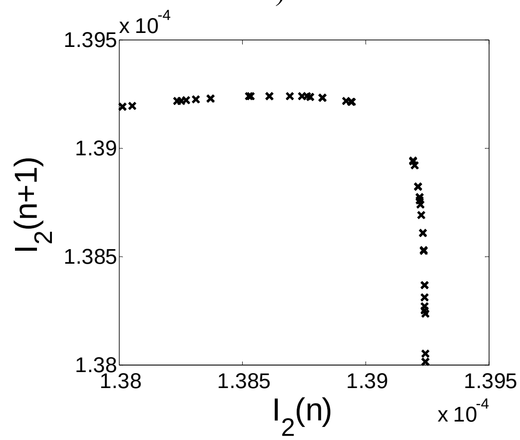



**Figure 2.** Attractor corresponding to flow rate $j_0 = 4.829250 \times 10^{-3}$ min$^{-1}$.

a)




b)




c)

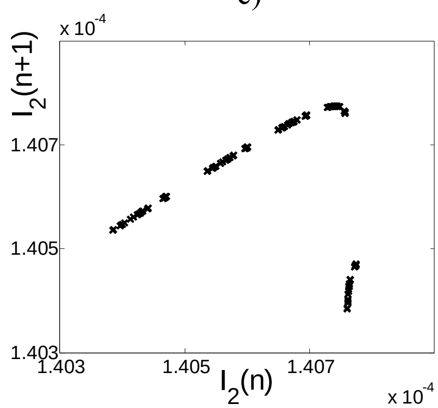



**Figure 3.**
The corresponding Poincaré 1D maps made by planes intersecting attractor at:
(a) large cycle only
([I$^-$] = 3,50 × 10$^{-8}$ mol dm$^{-3}$),
(b) small cycle only
([I$^-$] = 4.80 × 10$^{-8}$ mol dm$^{-3}$),
(c) both large and small cycle
([I$^-$] = 4.80 × 10$^{-8}$ mol dm$^{-3}$).

**References**

[1] W. C. Bray, H. A. Liebhafsky, J. Am. Chem. Soc., **53**, (1931), pp. 38–44.
[2] Lj. Kolar–Anić, Đ. Mišljenović, S. Anić, G. Nikolis, React. Kinet. Catal. Lett., **54**, (1995), pp. 35–41.
[3] A. Ivanovic–Šašić, V. Marković, S. Anić, Lj. Kolar–Anić, Ž. Čupić, Phys. Chem. Chem. Phys., **13**, (2011), pp. 20162–20171.

**Acknowledgements** The present investigations were supported by the Ministry of Education and Sciences of the Republic of Serbia, Grants no. 172015 and no. 45001.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-36. ID-80.**

# STOCHASTIC PARAMETRICALLY EXCITED HEREDITARY SANDWICH MULTI BEAM DYNAMICAL SYSTEMS

**K. (Stevanović) Hedrih** [1]

[1] Mathematical Institute SANU, Department of Mechanics,
and Faculty of Mechanical Engineering University of Niš, Serbia,
Priv. address: 18000- Niš, Serbiam ul. Vojvode Tankosića 3/22,
e-mail: khedrih@eunet.rs

**ABSTRACT.** Transversal vibrations of a stochastic parametrically excited hereditary sandwich multi beam system are investigated. Influence of rotatory inertia and transverse shear on stochastic vibration and stochastic stability of deformable forms and vibration processes in parametric resonance frequency interval are investigated. The systems of coupled stochastic partial integro-differential equations of transversal stochastic vibrations of a parametrically excited hereditary sandwich multi beam system were derived. The beams are graded by a hereditary material with known relaxation kernels, and each is subject to axial stochastic external excitations. The influence of rotatory inertia of beam cross sections and transverse shear of each beam cross sections under the transverse forces, and the corresponding terms in the partial integro-differential equations are taken into account. Bernoulli particular integral method and Lagrange method of variation constant are used for the transformation problem. The asymptotic averaged method Krilov-Bogolyuvov-Mitropolskiy is used for obtaining the first approximations of Itô stochastic differential equations and Stratonovich results. By using idea of Ariaratnam the sets of Lyapunov exponents are obtained.

**Keywords**: multi beam system, hereditary, relaxation kernel, rotatory inertia of beam cross sections, transverse shear of beam cross section, partial integro-differential equations, asymptotic averaged method, stochastic Itô differential equations, Lyapunov exponents, multi-frequency random oscillations, parametric resonance.

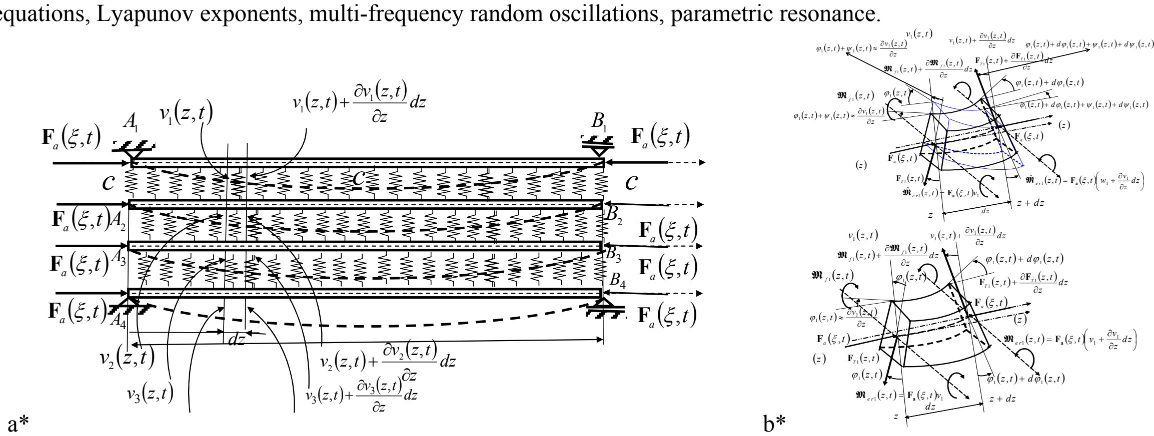



**Figure 1. (a*)** Sandwich milti (four) beam system with discrete continuum elastic layers. **(b*)** Cross section surface forces and moments acting on a beam element: The influence of rotatory inertia of beam cross sections and and cross section displacement of a beam element .

**BASIC DESCRIPTION:** Constitutive stress-strain relation, $\sigma_z(z,y,t)$- $\varepsilon_z(z,y,t)$ of visco-elastic hereditary beam material is in the form (for detail explanation see References [2-6,800]):

$$\sigma_z(z,y,t)=E\left[\varepsilon_z(z,y,t)-\int_0^t \mathfrak{R}(t-\tau)\varepsilon_z(z,y,\tau)d\tau\right],\ \mathfrak{R}(t-\tau)=\frac{E-\tilde{E}}{nE}e^{-\frac{t-\tau}{n}} \tag{1}$$

where $\mathfrak{R}(t-\tau)$ is kernel of relaxation of beam viscoelastic material with hereditary properties.

The governing equations of the dynamic equilibrium of the beams from sandwich multi hereditary beam system (see Figures 1.a* and 2.) subjected by corresponding number of same pairs of the opposite direction axial stochastic forces $F_a(\Xi,z,t)$ are (for detail explanation see References [2-9]):

$$d\,J_{xi}\frac{\partial^2\varphi_i(z,t)}{\partial t^2}=-d\mathfrak{M}_{fi}(z,t)+F_{Ti}(z,t)dz+F_{ai}(\Xi,z,t)dv_i(z,t)\quad,\ i=1,2,3,4,...N \tag{2}$$

$$dm_i\frac{\partial^2 v_i(z,t)}{\partial t^2}=d\,F_{Ti}(z,t)+c_i[v_i(z,t)-v_{i-1}(z,t)]dz-c_{i+1}[v_{i+1}(z,t)-v_i(z,t)]dz\ ,i=1,2,3,4,...N$$

From previous system, after eliminations angle coordinates $\varphi_i(z,t)$ we obtain system of $N$ stochastic partial integro-differential equations along transversal corresponding beam displacement $v_i(z,t)$. These equation contain terms $\frac{\partial^4 v_i(z,t)}{\partial t^4}$ with fourth order partial derivative along time. After applying Bernoulli method of particular integral we obtain infinite number of the sets, each of $N$ stochastic ordinary integro-differential equations along eigen time functions $T_{(i)s}(t)$ each with corresponding terms with $\overset{::}{T}_{(i)s}(t)$ fourth order derivative along time. Now, following the idea presented by S.T. Ariaratnam (1995) in reference [1], for solving the previous obtained equations, we can propose that random, bonded noise axial excitation $\xi(t),|\xi(t)|\le 1$, is taken in the form: $\hat{F}_{a1}(t)=\hat{F}_{a1}\,\xi(t)=\hat{F}_{a1}\,\mu\sin[\Omega t+\sigma B(t)+\gamma]$, where $B(t)$ is the standard Wiener process, and $\gamma$ is a random uniformly distributed variable in interval $[0,2\pi]$, then $\xi(t)$ is a stationary process having autocorrelation function and spectral density function in the forms:

$$R(\tau)=\frac{1}{2}\mu^2 e^{-\frac{\sigma^2\tau}{2}}\cos\Omega\tau \quad \text{and} \quad S(\omega)=\int_{-\infty}^{+\infty}R(\tau)e^{i\omega\tau}d\tau=\frac{1}{2}\mu\sigma^2\frac{\omega^2+\Omega^2+\frac{\sigma^2}{4}}{\left[\left(\omega^2-\Omega^2-\frac{\sigma^2}{4}\right)^2+\sigma^2\omega^2\right]} \tag{3}$$

The Lyapunov exponents of system mode processes $\lambda_s^k$, $s=1,2,3,4,...$, $k=1,2,...,2N$, [1] may be introduced by using the eigen time function modes as „new time component coordinates" $T_s^k(t)$ and which by making use of the averaged equations becomes (for detail explanation see References [1,4,5]):

$$\lambda_s^k=\lim_{t\to\infty}\frac{1}{2t}\ln\left\{\left[T_s^k(t)\right]^2+\frac{1}{\omega_{(s)k}^2}\left[\dot{T}_s^k(t)\right]^2\right\}=\lim_{t\to\infty}\frac{1}{2t}\ln\left\{\left[C_s^k(t)\right]^2\right\}=\lim_{t\to\infty}\frac{1}{t}\rho_s^k(t),\ k=1,2,3,4,...,2N$$

where $d\rho_s^k(t)=\frac{1}{C_s^k}\dot{C}_s^k dt$ and $d\vartheta_s^k(t)=\dot{\phi}_s^k(t)dt-\frac{1}{2}d\psi$ , $k=1,2,3,...,1N$ .

**Acknowledgement.** Parts of this research were supported by the Ministry of Sciences of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 "Dynamics of hybrid systems with complex structures. Mechanics of materials"

## References

[1] Ariaratnam S. T**.,** Stochastic Stability of Viscoelastic Systems under bounded Noise Excitation, *IUTAM Symp. on Advances in Nonlinear Stochastic Mechanics,* Kluwer Acad. Publ. Dordrecht, 1996, pp. 11-18.

[2] Goroshko, O.A., Hedrih (Stevanović), K., *Analitical Dynamics of Discrete Hereditary Systems* (in Serbian) , University of Niš, 2001, pp. 426.

[3] Hedrih (Stevanović), K., Two-Frequencies Forced Nonstationary Vibrations of the beam, *Matematieskaya fizika*, Vol. 12, Kiev, 1972, pp. 127-140 (in Russian).

[4] Hedrih (Stevanović) K., Transversal Vibration of a Parametrically Excited Beam: Influence of Rotatory Inertia and Transverse Shear on Stochastic Stability of Deformable Forms and Processes**,** © *International Journal of Nonlinear Sciences and Numerical Simulation***, 7**(1), 117-124, 2006**.**

[5] Hedrih (Stevanović K., Transversal vibration of a parametrically excited hereditary beam: Influence of rotatory inertia and transverse shear on stochastic stability of deformable forms and processes , Inter. IFNA-ANS Journal "Problems of nonlinear analysis in engineering systems", ISSN 1727-687X, is published (in two languages, in English and in Russian): (No.2(30), v.14, 2008,115-140).

[6] Nowatski W., *Dynamics of Structured,* Arcady, 1972 (in Polish).

[7] Mitropolskiy Yu.A. and Moseenkov B.I.: *Asymptotic solutions of the Partial Differential Equations*, Kiev 1976. (in Russian)

[8] Rašković, D., *Theory of Oscillations,* Naučna knjiga, Beograd, 1965, p. 503. (in Serbian)

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-37. ID-79.**

# HYBRID SYSTEM DYNAMICS ON LAYER WITH NONLINEAR ELASTIC AND INERTIA PROPERTIES

**K. (Stevanović) Hedrih [1], M. Stamenković [2*] , N. Nešić [2**]**

[1] Mathematical Institute SANU, Department of Mechanics,
and Faculty of Mechanical Engineering University of Niš, Serbia,
Priv. address: 18000- Niš, Serbiam ul. Vojvode Tankosića 3/22,
e-mail: khedrih@eunet.rs
[2] Mathematical Institute SANU, Department of Mechanics,
11 000-Belgrade, Kneza Mihaila 36/III, Serbia,
[2*]e-mail: s_marija86@yahoo.com, [2**]e-mail: nnesic@gmail.com

**ABSTRACT.** Considered hybrid system contain series of equal type deformable bodies ( beams, plates or membranes) coupled by discrete continuum layers with linear or nonlinear-elastic and inertia properties. This hybrid system vibrate on the discrete continuum (see Figure 1. a* and b*) layer with nonlinear-elastic and inertia properties for different cases that down rigid surface is fixed or in vibration state. The mathematical description of hybrid system dynamics on discrete continuum layer with nonlinear-elastic and inertia properties is done. The expressions for obtaining generalized forces of interactions between deformable bodies as well as between hybrid system and discrete continuum layer of foundation correspond to independent generalized coordinates of hybrid system dynamics are presented. By use phenomenological mapping a mathematical analogy between time functions in one mode of hybrid system dynamics is identified for corresponding multi beam, multi plate and multi membrane system dynamics on foundation presented by discrete continuum layer with different properties. Then, on the basis of this phenomenological mapping and mathematical analogy, we present that solutions for one type of the hybrid system dynamics is possible to use for other for qualitative analysis of linear or nonlinear phenomena appeared in dynamics.

**Keywords**: hybrid system, deformable body, discrete continuum layer, foundation, standard element, nonlinear elastic, inertia properties, partial differential equations, phenomenological mapping.

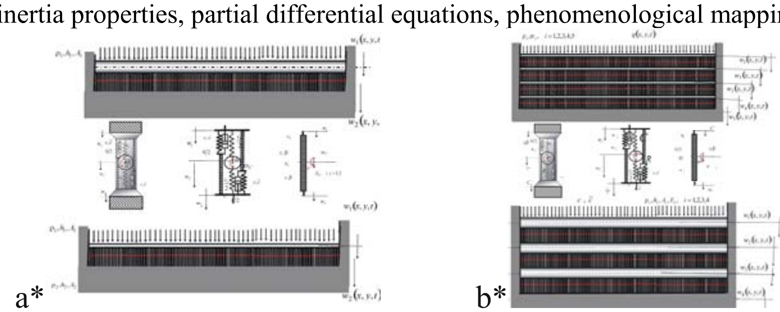



**Figure 1. (a*)** Mambrane and beam, and (b*) three membrane system and three beam system on discrete continuum nonlinear layer with translator and rotator inertia properties.

### Basic constitutive expressions and governing partial differential equations

Expressions of the generalized forces of interactions between deformable bodies as well as between hybrid system and discrete continuum layer of foundation correspond to independent generalized coordinates $w_k(x,y,t)$ (or $w_k(x,y,t)$) of hybrid system dynamics are in the form:

$$Q_{w_k}^{elem} = -\left\langle \frac{d}{dt}\frac{\partial \mathbf{E}_K{}^{elem}}{\partial\left(\frac{\partial w_k}{\partial t}\right)} - \frac{\partial \mathbf{E}_K{}^{elem}}{\partial w_k} \right\rangle = -\frac{1}{4}m\left[\left(\frac{\partial^2 w_{k+1}}{\partial t^2}+\frac{\partial^2 w_k}{\partial^2 t}\right) - \frac{i_C^2}{R^2}\left(\frac{\partial^2 w_{k+1}}{\partial t^2}-\frac{\partial^2 w_k}{\partial^2 t}\right)\right] - \tag{1}$$

$$-\left\{c_{0k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]+c_{0N,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]^3\right\} - b_{k,k+1}\left[\frac{\partial w_{k+1}(x,y,t)}{\partial t}-\frac{\partial w_k(x,y,t)}{\partial t}\right]$$

$$Q_{w_{k+1}}^{elem} = -\left\langle \frac{d}{dt}\frac{\partial \mathbf{E}_K{}^{elem}}{\partial\left(\frac{\partial w_{k+1}}{\partial t}\right)} - \frac{\partial \mathbf{E}_K{}^{elema}}{\partial w_{k+1}} \right\rangle = -\frac{1}{4}m\left[\left(\frac{\partial^2 w_{k+1}}{\partial t^2}+\frac{\partial^2 w_k}{\partial^2 t}\right) + \frac{i_C^2}{R^2}\left(\frac{\partial^2 w_{k+1}}{\partial t^2}-\frac{\partial^2 w_k}{\partial^2 t}\right)\right] + \tag{2}$$

$$+\left\{c_{0,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]+c_{0N,k,k+1}\left[w_{k+1}(x,y,t)-w_k(x,y,t)\right]^3\right\} + b_{k,k+1}\left[\frac{\partial w_{k+1}(x,y,t)}{\partial t}-\frac{\partial w_k(x,y,t)}{\partial t}\right]$$

For the hybrid system dynamic constructed by three coupled membranes (Figure 1. b*, for details see References [1-8]) on the discrete continuum layer of foundation system of partial differential equations is in the form:

$$\rho_1\frac{\partial^2 w_1(x,y,t)}{\partial t^2} = \rho_1 c_1^2\Delta w_1(x,y,t) - \frac{1}{4}m_1\left[\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial^2 t}\right) - \kappa\left(\frac{\partial^2 w_2}{\partial t^2}-\frac{\partial^2 w_1}{\partial^2 t}\right)\right] +$$

$$+c\left[w_2(x,y,t)-w_1(x,y,t)\right] + b\left[\frac{\partial w_2(x,y,t)}{\partial t}-\frac{\partial w_1(x,y,t)}{\partial t}\right] + q_1(x,y,t)$$

$$\rho_2\frac{\partial^2 w_2(x,y,t)}{\partial t^2} = \rho_2 c_2^2\Delta w_2(x,y,t) - \frac{1}{4}m_1\left[\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial^2 t}\right) + \kappa\left(\frac{\partial^2 w_2}{\partial t^2}-\frac{\partial^2 w_1}{\partial^2 t}\right)\right] - c\left[w_2(x,t)-w_1(x,t)\right] -$$

$$-b\left[\frac{\partial w_2(x,t)}{\partial t}-\frac{\partial w_1(x,t)}{\partial t}\right] - \frac{1}{4}m_2\left[\left(\frac{\partial^2 w_3}{\partial t^2}+\frac{\partial^2 w_2}{\partial^2 t}\right) - \kappa\left(\frac{\partial^2 w_3}{\partial t^2}-\frac{\partial^2 w_2}{\partial^2 t}\right)\right] -$$

$$-c\left[w_2(x,y,t)-w_3(x,y,t)\right] - b\left[\frac{\partial w_2(x,y,t)}{\partial t}-\frac{\partial w_3(x,y,t)}{\partial t}\right] - q_2(x,y,t) = 0$$

$$\rho_3\frac{\partial^2 w_3(x,y,t)}{\partial t^2} = \rho_2 c_3^2\Delta w_3(x,y,t) - \frac{1}{4}m_2\left[\left(\frac{\partial^2 w_3}{\partial t^2}+\frac{\partial^2 w_2}{\partial^2 t}\right) + \kappa\left(\frac{\partial^2 w_3}{\partial t^2}-\frac{\partial^2 w_2}{\partial^2 t}\right)\right] - c\left[w_3(x,t)-w_2(x,t)\right] -$$

$$-b\left[\frac{\partial w_3(x,t)}{\partial t}-\frac{\partial w_2(x,t)}{\partial t}\right] - \frac{1}{4}m_0\left[\left(\frac{\partial^2 w_0}{\partial t^2}+\frac{\partial^2 w_3}{\partial^2 t}\right) - \kappa\left(\frac{\partial^2 w_0}{\partial t^2}-\frac{\partial^2 w_3}{\partial^2 t}\right)\right] -$$

$$-c_0\left[w_3(x,y,t)-w_0(x,y,t)\right] - c_{0N}\left[w_3(x,y,t)-w_0(x,y,t)\right]^3 - b\left[\frac{\partial w_3(x,y,t)}{\partial t}-\frac{\partial w_0(x,y,t)}{\partial t}\right] - q_3(x,y,t) = 0$$

**Acknowledgement.** Parts of this research were supported by the Ministry of Sciences of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 “Dynamics of hybrid systems with complex structures. Mechanics of materials”

## References

[9] Hedrih (Stevanović) K., (2006), Modes of the Homogeneous Chain Dynamics, Signal Processing, Elsevier, 86(2006), 2678-2702.. ISSN: 0165-1684

[10] Hedrih (Stevanović) K., (2006), Transversal vibrations of double-plate systems. Acta. Mech. Sin. 22:487-501, 2006.

[11] Hedrih (Stevanović) K., (2008), Energy transfer in double plate system dynamics. Acta. Mech. Sin. 24, 331–344, 2008.

[12] Hedrih (Stevanovi´c) K., (2009), Energy transfer in the hybrid system dynamics (energy transfer in the axially moving double belt system), Special Issue, Arch. of Applied Mech., (2009) vol.79, No.6-7 pp. 529-540. DOI 10.1007/s00419-008-0285-7.

[13] Hedrih (Stevanović) K R. (2012), Energy and Nonlinear Dynamics of Hybrid Systems, Book Chapter, in Dynamical Systems and Methods, Edited by Albert Luo, Tenreiro Machado and D Baleanu, , 2012, Part 1, Pages 29-83, 2012, DOI: 10.1007/978-1-4614-0454-5_2 , ISBN 978-1-4614-0453-8. e-ISBN 978-1-4614-0454-5, Springer New York Dordrecht Heidelberg London

[14] Hedrih (Stevanović) K., Simonović J., (2010), “Models of Hybrid Multi-Plates Systems Dynamics”, The International Conference-Mechanical Engineering in XXI Century, Niš, Serbia, 25-26 September 2010, Proceedings, pp.17-20.

[15] Hedrih (Stevanović) K R. and Simonović J., Multi-frequency analysis of the double circular plate system non-linear dynamics", Nonlinear Dynamics: Volume 67, Issue 3 (2012), Page 2299-2315.

[16] Rašković, D., (1965), Teorija oscilacija, (Theory of Oscillations), Naučna knjiga, 1965, 503.

**Naučno društvo Srbije** | **Serbian Scientific Society**
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-38. ID-22.**

# SYSTEM OF DOUBLE THIN PLATES CONNECTED WITH LAYER OF ROLLING VISCO-ELASTIC NONLINEAR PROPERTIES

**Julijana D. Simonović**

Faculty of Mechanical Engineering, University of Niš, Serbia.
ul. Bulevar Nikole Tesle 55/16, 18000 Niš, Serbia.
e-mail: bjulijana@masfak.ni.ac.rs

Multi frequency vibrations of a system of two isotropic circular thin plates interconnected by a rolling visco-elastic layer that has nonlinear characteristics of the third order are considered. The considered physical system should be of interest to many researches from mechanical and civil engineering and its model is presented on the Fig. 1 a).

The connecting layer is modeled by model of rheological elements with rolling visco-elastic properties continually distributed between plate surfaces. The idea of discretisation of the continuum of the layer goes from the [4, 5]. For standard rolling visco nonlinear elastic element, Fig.1.c) lighted on a way of the rheological models, Fig. 2. b), see [2], we write the constitutive relation in the form:

$$F_{1(2)} = \pm\left(c + \frac{c_1}{4}\right)[w_2 - w_1] \pm \beta[w_2 - w_1]^3 \pm b\left[\frac{\partial w_2}{\partial t} - \frac{\partial w_1}{\partial t}\right] - \frac{1}{4}m\left\langle\left(\frac{\partial^2 w_2}{\partial t^2} + \frac{\partial^2 w_1}{\partial^2 t}\right) \mp \frac{i_C^2}{R^2}\left(\frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial^2 t}\right)\right\rangle \quad (1)$$

where upper sign belongs to the force on the upper end $F_1$ and the lower sign to the force on the lower end $F_2$ of the element. The $i_C^2 = \mathbf{J}_C/m$ is the square of radius of inertia for the rolling element. If the rolling element is the disc then $i_C^2 = R^2/2$. Stiffness of the springs are $c$ and $c_1$, the linear ones, and $\beta$, nonlinear one, and $b$ is dumping coefficient as it is presented on the Fig. 1 c). The governing equations of the double plate system, are formulated in terms of two unknowns: the transversal displacement $w_i(r,\varphi,t)$, $i=1,2$ in direction of the axis $z$, of the point of plates middle surfaces. Since the elements are continually distributed on plates surfaces the generalized resulting forces (1) are also continually distributed onto middle plate points. Our assumptions for the plates are: they are thin with same contours and with equal type of the boundary conditions and they have small transversal displacements. The system of two coupled partial differential equations is derived using d'Alembert's principle of dynamic equilibrium, see [3, 6], in the following forms:

$$\begin{aligned}
&\frac{\partial^2 w_1}{\partial t^2}(1+\tilde{a}_{11}) + \tilde{a}_{12(1)}\frac{\partial^2 w_2}{\partial t^2} + c_{(1)}^4\Delta\Delta w_1 - 2\delta_{(1)}\left[\frac{\partial w_2}{\partial t} - \frac{\partial w_1}{\partial t}\right] - a_{(1)}^2[w_2 - w_1] = \varepsilon\beta_{(1)}[w_2 - w_1]^3 + \tilde{q}_{(1)} \\
&\frac{\partial^2 w_2}{\partial t^2}(1+\tilde{a}_{22}) + \tilde{a}_{12(2)}\frac{\partial^2 w_1}{\partial t^2} + c_{(2)}^4\Delta\Delta w_2 + 2\delta_{(2)}\left[\frac{\partial w_2}{\partial t} - \frac{\partial w_1}{\partial t}\right] + a_{(2)}^2[w_2 - w_1] = -\varepsilon\beta_{(2)}[w_2 - w_1]^3 - \tilde{q}_{(2)}
\end{aligned} \quad (2)$$

where are : $\tilde{a}_{ii} = \hat{a}_{ii}/\rho_i h_i$, $\tilde{a}_{12(i)} = \hat{a}_{12}/\rho_i h_i$, $\hat{a}_{12} = m/8$, $\hat{a}_{ii} = m/4 + \mathbf{J}_C/4R^2 = 3m/8$ and $a_{(i)}^2 = 1/\rho_i h_i (c + c_1/4)$, $\mathrm{D}_i = E_i h_i^3/12(1-\mu_i^2)$, $c_{(i)}^4 = \mathrm{D}_i/\rho_i h_i$, $2\delta_i = b/\rho_i h_i$ and $\varepsilon\beta_{(i)} = \beta/\rho_i h_i$, for $i=1,2$ and for the strong nonlinear characteristic. The $\tilde{q}_{(i)} = \tilde{q}_{(i)}(r,\varphi,t)$ are the known functions of the external excitation forces on plates surfaces. The first asymptotic approximation of the solutions describing stationary and no stationary behavior, in the regions around the resonances, is derived by using the Krilov-Bogolyubov-Mitropolskiy asymptotic method, see [7]. On the bases of those results the influence on the system dynamics of rolling coupling element was analyzed by changing their masses. A series of the amplitude-frequency and phase-frequency curves of the two-frequency like vibration regimes are presented for three cases of rolling elements masses values. That curves present the evolution of the first asymptotic approximation of solutions for different nonlinear harmonics obtained by changing external excitation frequencies through discrete as well as continuous values for different values of rolling elements masses.

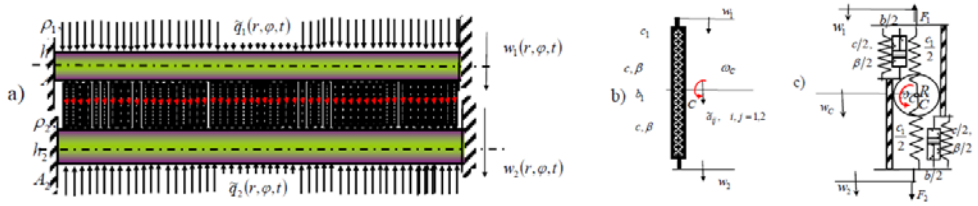


**Figure 1:** a) Model of the double circular plate system connected with a layer of the rheological elemets with properties of the nonlinear elasticity, dumping and inertia of translation and rotation of rolling part; b) a rheological scheme of rolling visco-elastic nonlinear discrete element; c) a model of rolling visco-elastic nonlinear discrete element.

Such an analyze prove that the presence of rolling coupling elements in the interconnected layer of two plates causes frequency overlap of resonant regions of nonlinear modes, which in the same time cause the enlargement of the mode mutual interactions. The phenomena of nonlinearity described in the paper [1] are all presented and lighted also in these results.

**Acknowledgment** *I extend my sincere and special appreciation to Professor Katica (Stevanović) Hedrih for all her comments and motivation that she gave to me. Parts of this research were supported by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU Belgrade Grant OI174001 - Dynamics of hybrid systems with complex structures. Mechanics of materials.*

## References

[1] Brennan M.J., Kovacic I., Carrella A., Waters T.P., On the jump-up and jump- down frequencies of the Duffing oscillator, J. Sound Vibration 318, pp. 1250-1261, 2008.

[2] Goroško O. A. i Hedrih (Stevanović) K., Analitička dinamika (mehanika) diskretnih naslednih sistema, (Analytical Dynamics (Mechanics) of Discrete Hereditary Systems), University of Niš, Monograph, pp. 426, YU ISBN 86-7181-054-2, 2001.

[3] Hedrih (Stevanović) K., Transversal Vibrations of Double-Plate Systems, Acta Mechanica Sinica, Springer, 22, pp. 487-501, 2006.

[4] Hedrih (Stevanović) K., Integrity of Dynamical Systems, Journal Nonlinear Analysis, 63, pp. 854 – 871, 2005.

[5] Hedrih (Stevanović) K., Frequency equations of small oscillations mixed systems of the coupled discrete and continuous subsystems, Mehanika tverdogo tela (Rigid Body Mechanics), Donetsk, UDC 531.1:534.012:534.013:, ISSN 0321-1975, vip. 33, pp. 174-189. Ukraine

[6] Hedrih (Stevanović) K. and Simonović J., Multi-frequency analysis of the double circular plate system non-linear dynamics, Nonlinear Dynamics: Volume 67, Issue 3,pp. 2299-2315, Springer, 2012.

[7] Mitropolyskiy, Yu. A., Nelinyeynaya mehanika-Asimptoticcheskie metodi, Institut matematiki NAN Ukraini, Kiev, pp. 397, 1995.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-39. ID-78.**

# FORCED OSCILLATIONS OF A MEMBRANE ON NONLINEAR ELASTIC FOUNDATION

**Nikola Nešić** [1]

[1] Mathematical Institute SANU, Department of Mechanics,
11 000-Belgrade, Kneza Mihaila 36/III, Serbia,
e-mail: nnesic@gmail.com

**ABSTRACT.** In this research, forced transversal oscillations of a rectangular membrane on nonlinear elastic foundation are considered, and for special case an analytical approximations of the solutions are given. Based on numerical experiment specific visualizations of the asymptotic approximation of amplitude-frequency and phase frequency curves which correspond to the asymptotic approximation of solutions are conducted, which describes qualitative properties of one frequency nonlinear oscillation stationary and no stationary regimes.

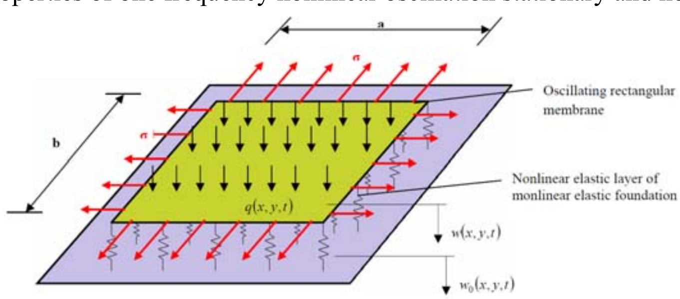



**Figure 1.** Membrane on the nonlinear elastic foundation loaded by external distributed excitation

On the basis of the vibration analysis of the multi deformable body (beams, plates or membranes) systems (see Refs. [1-4]) with coupling by elastic layers, we analyzed vibrations of the deformable membrane on deformable nonlinear elastic foundation. In Figure 1. are displayed analyzed forced nonlinear vibrations of the membrane system on a nonlinear elastic foundation. Membrane is loaded along contour in middle surface by stress extension $\sigma$ . Nonlinear elastic foundation is in the form of the discrete continuum layer consisting of the standard light ideally elastic element with material properties described by constitutive force-strain relation in the form: $F_e = -c[w(x,y,t) - w_0(x,y,t)] - \tilde{c}[w(x,y,t) - w_0(x,y,t)]^3$, where $w(x,y,t)$ is transversal displacement of the membrane point $N(x,y)$, and $w_0(x,y,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} w_{0nm} W_{nm}(x,y)\cos\Omega_{nm}t$ known transversal displacement of down rigid surface of nonlinear elastic foundation. Partial differential equation of the nonlinear vibrations of membrane on nonlinear foundation is in the following form (for details see References [1-7]):

$$\frac{\partial^2 w(x,y,t)}{\partial t^2} = c_0^2 \Delta w(x,y,t) - \frac{c}{\rho}[w(x,y,t) - w_0(x,y,t)] - \frac{\tilde{c}}{\rho}[w(x,y,t) - w_0(x,y,t)]^3 + \frac{q(x,y,t)}{\rho} \qquad (1)$$

where $\frac{q(x,y,t)}{\rho}=\sum_{m=1}^{M}\sum_{n=1}^{N}h_{0nm}W_{nm}(x,y)\cos(\Omega_{nm}t+\vartheta_{nm})$ external distributed transversal excitation. For one frequency forced nonlinear vibrations in the form of eigen amplitude function $W_{NM}(x,y)$, for $w_0(x,y,t)=w_{0NM}W_{NM}(x,y)\cos\Omega_{NM}t$ and $\frac{q(x,y,t)}{\rho}=h_{0NM}W_{NM}(x,y)\cos(\Omega_{NM}t+\vartheta_{NM})$ for obtaining corresponding eigen time function $T_{NM}(t)$, the differential equation is obtained using Bernoulli's particular integral method (see Ref. [6]) or Galerkin method in the following form

$$\begin{aligned}&\ddot{T}_{NM}(t)+(\omega_0^2+c_0^2k_{NM}^2)T_{NM}(t)=-\tilde{\omega}_{0F}^2\tilde{g}_{NM}T_{NM}^3(t)+h_{0NM}\cos(\Omega_{NM}t+\vartheta_{NM})+\\&+w_{0NM}\omega_0^2\cos\Omega_{NM}t-\tilde{\omega}_{0n}^2\tilde{g}_{NM}T_{NM}^3(t)+3\tilde{\omega}_{0n}^2\tilde{g}_{NM}T_{NM}^2(t)w_{0NM}\cos\Omega_{NM}t-\\&-\tilde{\omega}_{0n}^2\tilde{g}_{NM}3T_{NM}(t)w_{0NM\,0NM}^2\cos^2\Omega_{NM}t+\tilde{\omega}_{0n}^2\tilde{g}_{NM}w_{0NM\,0NM}^3\cos^3\Omega_{NM}t\end{aligned} \qquad (2)$$

where $\omega_0^2=\frac{c}{\rho}$, $\tilde{\omega}_{0F}^2=\frac{\tilde{c}}{\rho}$, $\tilde{g}_{NM}=\frac{\iint_A[W_{NM}(x,y)]^4\,dxdy}{\iint_A[W_{NM}(x,y)]^2\,dxdy}$, and $k_{mn}^2$ is characteristic number depending on membrane boundary conditions. Using Krilov-Bogolybov-Mitropolyski method (see Ref. [5]) for forced one frequency nonlinear vibrations for $w_0(x,y,t)=0$, in frequency main resonant interval $\Omega_{NM}\approx\omega_{NM}$, small external excitation amplitude $h_{0NM}\approx\varepsilon\tilde{h}_{0NM}$, and for time function $T_{NM}(t)$ described by:

$$\ddot{T}_{NM}(t)+(\omega_0^2+c_0^2k_{NM}^2)T_{NM}(t)=-\tilde{\omega}_{0F}^2\tilde{g}_{NM}T_{NM}^3(t)+h_{0NM}\sin(\Omega_{NM}t+\vartheta_{NM}) \qquad (3)$$

the system of ordinary differential equations along amplitude $a_{NM}(t)$ and phase $\phi_{NM}(t)=\Phi_{NM}(t)-\vartheta_{NM}(t)$, $\vartheta_{NM}(t)=\Omega_{NM}t+\vartheta_{NM}$ in first asymptotic approximation of first asymptotic approximation $T_{NM}(t)=a_{NM}(t)\cos\Phi_{NM}(t)$ of solution, is obtained in following form:

$$\begin{aligned}&\frac{da_{NM}(t)}{dt}=-\frac{h_{0NM}}{\omega_{NM}+\Omega_{NM}}\cos\phi_{NM}(t),\\&\frac{d\phi_{NM}(t)}{dt}=\omega_{NM}-\Omega_{NM}+\frac{3}{8}\frac{\tilde{\omega}_{0F}^2\tilde{g}_{nm}}{\omega_{NM}}[a_{NM}(t)]^2+\frac{h_{0NM}}{a_{NM}(t)[\omega_{NM}+\Omega_{NM}]}\sin\phi_{NM}(t)\end{aligned} \qquad (4)$$

By use system (4) series of the amplitude-frequency and phase frequency curves for stationary and no stationary vibration regimes in resonant frequency interval are obtained.

By differential equation (2) is rheononlinear we open a new research task for next investigation of different types of vibration regimes.

**Acknowledgement**. Author highly acknowledges Professor Katica (Stevanović) Hedrih, Leader of the Project OI174001, for all her comments, valuable consultations and motivation that she gave him to submit this paper. Parts of this research were supported by the Ministry of Sciences and Technology of the Republic of Serbia through Mathematical Institute SANU Belgrade Grant OI174001 Dynamics of hybrid systems with complex structures. Mechanics of materials.

## References

[17] Hedrih (Stevanović) K., (2006), Transversal vibrations of double-plate systems. Acta. Mech. Sin. 22:487-501, 2006.
[18] Hedrih (Stevanović) K., (2008), Energy transfer in double plate system dynamics. Acta. Mech. Sin. 24, 331–344, 2008.
[19] Hedrih (Stevanović) K., (2009), Energy transfer in the hybrid system dynamics (energy transfer in the axially moving double belt system), Special Issue, Arch. of Applied Mech., (2009) vol.79, No.6-7 pp. 529-540. DOI 10.1007/s00419-008-0285-7.
**[20]** Hedrih (Stevanović) K. and Simonović J., (2011), „*Multi-frequency analysis of the double circular plate system non-linear dynamics*", Nonlinear Dynamics, DOI 10.1007/s11071-011-0147-7.
[21] Mitropolyskiy, Yu.A and Mosseenkov, B.I., (1976), *Assimptoticheskie recheniya uravnyeniya v chastnih proizvodnih*, Vichaya chkola Kiev. (in Russian)
[22] Rašković D., Theory of oscilations. Naučna knjiga, Belgrade, 1965.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-40. ID-59.**

# VIBRO-IMPACT SYSTEM BASED ON OSCILLATOR WITH TWO HEAVY MASS PARTICLES ALONG HORIZONTAL ROUGH LINE

**Srdjan Jović[1], Vladimir Raičević[1]**

[1]Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština
38 220 Kosovska Mitrovica, Ul. Kralja Petra I br. 149/12, Serbia,
e-mail: jovic003@gmail.com

**ABSTRACT.** This work is based on the analysis of vibro-impact system based on oscillator with two degrees of freedom of motion moving along rough horizontal plane and having Coulomb's sliding friction coefficient of $\mu = tg\alpha_0$. The oscillator is consisted by two heavy mass particles, mass $m_1$ and $m_2$, bonded by springs $c_1$ and $c_2$ for the stabile points $A_1$ and $A_2$. Free motion of heavy mass particles is limited by their mutual impacts. The analytical-numerical results for defined kinetic parameters of the observed vibro-impact system are the base for the energy analysis visualization. In this paper, the investigation methodology for the energy transfer between impact elements of the observed vibro-impact system is presented. The procedure of the determination of time and position of the heavy mass particle collision, and determination of their input and output velocities immediately before and after the impact. The graphic visualization of the motion analysis of the representative point of the system kinetic during the kinetic (dynamic) is made by application of software package MathCad and user's package CorelDraw.

**Keywords**: Heavy mass particle, friction, vibro-impact, phase trajectory, singular points, total energy, kinetic and potential energy, graphical presentation, representative point.

## References

[1] Babickii V. I.: ***Theory of vibro-impact systems***, Moskva, "Nauka", 1978. (in Russian).

[2] Hedrih (Stevanović) K., ***Vibrations of a Heavy Mass Particle Moving along a Rough Line with Friction of Coulomb Type***, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation ISSN: 1565-1339, Volume 11, NO.3, pp.203-210, 2010.

[3] Hedrih (Stevanović) K., ***Free and forced vibration of the heavy material particle along line with friction***: Direct and inverse task of the theory of vibrorheology, 7th EUROMECH Solid Mechanics Conference, J. Ambrósio et.al. (eds.), Lisbon, Portugal, September 7-11, 2009, CD –MS-24, Paper 348, pp. 1-20.

[4] Hedrih (Stevanović) K., Jović S., ***Models of Technological Processes on the Basis of Vibro-impact Dynamics***, Scientific Technical Review, Vol.LIX,No.2,2009,pp.51-72 **.**

[5] Hedrih (Stevanović) K., Raičević V., Jović S., ***Vibro-impact of a Heavy Mass Particle Moving along a Rough Circle with Two Impact Limiters***, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation ISSN: 1565-1339, Volume 11, NO.3, pp.211-224, 2010.

[6] Hedrih (Stevanović) K., Raičević V., Jović S., ***Phase Trajectory Portrait of the Vibro-imact Forced Dynamics of Two Mass Particles along Rough Circle***, Communications in Nonlinear Science and Numerical Simulation ISSN: 1007-5704, Volume 16, Issue 12, December 2011, Pages 4745-4755.

[7] Jović S., ***The phase portrait of the vibro-impact dynamics of two mass particle motions along rough circle***, DYNAMICAL SYSTEMS - Nonlinear Dynamics and Control, EDITORS: J. Awrejcewicz, M. Kazmierczak, P. Olejnik, J. Mrozowski, pp. 429-434, Łódź, December 5-8, 2011, Poland.

[8] Jović S., ***Energijska analiza dinamike vibroudarnih sistema***, magistarski rad, str. 239, odbranjen 06. Novembra 2009. Fakultet tehničkih nauka u Kosovskoj Mitrovici Univerziteta u Prištini.

[9] Jović S., ***Energijska analiza dinamike vibroudarnih sistema sa krivolinijskim putanjama i neidealnim vezama***, doktorska disertacija, str. 335, odbranjena 15. Februara 2011. Fakultet tehničkih nauka u Kosovskoj Mitrovici Univerziteta u Prištini.

[10] Jović S., Raičević V., ***Vibro-impact forced oscillations of a heavy mass particle along a rough circle excited by a single-frequency force***, Acta Mechanica, ISSN: 0001-5970, Volume 223, Number 6, pp. 1153-1168, June 2012. DOI: 10.1007/s00707-012-0623-2v.

[11] Jović S., Raičević V., ***Energy analysis of vibro-impact system dynamics based on a heavy mass particle free oscillations along curvilinear rough trajectories***, Scientific Technical Review, Vol.60, No.3-4, 2010, pp.9-21.

[12] Jović S., Raičević V., Garić Lj., ***Vibro-Impact System Based on Forced Oscillations of Heavy Mass Particle Along a Rough Parabolic Line***, Hindawi Publishing Corporation, Mathematical Problems in Engineering, Volume 2012, Article ID 846390, 17 pages doi:10.1155/2012/846390, ISSN: 1024-123x, (Online First), 2012.

[13] Kobrinskii A. E., Kobrinskii A. A., ***Vibroudrni sistemi***, Moskva, Nauka, 1973.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-41. ID-71.**

# THE PHASE PORTRAIT OF THE VIBRO-IMPACT SYSTEM BASED ON OSCILLATOR, WITH THREE HEAVY MASS PARTICLES MOVING ALONG A ROUGH CIRCLE

**Srdjan Jović[1], Vladimir Raičević[1]**

[1]Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština
38 220 Kosovska Mitrovica, Ul. Kralja Petra I br. 149/12, Serbia,
e-mail: jovic003@gmail.com

**ABSTRACT.** The paper is presentation of an analysis of vibro-impact system motion based on oscillator with three degrees of freedom moving in a circular rough line in the vertical plane. Oscillator consists of three heavy mass particles whose free motion is limited by two fixed elongation limiters. Analytical and numerical results for the specific kinetic parameters of observed vibro-impact systems are the basis for visualization of the energy analysis which was subject of this analytical research. This paper deals with study of energy transfer between elements of the analyzed vibro-impact system. The process of determining time interval and angle at which there is an impact of heavy mass particles appeared and the determination of their incoming and outgoing velocities immediately before and after the impact was studied here.

Free movement of heavy mass particles was divided into appropriate intervals. Each motion interval corresponds to the differential equation of motion from ordinary non-linear homogeneous second order differential equations with variable coefficients. These differential equations are solved in analytical form. Differential equations of motion for the corresponding motion intervals are matched with the corresponding initial conditions of motion, impact conditions to the elongation limiters, impact conditions of heavy mass particles, and alternation conditions of the direction that cause an alternation of friction force direction. By the analytical solution of differential equations of motion, we came to the analytical expression for the equation of phase trajectory in plane $(\varphi_i, \dot{\varphi}_i)$, $i = 1,2,3$ where $i$ is number of degfees of freedom, with energy equation curves necessary for energy analysis of the dynamics of vibro-impact system. Graphical visualization and analysis representative kinetics state point of the system during the kinetics (dynamics) was performed using the software package MathCad and the users package CorelDraw.

**Keywords**: Heavy mass particle, rough circle, friction, two impact limiters, vibro-impact, phase trajectory, singular points, representative point.

## References

[14] Babickii V. I.: ***Theory of vibro-impact systems***, Moskva, "Nauka", 1978. (in Russian).

[15] Hedrih (Stevanović) K., ***Vibrations of a Heavy Mass Particle Moving along a Rough Line with Friction of Coulomb Type***, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation ISSN: 1565-1339, Volume 11, NO.3, pp.203-210, 2010.

[16] Hedrih (Stevanović) K., ***Free and forced vibration of the heavy material particle along line with friction***: Direct and inverse task of the theory of vibrorheology, 7th EUROMECH Solid Mechanics Conference, J. Ambrósio et.al. (eds.), Lisbon, Portugal, September 7-11, 2009, CD –MS-24, Paper 348, pp. 1-20.

[17] Hedrih (Stevanović) K., Jović S., ***Models of Technological Processes on the Basis of Vibro-impact Dynamics***, Scientific Technical Review, Vol.LIX,No.2,2009,pp.51-72 **.**

[18] Hedrih (Stevanović) K., Raičević V., Jović S., ***Vibro-impact of a Heavy Mass Particle Moving along a Rough Circle with Two Impact Limiters***, ©Freund Publishing House Ltd., International Journal of Nonlinear Sciences & Numerical Simulation ISSN: 1565-1339, Volume 11, NO.3, pp.211-224, 2010.

[19] Hedrih (Stevanović) K., Raičević V., Jović S., ***Phase Trajectory Portrait of the Vibro-imact Forced Dynamics of Two Mass Particles along Rough Circle***, Communications in Nonlinear Science and Numerical Simulation ISSN: 1007-5704, Volume 16, Issue 12, December 2011, Pages 4745-4755.

[20] Jović S., ***The phase portrait of the vibro-impact dynamics of two mass particle motions along rough circle***, DYNAMICAL SYSTEMS - Nonlinear Dynamics and Control, EDITORS: J. Awrejcewicz, M. Kazmierczak, P. Olejnik, J. Mrozowski, pp. 429-434, Łódź, December 5-8, 2011, Poland.

[21] Jović S., ***Energijska analiza dinamike vibroudarnih sistema sa krivolinijskim putanjama i neidealnim vezama***, doktorska disertacija, str. 335, odbranjena 15. Februara 2011. Fakultet tehničkih nauka u Kosovskoj Mitrovici Univerziteta u Prištini.

[22] Jović S., Raičević V., ***Vibro-impact forced oscillations of a heavy mass particle along a rough circle excited by a single-frequency force***, Acta Mechanica, ISSN: 0001-5970, Volume 223, Number 6, pp. 1153-1168, June 2012. DOI: 10.1007/s00707-012-0623-2v.

[23] Jović S., Raičević V., ***Energy analysis of vibro-impact system dynamics based on a heavy mass particle free oscillations along curvilinear rough trajectories***, Scientific Technical Review, Vol.60, No.3-4, 2010, pp.9-21.

[24] Jović S., Raičević V., Garić Lj., ***Vibro-Impact System Based on Forced Oscillations of Heavy Mass Particle Along a Rough Parabolic Line***, Hindawi Publishing Corporation, Mathematical Problems in Engineering, Volume 2012, Article ID 846390, 17 pages doi:10.1155/2012/846390, ISSN: 1024-123x, (Online First), 2012.

[25] Kobrinskii A. E., Kobrinskii A. A., ***Vibroudarni sistemi***, Moskva, Nauka, 1973.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-42, ID-39.**

# ANALYSIS OF A RIGID BODY ROTATION AROUND TWO NO INTERSECTING AXES – VECTOR METHOD AND PARAMETER ANALYSIS OF PHASE TRAJECTORIES

**Ljiljana Veljović**
Faculty of Mechanical Engineering University of Kragujevac,
Sestre Janjić 6, 34000 Kragujevac, Serbia
E-mail: veljovicljilja@yahoo.co.uk

**ABSTRACT** Vector expressions for linear momentum and angular momentum and their corresponding derivatives with respect to time describe rigid body nonlinear dynamics with coupled rotations around axes without intersection**.** Vector method based on mass moment vectors coupled for pole and oriented axes was defined by K. Hedrih in 1991. and it is applied for analysis of a heavy gyrorotor nonlinear dynamics. Series of graphical presentations and their parametric transformations in realiation with changing orthogonal distance beetween axes of coupled rotations is presented.

## Introduction

Mass moment vectors for axis and pole was introduced and defined by Hedrih (Stevanovic) K., and presented in the monograph [1] as well as in the series of papers [2-4] listed here.
Here is considered a heavy rigid body which rotates around two coupled axes without intersection and with orthogonal distance defined by vector $\vec{r}_0 = \overrightarrow{O_2O_1}$. The body is eccentrically and inclined positioned on the self–rotation axis. Two angular velocities around two axes oriented by unit vectors $\vec{n}_1$ and $\vec{n}_2$ are denoted as $\vec{\omega}_1 = \omega_1\vec{n}_1$ and $\vec{\omega}_2 = \omega_2\vec{n}_2$. By using basic definitions for linear momentum and angular momentum as well as expression for velocity of elementary body mass particle rotation $\vec{v} = [\vec{\omega}_2, \vec{r}_0] + [\vec{\omega}_1 + \vec{\omega}_2, \vec{\rho}]$, the following vector expressions are presented: a* for linear momentum and b* for angular momentum.

a* $$\vec{K} = [\vec{\omega}_2, \vec{r}_0]M + \vec{\omega}_1 \vec{S}^{(O_1)}_{\vec{n}_1} + \vec{\omega}_2 \vec{S}^{(O_1)}_{\vec{n}_2} \tag{1}$$

b* $$\vec{L}_{O_1} = \omega_1 \vec{n}_1 r_0^2 M + \omega_1 [\vec{\rho}_C, [\vec{n}_1, \vec{r}_0]]M + \omega_1 \left[\vec{r}_0, \vec{S}^{(O_2)}_{\vec{n}_1}\right] + \omega_2 \left[\vec{r}_0, \vec{S}^{(O_2)}_{\vec{n}_2}\right] + \omega_1 \vec{J}^{(O_2)}_{\vec{n}_1} + \omega_2 \vec{J}^{(O_2)}_{\vec{n}_2} \tag{2}$$

Where $\vec{S}_{\vec{n}_1}^{(O_1)} = \iiint\limits_V [\vec{n}_1,\vec{\rho}]dm$ and $\vec{S}_{\vec{n}_2}^{(O_1)} = \iiint\limits_V [\vec{n}_2,\vec{\rho}]dm$, $dm = \sigma dV$ are coresponding body mass linear moments of the rigid body for the axes oriented by direction of component angular velocities of coupled rotations through the moveable pole $O_1$ on self rotating axis .

Using first derivatives of linear momentum and angular momentum two vector equations of rigid body coupled rotation around axes without intersection are obtained. These vector expressions can be used in general case when considered system has two degree of freedom. Differential equation of self rotation and equation of phase expressions for kinetic pressures to bearings of self-rotation shaft with corresponding components as well as for corresponding vector rotators are obtained [5-7]. Using obtained vector expressions by applying software tool, series of graphical presentations depending on the orthogonal distance between two axes of coupled disk rotations are obtained. These graphical presentations are: phase portraits, intensity of kinematics vector rotators, kinematics vector rotator hodographs and angular velocities of its rotations around self –rotation axis; series of intensity of kinetic pressures components on self rotation shaft bearings.

## Concluding remarks

By use derived vector expressions as well as graphical presentations of the system nonlinear dynamics, we can
conclude that body mass inertia moment vectors and vector rotators are very suitable for obtaining linear momentum and angular momentum and their derivatives of the system with coupled numerous rotations around axes without intersections, as well as for vector method analysis of system vector parameter dynamics. Analysis of rotation of a heavy gyro-rotor show us that in graphical presentations of the system kinetic parameters exits a set of the fixed points not depending of change of rigid body eccentricity or angle of inclination or of the orthogonal distance between axes of rigid body coupled rotations.

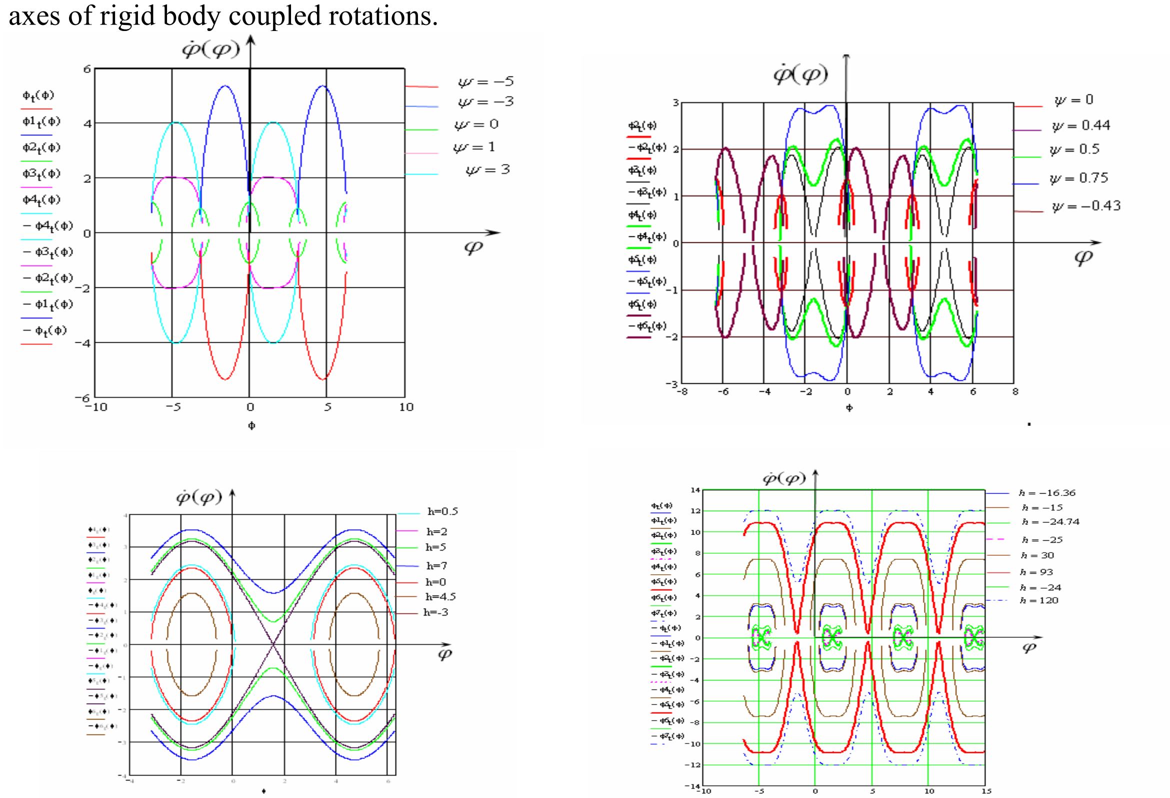



**Figure** 1. Parametric transformation of phase trajectory for different values of orthogonal distance between axes

***Acknowledgment.*** *My sincerely thanks to Professor Katica R. (Stevanović) Hedrih, Leader of the Project OI174001 and my supervisor for all her comments, valuable consultations and motivation that she gave me to submit this paper. Parts of this research were supported by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 and Faculties of Mechanical Engineering in Niš and Kragujevac.*

**Keywords** nonlinear dynamics, gyro-rotor, phase portraits, vector rotators

## References

[1] Hedrih (Stevanović), K., *Vector Method of the Heavy Rotor Kinetic Parameter Analysis and Nonlinear Dynamics*, University of Niš 2001,Monograph, p. 252. (in English), YU ISBN 86-7181-046-1.
[2] Hedrih (Stevanović), K. (1998), Vectors of the Body Mass Moments, Monograph paper, Topics from Mathematics and Mechanics, Mathematical institute SANU, Belgrade, Zbornik radova 8(16), 1998, pp. 45-104. published in 1999 .(in English), (Zentralblatt Review).
[3] Katica (Stevanović) Hedrih, (*Derivatives of the Mass Moment Vectors at the Dimensional Coordinate System N*, dedicated to memory of Professor D. Mitrinović, Facta Universitatis Series Mathematics and Informatics, 13 (1998), pp. 139-150. (1998, published in 2001. Edited by G. Milovanović).
[4] Hedrih (Stevanović), K., *Some vectorial interpretations of the kinetic parameters of solid material lines*, ZAMM. Angew.Math. Mech.73(1993) 4-5, T153-T156.
[5] Katica (Stevanović) Hedrih and Ljiljana Veljović, *Nonlinear dynamics of the heavy gyro-rotor with two skew rotating axes*, Journal of Physics: Conference Series, 96 (2008) 012221 DOI:10.1088/1742-6596/96/1/012221, IOP Publishing http://www.iop.org/EJ/main/-list=current/
[6] Hedrih (Stevanović) K, Veljović Lj., *The Kinetic Pressure of the Gyrorotor Eigen Shaft Bearings and Rotators*, The Third International Conference Nonlinear Dynamics – 2010, pp. 78-83
[7] Hedrih (Stevanović) K, Veljović Lj., *Vector Rotators of a Rigid Body Dynamics with Coupled Rotations around Axes without Intersection,* Mathematical Problem in Engineering, Volume 2011, Article 351269, 26 pages, doi:10.1155/2011/351269

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-43. ID-33.**

# THREE-PARAMETRIC TESTING OF SINGULARITY AND POSITION OF NON-LINEAR DYNAMICS RELATIVE BALANCE OF HEAVY MATERIAL PARTICLE ON ECCENTRICALLY ROTATING SMOOTH CIRCLE LINE

**Marija Stamenković, Marija Mikić**

Mathematical Institute SANU, Department of Mechanics,
11 000-Belgrade, Kneza Mihaila 36/III, Serbia,
e-mail: s_marija86@yahoo.com

Mathematical Institute SANU, Department of Mechanics,
11 000-Belgrade, Kneza Mihaila 36/III, Serbia, and Faculty of Mathematics,
11 000-Belgrade, Studentski trg 16, Serbia
e-mail: marijam@matf.bg.ac.rs

## ABSTRACT

The paper contains analytical descriptions of heavy material particle which moves on a rotating circular smooth line, radius $R$, which rotate around vertical axis, eccentrically positioned in relation to center of circle line on distance $e$, angular velocity $\Omega$, (see Figure 1).
By using software MathCad, three-parametric testing of singularity and position of non-linear dynamics relative balance of heavy material particle on eccentrically rotating smooth circle line is examined.

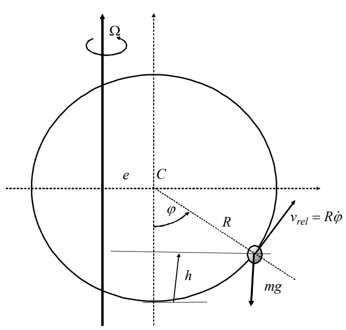



**Figure 1.** Moving of heavy material particle on circle line, radius $R$, which rotates around vertical axis, eccentrically positioned in relation to center of circleline on distance $e$,by angular velocity $\Omega$

**Introduction.** Dynamics of heavy material particle is a very old engineering problem with many different research results and discoveries of new nonlinear phenomena (see Refs. [1], [2], [3]). Many researchers pay attention and interest for research the nonlinear dynamics (see Refs. [4], [5], [6], [7]) by using new analytical, numerical and experimental methods to discover the properties of nonlinear dynamics.
In Figure 1, for generalized coordinate we will take the angle φ, by which we mark relative position of material particle on circle line.

The system has one degree of moving freedom, and two degrees of moving freedom because one reonomic relation is imposed- rotating by constant angular velocity.

Differential equation of heavy material particle moving illustrated in figure is:

$$\ddot{\varphi}+\Omega^2\left(\frac{g}{R\Omega^2}-\cos\varphi\right)\sin\varphi-\Omega^2\frac{e}{R}\cos\varphi=0$$

If we introduce following symbols $\lambda=\frac{g}{R\Omega^2}$ $\varepsilon=\frac{e}{R}=\psi$ , previous differential equation is in the following form $\ddot{\varphi}+\Omega^2(\lambda-\cos\varphi)\sin\varphi-\Omega^2\psi\cos\varphi=0$. Using the previous ordinary nonlinear differential equations of second order, form non-linear system of differential equations of first order in the following form: $\frac{d\varphi}{dt}=u,\ \frac{du}{dt}=-\Omega^2(\lambda-\cos\varphi)\sin\varphi-\Omega^2\psi\cos\varphi$.

Then we determine the stationary points of the previous system of differential equations, which correspond to positions of relative equilibrium of a heavy material point on the eccentric-rotating circular smooth line. Depending on the parameter values **Error! Objects cannot be created from editing field codes.** and **Error! Objects cannot be created from editing field codes.**, system will have a different number of stationary points, as shown in this paper. To determine the behavior of solutions near singular points we linearize the system around the stationary points of system **Error! Objects cannot be created from editing field codes.**:

$$\frac{d\varphi}{dt}=u,\ \frac{du}{dt}=-\Omega^2(\lambda\cos\varphi_s-\cos^2\varphi_s+\sin^2\varphi_s-\psi\sin\varphi_s)\varphi.$$

We are interested in the values of the parameters $\lambda$ and $\psi$ leads to changes in dynamic systems. The paper deals with the specific parameter values $\lambda$ and $\psi$ for which there is a substantial change in the dynamical system, ie. we determined the bifurcation point. The example is illustrated (for fixed values of parameters $\lambda$ and change the parameter $\psi$ ) change in the dynamic system. (Figure 2 and Figure 3)

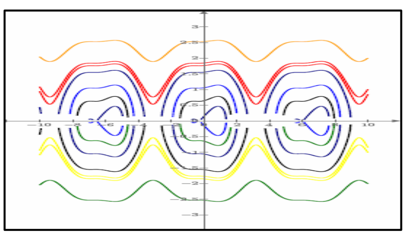


Figure 2.

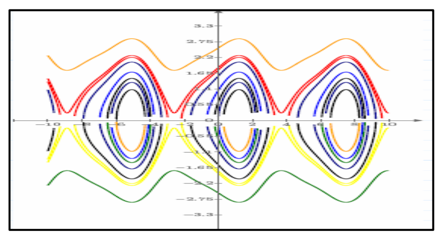


Figure 3.

***Acknowledgement****. We want to express our sincere and special appreciation to Professor Katica (Stevanović) Hedrih Project OI174001 Leader , for all her comments and motivation that she gave to us. This research was supported through Project OI174001 by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU.*

## References

[1] Andronov, A.A., Vitt, A.A., Haykin, S.E., (1981), *Teoriya kolebaniy,* Nauka, Moskva., pp. 568

[2] Gerard I. and Daniel J., *Elementary Stability and BIfurcation Theory*, Springer Verlag, 1980.

[3] Guckenheimer, J. and Holmes, Ph. , (1983), Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Fields , Springer-Verlag, pp. 461.

[4] Hedrih (Stevanović), K., *Trigger of Coupled Singularities (invited plenary lecture)* , Dynamical Systems-Theory and Applications, Edited By J. Awrejcewicz and all, Lodz 2001, pp. 51-78.

[5] Hedrih (Stevanović), K., *Nonlinear Dynamics of a Gyrorotor, and Sensitive Dependence on initial Conditions of a Heav Gyrorotor Forced Vibration/Rotation Motion*, Semi-Plenary Invited Lecture, Proceednings: COC 2000, Edited by F.L. Chernousko and A.I. Fradkov, IEEE, CSS, IUTAM, SPICS, St. Petersburg, Inst. for Problems of Mech. Eng. of RAS, 2000., Vol. 2 of 3, pp. 259-266.

[6] Hedrih (Stevanoviж), K., *On Rheonomic Systems with Equaivalent Holonomic Conservative System,* Int. Conf. ICNM-IV, 2002, Edited by W. Chien and all. Shanghai, *T.* Nonlinear dynamics. pp. 1046-1054.

[7] Hedrih (Stevanović), K., *The Vector Method of the Heavy Rotor Kinetic Parameter Analysis and Nonlinear Dynamics*, Monograph, University of Niš, 2001, pp. 252., YU ISBN 86-7181-046-1.

Naučno društvo Srbije — Serbian Scientific Society

http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-44. ID-49.**

# OPTIMAL DESIGN OF THIN-WALLED AIRCRAFT STRUCTURES USING TWO-LEVEL OPTIMIZATION APPROACH

**S. Maksimovic[1], K. Maksimovic[2] I. Vasovic[3],**
[1] Military Technical Institute,
Ratka Resanovića 1, 11000 Belgrade, Serbia
e-mail: s.maksimovic@open.telekom.rs
[2]Republic Serbia, City Administration of City of Belgrade,
Secretariat for Utilities and Housing Services Water Management,
Kraljice Marije 1, 11000 Belgrade, Serbia
e-mail: katarina.maksimovic@beograd.gov.rs
[3] Institute Goša, Milana Rakica 35, Belgrade, Serbia
e-mail: ivanavvasovic@gmail.com

**ABSTRACT.** An attention in this investigation is focused on developing efficient optimization method for minimum weight design of thin-walled structures. Today, it is a common practice to use numerical optimization methodologies to deal with multidisciplinary industrial design problems. One of the major tasks in the design of aircraft structures is the sizing of the structural members to obtain the desired strength, weight, and stiffness characteristics. Optimization algorithms have been coupled with structural analysis programs for use in this sizing process. Most of the difficulties associated with large structural design are solution convergence and computer resources requirements. Structural optimization problems traditionally have been solved by using either the mathematical programming (MP) or the optimality criteria (OC) approach. More recently, the works in Refs [1-4] have illustrated the uniformity of the methods. Each approach offers certain advantages and disadvantages. The MP methods are extremely useful in defining the design problem in proper mathematical terms. When the design variables are few the these methods can be used quite effectively for optimization. However, in the presence of a large number of variables these methods are very slow. The rate of convergence for OC methods is initially very fast, step size determination is critical closer to the local optimum where the number of active constraints' increases and the computations of Lagrange multipliers becomes more complex. Ideally, a methodology that exploits the strength of both approaches could be employed in a practical system. The object of the present investigation is to develop such design method that can efficiently optimize large structures that exploit power of the MP and OC methods. The motivation of this investigation is to come up with two-level optimization approach using optimality criteria and mathematical programming techniques. Two-level optimization approach permits a large problem to be broken down into a number of smaller ones, at different levels according to the type of problem being solved. This approach breaks the primary problem statement into a system level design problem and set of uncoupled component level problems. Results are obtained by iteration

between the system and component level problems. The decomposition of a complex optimization problem into a two-level hierarchy of simpler problems often has computational advantages. It makes the whole problem more tractable, especially for the large aircraft structures, because the number of design variables and constraints are so great that the optimization becomes both intractable and costly. The nature of an aircraft structure makes two-lewel optimization highly practical, not only in terms of reducing the computing cost but also because the individual tasks in the traditional design process are preserved. The suitability of multilevel optimization in more complex design problem tested on a structure representative of a wing box in composite material, with buckling limitations in each panel, and another problem in which reliability requirements are included. Two-level approach for optimization of the composite structures subject to stress, displacement, buckling and local failure constraints is developed.
The efficiency of method is based on application of the two-level approach in optimization structural systems. This approach breaks the primary problem statement into a system level design problem and set of uncoupled component level problems. Two-level optimization approach is applied to structural design problems like: minimum weight of the complex aircraft composite structure under various strength, buckling and stiffness constraints. This paper considers a discrete model the buckling sensitivity analysis of thin multi-layered angle-ply composite structures. Angle-ply design variables and the thickness of each layer considered as design variables.
Optimization method presented here is based on combining optimality criterion (OC) and mathematical programming (MP) algorithms. Finite element analysis (FEA) are used to compute internal forces at the system level. The local stress and local initial failure load in each independent element are defined as component constraints. The use of this MP algorithm is essential to two-level approach and local level, since it can handle the highly nonlinear component problem, such as local buckling or special initial failure constraints in mechanical fastened joints. Optimality criterion method significantly reduces time and cost the optimization process. The two-level optimization approach is applied to minimum-weight design of aircraft structural components such as aircraft nose landing gear, wing skins and parashute composite beam subject to multiple constraints. Finally, several examples are shown to illustrate effectiveness of the two-level optimization model.

**Key words**: Optimization methods, multi-level approach, optimality criterions, displacement constraints, stress constraints, buckling constraints, finite elements

## References

[1] Arora, J. S and Haug, E. J., Efficient Hybrid Methods for Optimal Structural Design, Journal of Engineering Mechanics Division, Proc. ASCE, 103 (EM3), pp 663-680, June 1974.

[2] Maksimović, S., Some Computational and Experimental Aspects of Optimal Design Process of Composite Structures; J. Composite Structures, 16 (1990) 237-258

[3] Maksimovic, S. and Zeljkovic, V., Multilevel Optimization of Large Scale Structures Based on Combining Optimality Criterion and Mathematical Programming Algorithms, Ed. B.H.V. Topping, Advances in Structural Engineering Optimization, Civil-Comp Press, Edinburgh, 1996.

[4] Maksimovic, S., Zeljkovic, V., Multilevel Optimization Approach Applied to Aircraft Structures, *The First International Conference on Computational Mechanics (CM'04)*, November 15-17, 2004, Belgrade, Paper 55, 1-10, (Eds Mijuca D. and Maksimovic, S.)

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-45. ID-18.**

# OPTIMIZATION OF SANDWICH PLATES WITH PRISMATIC CORES

**J. Djoković[1], K.Veljković[2]**

[1] Technical Faculty of Bor, University of Belgrade, Serbia
e-mail: jelenamdjokovic@gmail.com
[2] Polytechnical school Kragujevac, Kragujevac, Serbia
e-mail: kaja76@gmail.com

**ABSTRACT.** In this paper have been analyzed multifunctional sandwich panels with prismatic cores. Their behavior have been compared with panels designed using honeycomb cores. The optimal dimensions and the minimum weight of sendwich panels with prismatic cores have been evaluated. Two loading direction were studied. Failure mechanism have been devised that account for interaction between core and face member during buckling in each loading direction. Non-dimensional expressions are obtained for combination of bending moment and shear force that activates each failure mechanisms. A quadratic optimizer is used to ascertain the dimensions that minimize the panel weight. The prismatic core panels performs best when loaded longitudinally, because the performance is limited by plate buckling, rather than beam buckling. Honeycomb core panels are more weight efficient than prismatic core panels at low load capacity. The benefit diminishes as the load increases. The large the yield strain of the material used to manufacture the panel, the grater the performance, and the larger the benefits of the prismatic core.

**Keywords**: Prismatic sandwich panels, Buckling, Optimal design

## References

[1] Valdevit L., Hutchinson J.W. and Evans A.G., Structurally optimized sandwich panels with prismatic cores, *International Journal of Solid and Structures*, **41** **(**2004), pp. 5105-5124.

[2] Allen, H. G.: Analysis and Design of Structural Sandwich Panels, Pergamon Press, 1969.

[3] Wicks N. and Hutchinson J. W., Performance of sandwich plates with truss cores, Mechanics of Materials, **36** **(**2004), pp. 739-751.

[4] Wicks N. and Hutchinson J. W., Optimal truss plates, International Journal of Solid and Structures, **38** **(**2001), pp. 5165-5183.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-46. ID-76.**

# NONLINEAR FREE VIBRATIONS OF AN ELASTICALLY CONNECTED CIRCULAR DOUBLE-MEMBRANE COMPOUND SYSTEM (PART I)

**Milan Cajić*, Danilo Karličić****

Mathematical Institute SANU, Department of Mechanics
Uneversity of Belgrade, Kneza Mihaila 36, Serbia,
*email: mcajic@mi.sanu.ac.rs
**email: danilok@mi.sanu.ac.rs

**ABSTRACT**. A few studies are available in the literature on vibrations and energy analysis of elastically connected circular double-membrane compound system [4], [6], [7] and [8] and no one on nonlinear vibrations of such a system. In this research, free transversal oscillations of elastically connected circular double-membrane compound system are considered, and analytical approximations of the solutions are given.

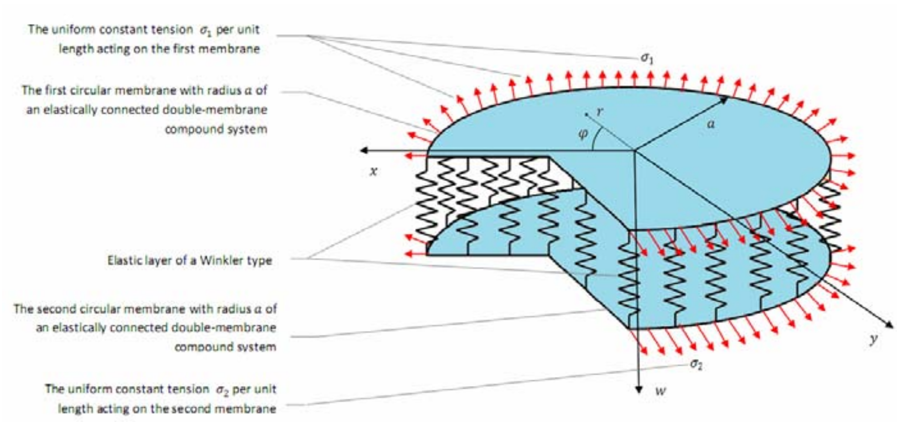



Figure 1. An elastically connected double-membrane system.

In this communication, we analyzed nonlinear vibrations of two deformable membranes with deformable elastic foundation in the middle (Fig.1.). Membranes are loaded along contour by constant tension per unit area $\sigma_i$ , $i=1,2$. The layer is in the form of discrete continuum. This layer consists of the standard light ideally elastic element with material properties described by constitutive force-strain relation in the form: $F_e = -c\,w(x,y,t) - \tilde{c}\,[w(x,y,t)]^3$ where $w(x,y,t)$ is transversal displacement of the membrane point $N(x,y)$. The governing system of the coupled partial differential equations for frees double-membrane vibrations are expressed in the following form, see [1], [2], [4] and [9]:

$$\frac{\partial^2 w_1(x,y,t)}{\partial t} = c_0^2 \Delta w_1(x,y,t) + \frac{c}{\rho}\left[w_2(x,y,t) - w_1(x,y,t)\right] + \frac{\widetilde{c}}{\rho}\frac{c}{\rho}\left[w_2(x,y,t) - w_1(x,y,t)\right]^3 \quad (1)$$

$$\frac{\partial^2 w_2(x,y,t)}{\partial t} = c_0^2 \Delta w_2(x,y,t) - \frac{c}{\rho}\left[w_2(x,y,t) - w_1(x,y,t)\right] - \frac{\widetilde{c}}{\rho}\frac{c}{\rho}\left[w_2(x,y,t) - w_1(x,y,t)\right]^3 \quad (2)$$

Coupled partial differential equations are base for the description of dynamical free processes and the general solution is obtained using Bernoulli's particular integral method. For the systems under consideration, the eigen frequencies and eigen mode shapes of vibrations are determined. It is shown that free vibrations of double-membrane systems perform synchronous and asynchronous two frequency vibrations in each of infinite number eigen amplitude shapes. Also, it is shown that one-mode vibrations correspond to two-frequency regime for free vibrations induced by initial perturbation of equilibrium system configuration. Frequencies in one mode of vibration are obtained using Krilov-Bogolybov-Mitropolyski method for free nonlinear vibrations (see [5]).

It is presupposed that nonlinearity is small and that it is possible to choose a small parameter of nonlinearity. A first asymptotic approximation of solution is obtained in two-frequency regime and corresponding differential equations of the first approximation of the amplitude and phase too. By using these approximations of the ordinary differential equations using numerical experiment we obtain amplitude frequency and phase-frequency graphs for stationary and no stationary one frequency regimes of nonlinear vibrations of our system. In this research are compared: linearity and nonlinearity of the system and analytical and numerical methods (see [3]).

**Acknowledgement**. We sincerely thank to Professor Katica (Stevanović) Hedrih, Leader of the Project OI174001 for all her comments, valuable consultations and motivation that she gave us to submit this paper. Parts of this research were supported by the Ministry of Sciences and Technology of Republic of Serbia through Mathematical Institute SANU Belgrade Grant OI174001 Dynamics of hybrid systems with complex structures. Mechanics of materials.

### References

[1] Hedrih S. K, Transversal vibrations of double-plate systems, *Acta Mechanica Sinica*, **22** (2006), pp. 487-501.

[2] Hedrih S. K, Energy transfer in double plate system dynamics, *Acta Mechanica Sinica*, **24** (2008), pp. 331–344.

[3] Hedrih S. K, Simonović J, Multi-frequency analysis of the double circular plate system non-linear dynamics, N*onlinear Dynamics,* **67** (2012), pp. 2299-2315.

[4] Karličić D, Cajić M, Energy Transfer Analysis of an Elastically Connected Circular Double-Membrane Compound System*, 8th European Solid Mechanics Conference* (July 9-13, 2012. Graz, Austria), Book of Abstracts, pp. 158.

[5] Mitropolyskiy A. Yu, Mosseenkov I. B, Assimptoticheskie recheniya uravnyeniya v chastnih proizvodnih, *Vichaya chkola Kiev* (1976), (book in Russian).

[6] Noga S, Free transverse vibration analysis of an elastically connected annular and circular double-membrane compound system, *Journal of Sound and Vibration,* **329** (2010), pp. 1507–1522.

[7] Oniszzuk Z, Vibrations of elastically connected circular double-membrane compound system*, Scientific Works of Warsaw University of Technology*, Civil Engineering **132 (**1998**)**, pp. 61-81 (in Polish).

[8] Oniszzuk Z, Free vibrations of elastically connected circular double-membrane system*, Proceedings of XVIIth Symposium "Vibrations in Physical Systems"*, Poznan-Blazejevko (1996)*,* pp. 208-209.

[9] Rašković D, Theory of Oscilations, *Naučna knjiga* (1965), Belgrade (book in Serbian).

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-47. ID-77.**

# NONLINEAR FORCED VIBRATIONS OF AN ELASTICALLY CONNECTED CIRCULAR DOUBLE-MEMBRANE COMPOUND SYSTEM

**Danilo Karličić*, Milan Cajić****

Mathematical Institute SANU, Department of Mechanics
Uneversity of Belgrade, Kneza Mihaila 36, Serbia,
*e-mail: danilok@mi.sanu.ac.rs
** e-mail: mcajic@mi.sanu.ac.rs

**ABSTRACT**. A few studies are available in the literature on vibrations and energy analysis of elastically connected circular double-membrane compound system [4], [6], [7] and [8] and no one on nonlinear vibrations of such a system. In this research, forced transversal oscillations of elastically connected circular double-membrane compound system are considered, and analytical approximations of the solutions are given.

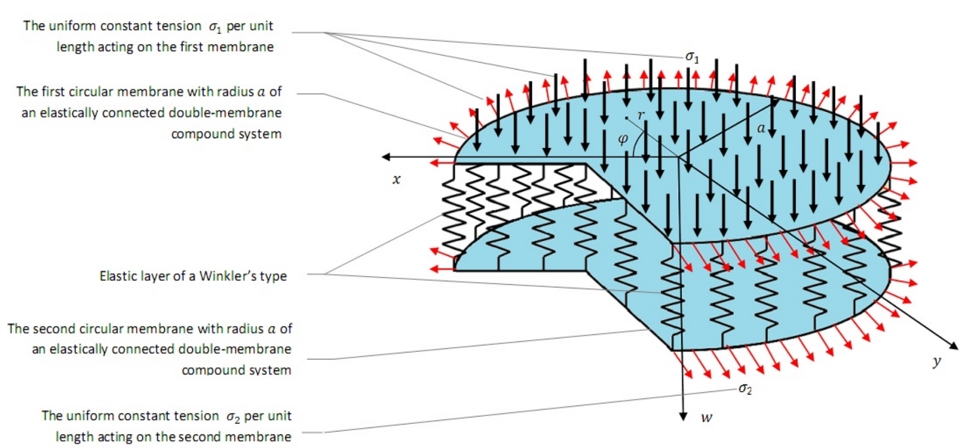



Figure 1. An elastically connected double-membrane system.

In the first part of our communication, we analyzed nonlinear free vibrations of our system. In the second part we analyzed nonlinear forced vibrations of two deformable membranes with deformable elastic foundation in the middle (Fig.1.). Membranes are loaded along contour by constant tension per unit area $\sigma_i$, $i=1,2$. The layer is in the form of discrete continuum. This layer consists of the standard light ideally elastic element with material properties described by constitutive force-strain relation in the form: $F_e=-c\,w(x,y,t)-\tilde{c}\,[w(x,y,t)]^3$ where $w(x,y,t)$ is transversal displacement of the membrane point $N(x,y)$. The governing system of the coupled partial differential equations for forced double-membrane vibrations are expressed in the following form, see [1], [2], [4] and [9]:

$$\frac{\partial^2 w_1(x,y,t)}{\partial t} = c_0^2 \Delta w_1(x,y,t) + \frac{c}{\rho}[w_2(x,y,t) - w_1(x,y,t)] + \frac{\tilde{c}}{\rho}\frac{c}{\rho}[w_2(x,y,t) - w_1(x,y,t)]^3 + \frac{q_1(x,y,t)}{\rho} \tag{1}$$

$$\frac{\partial^2 w_2(x,y,t)}{\partial t} = c_0^2 \Delta w_2(x,y,t) - \frac{c}{\rho}[w_2(x,y,t) - w_1(x,y,t)] - \frac{\tilde{c}}{\rho}\frac{c}{\rho}[w_2(x,y,t) - w_1(x,y,t)]^3 + \frac{q_2(x,y,t)}{\rho} \tag{2}$$

It is assumed that the membranes are thin and homogeneous and perfectly elastic and they have constant thickness .The membranes are subjected to arbitrarily distributed continuous loads. The forced vibrations of two membranes subjected to arbitrarily distributed continuous loads are determined by using the classical method of the expansion in a series of the normal modes of vibrations. For the forced one frequency nonlinear vibrations in resonant frequency regime partial differential eq. (1) and (2) are used and asymptotic method of Krilov-Bogolyubov-Mitropolyski is applied [5]. It is presupposed that nonlinearity is small and that it is possible to choose a small parameter of nonlinearity. A first asymptotic approximation of solution is obtained in two frequency regime and corresponding differential equations of the first approximation of the amplitude and phase too. By using these approximations of the ordinary differential equations using numerical experiment we obtain amplitude frequency and phase-frequency graphs for stationary and no stationary one frequency regimes of nonlinear vibrations of our system. In this research are compared: linearity and nonlinearity of the system; free and forced oscillations, analytical and numerical methods (see [3]). From the amplitude-frequency and phase-frequency graphs we made a qualitative analysis of: stability of the stationary amplitudes and phases depending of external excitation frequency; appearance of the resonant jumps in one frequency regimes; the influence of the velocity of external excitation frequency change to the no stationary amplitude and phase in the first asymptotic approximation of the one frequency forced vibration, when this frequency is close to the eigen frequency of the liberalized system corresponding to nonlinear.

**Acknowledgement**. We sincerely thank to Professor Katica (Stevanović) Hedrih, Leader of the Project OI174001 for all her comments, valuable consultations and motivation that she gave us to submit this paper. Parts of this research were supported by the Ministry of Sciences and Technology of Republic of Serbia through Mathematical Institute SANU Belgrade Grant OI174001 Dynamics of hybrid systems with complex structures. Mechanics of materials.

## References

[1] Hedrih S. K, Transversal vibrations of double-plate systems, *Acta Mechanica Sinica*, **22** (2006), pp. 487-501.
[2] Hedrih S. K, Energy transfer in double plate system dynamics, *Acta Mechanica Sinica*, **24** (2008), pp. 331–344.
[3] Hedrih S. K, Simonović J, Multi-frequency analysis of the double circular plate system non-linear dynamics, N*onlinear Dynamics,* **67** (2012), pp. 2299-2315.
[4] Karličić D, Cajić M, Energy Transfer Analysis of an Elastically Connected Circular Double-Membrane Compound System*, 8th European Solid Mechanics Conference* (July 9-13, 2012. Graz, Austria), Book of Abstracts, pp. 158.
[5] Mitropolyskiy A. Yu, Mosseenkov I. B, Assimptoticheskie recheniya uravnyeniya v chastnih proizvodnih, *Vichaya chkola Kiev* (1976), (book in Russian).
[6] Noga S, Free transverse vibration analysis of an elastically connected annular and circular double-membrane compound system, *Journal of Sound and Vibration,* **329** (2010), pp. 1507–1522.
[7] Oniszzuk Z, Vibrations of elastically connected circular double-membrane compound system*, Scientific Works of Warsaw University of Technology*, Civil Engineering **132 (**1998**)**, pp. 61-81 (in Polish).
[8] Oniszzuk Z, Free vibrations of elastically connected circular double-membrane system*, Proceedings of XVIIth Symposium "Vibrations in Physical Systems"*, Poznan-Blazejevko (1996)*,* pp. 208-209.
[9] Rašković D, Theory of Oscilations, *Naučna knjiga* (1965), Belgrade (book in Serbian).

Naučno društvo Srbije — Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-51. ID-62.**

# COMPATIBILITY OF ENDURANCE LIMIT AND FATIGUE CRACK GROWTH PARAMETERS IN BEHAVIOR OF WELDED JOINT OF LOW ALLOYED STEELS

**Ivica Čamagić[1], Zijah Burzić[2], Nemanja Vasić[1], Srđan Jović[1], Slobodan Makragić[1], Predrag Živković[1]**

[1] Faculty of Technical Sciences, University of Pristina, 7 Kneza Milosa Street, Kosovska Mitrovica, Serbia
e-mail: ivica.camagic@pr.ac.rs
[2] Military Institute of Techniques, 1 Ratka Resanovića Street, Belgrade, Serbia
e-mail: zijah_burzic@vektor.net

**ABSTRACT** Properties that describe crack initiation and propagation, under variable load, are the most important for the exploitation safety of welded structures. Fatigue is described as cumulative damage due to variable load, which is shown with fatigue crack initiation and propagation. Fatigue cracks initiation on smooth and homogenous shapes, due to the local stress concentration on inevitable constructional crossings and cross-section changes, can't be described with the simple dependences of load, stress, material properties and cross-section dimension. Therefore, the empirically derived dependencies are used, based on massive experimental and laboratory testing. The strength of welded joint under variable loads, such is occurring in nonstationary modes of welded structures is an important feature of century. Endurance limit of welded joint is determined by examining specimens under variable load to the occurrence of crack or fracture [1]. It should be noted that the damage in the form of cracks occurs after a large number of load changes at tensions which are lower than the yield strenght (highly cyclical fatigue), or after a relatively small number of load changes,with tensions which are close to the yield strenght (low cycle fatigue). The subject of experimental explorations in this paper was highly cyclical fatigue [2]. Part of the research in this paper focuses on the analysis of the impact of variable loads on the behavior of welded joints in the presence of defect like crack, and determining the fatigue crack growth parameters by testing the standard Charpy specimens using the tube bending three-point method [3].

**Keywords**: welded joint, crack, fracture, endurance limit, fatigue crack growth parameters

## References

[1] ASTM E466-89, Standard Practice for Conducting Constant Amplitude Axial Fatigue Tests of Metallic Materials, Annual Book of ASTM Standards, Vol. 03.01, p. 571, 1989.
[2] I. Čamagić, N. Vasić, B. Ćirković, S. Cvetković, Z. Vasić, Variable and impact load testing of HSLA steel welded joint, Struktural integrity and life, Vol. 10, No. 3, 2010, pp. 231-234.
[3] C. P. PARIS, and B. R. HAYDEN, A New System for Fatigue Crack Growth Measurement and Control, ASTM Symposium on Fatigue Crack Growth, Pittsburg, 1989.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-52. ID-64.**

# CHARACTERIZATION OF ALLOYS AND LIQUIDUS PROJECTIONS OF THE TERNARY Bi–Cu–In SYSTEM

**M. Premović[1*] and D. Minić[1]**

[1] The University of Pristina, Faculty of Technical Sciences, 38220 Kosovska Mitrovica, Serbia
e-mail: milena.premovic@gmail.com

**ABSTRACT.** Over the years numerous investigations of the low temperature ternary systems were carried out around the world in order to substitute low temperature lead based solders. In recent years these investigations were expanded to the high temperature ternary systems with copper. The great interest for these systems is a consequence of the wide application of these alloys in various industries.

Lead-free solders with copper are considered as a possible substitution for standard lead-tin solders. We used to investigated the ternary Bi-Cu-In system.

The ternary Bi–Cu–In system was previously studied by Itabashi et al., who determined the activities of indium in the presented system. Aljilji et al. defined several quasi binary sections for the system, by comparing calculated and experimental values obtained using Differential Thermal Analysis.

For a more complete characterization of the ternary Bi–Cu–In system, alloys from the three vertical sections were investigated using microstructural analysis as well as by electrical conductivity and Brinell hardness measurements. The microstructures of the alloys were investigated at room temperature by application of Scanning Electron Microscopy (SEM) combined with Energy Dispersive Spectrometry (EDS) and optical microscopy. Isolines of electrical conductivity and hardness for the entire Bi–Cu–In system were calculated using regression models. In addition, thermodynamic assessment of the system has been carried out by calculation of the isothermal section at 25$^0$C and description of the liquidus surface using CALPHAD method. Experimental and analytical results were found to be in a good agreement.

**Keywords**: Bi–Cu–In system, microstructure, hardness, electrical conductivity, liquidus

**References**

[5] V. Gandova, K. Lilova, H. Malakova, B. Huber, N. Milcheva, H. Ipser, J. Vrestal, G. Vassilev, *J. Min. Metall. Sect. B-Metall*., 46 (1) B (2010) pp. 11-23.
**[6]** D. Živković, D. Minić, D. Manasijević, A. Kostov, N. Talijan, Lj. Balanović A. Mitovski Ž. Živković, *J. Min. Metall. Sect. B-Metall*., 46 (1) B (2010) pp. 105-111.
[7] O. Teppo, J. Niemela, P. Taskinen, Report TKK-V-B50, Helsinki University of Technology, 1989.
[8] A.T. Dinsdale, A. Kroupa, J. Vízdal, J. Vrestal, A. Watson, A. Zemanova, COST531 Database for Lead-free Solders, Ver. 3.0, 2008, Unpublished Research.
[9] D. Boa, I. Ansara, *Thermochim. Acta*, 314 (1) (1998) pp. 79–86.
[10] [6] X.J. Liu, H.S. Liu, I. Ohnuma, R. Kainuma, K. Ishida, S. Itabashi, K. Kameda, K. Yamaguchi, *J. Electron. Mater*., 30 (9) (2001) pp. 1093–1103.
[11] H.S. Liu, X.J. Liu, Y. Cui, C.P. Wang, I. Ohnuma, R. Kainuma, J.P. Jin, K. Ishida, *J. Phase Equilibr.,* 23 (5) (2002) pp. 409–415.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-53.ID-65.**

# CHARACTERIZATION OF ALLOYS OF THE TERNARY Ag–Bi–Zn SYSTEM

**D. Minić[1*] and M. Premović[1]**

[1] The University of Pristina, Faculty of Technical Sciences, 38220 Kosovska Mitrovica, Serbia
e-mail: dminic65@open.telekom.rs

**ABSTRACT.** Recent investigations of high temperature lead-free solder systems in the frame of COST MP0602 project, indicated different silver-based systems as possible new more environmentally friendly substitue materials to be applied in electronics, one of the promising candidates is the Ag–Bi–Zn system. The results of investigation of the alloys in high-temperature Ag–Bi–Zn ecological solder system are presented in this paper. Structural, mechanical and electrical properties were examined using optical microscopy, hardness and electrical conductivity measurements.. Investigated samples were chosen from three vertical sections Ag–BiZn, Bi–AgZn and Zn–AgBi for the ternary Ag–Bi–Zn system.

Microstructural analysis of investigated samples was performed by optical microscopy, using Reichert MeF2 microscope. Hardness measurements were done using standard procedure according to Brinell, with ball diameter of 20 mm and load of 15.6 kP. Electrical conductivity of investigated materials was measured using SIGMATEST 2.069 (Foerster) eddy current instrument for measurements of electrical conductivity of nonferromagnetic metals based on complex impedance of the measuring probe with 8mm probe diameter. Series of measurements were carried out to determine the electrical conductivity of the investigated alloys. Reasonable agreement between calculated and experimental data was noticed.

**Keywords**: Ag–Bi–Zn alloys, lead-free solders, microstructure, hardness, electrical conductivity

## References

[1] http://www.cost602.ipm.cz/
[2] H. Ipser, J. Min. Metall., Basic research on leaf-free soldering, Sect.B – Metall., 43 B (2) (2007), pp. 109-112.
[3] M. Abtew, G. Selvaduray, Lead-free solders in microelectronics, Mater. Sci. Eng. Reports, 27 (5) (2000), pp. 95-141.
[4] P. Terzieff, Z. Li, S. Knott, A. Mikula, Physicochemical properties of liquid Ag-Bi-Sn, Physica B, 388 (2007), pp. 312-317.
[5] E. Zoro, E. Dichi, C. Servant, B. Legendre, Phase equilibria in the Ag–Au–Bi ternary system, Journal of Alloys and Compounds, 400 (2005), pp. 209–215.
[6] S. Hassam, Z. Bahari, B. Legendre, Phase diagram of the Ag–Bi–Sb ternary system, Journal of Alloys and Compounds, 315 (2001), pp. 211–217
[7] O. Kubaschewski O., Silver-Bismuth-Zinc, Ternary Alloys, VCH, Vol.1, 1988, pp 335-340.

[8] http://www.asminternational.org/AsmEnterprise/APD (ASM Alloy Phase Diagrams Center, P. Villars, Editor-in-chief; H. Okamoto and K. Cenzual, Section editors; ASM International, Materials Park, OH, USA, 2007, DOI: 10.1361/apd-ag-bi-zn-950342)

[9] H. Ohtani, S. Ono, K. Doi, M. Hasebe, Thermodynamic study of phase equilibria in the Sn- Ag-Bi-Zn quaternary system, Materials Transactions, 45 (3) (2004) 614-624.

[10] D. Minić, D. Živković, D. Manasijević et al., Thermodynamic predicting and experimental investigation of the Ag-Bi-Zn system, *in print*

[11] Y. Miyazawa, T. Ariga, Influences of aging treatment on microstructure and hardness of Sn-(Ag, Bi, Zn) eutectic solder alloys, Materials Transactions, 42 (5) (2001) 776-782.

[12] T.C. Hsuan, K.L. Lin, Microstructural evolution of ε-AgZn3 and h-Zn phases in Sn–8.5Zn–0.5Ag–0.01Al–0.1Ga solder during aging treatment, Journal of Alloys and Compounds 469 (2009) 350–356.

[13] G. Vassilev, E. Dobrev, S. Evtimova, J.C. Tedenac, Studies of the phase equilibria in the Ag–Sn–Zn system, Journal of Alloys and Compounds 327 (2001) 285–291.

[14] http://en.wikipedia.org/wiki/Bismuth

[15] M. Kolarevic, Brzi razvoj proizvoda, Zadužbina Andrejevic, Beograd,

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL-54. ID-63.**

# HIGH- TECH ARCHITECTURE AND SYNONYMS FOR A PREFABRICATED MODEL

**Julija Aleksić**

Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština
38 220 Kosovska Mitrovica, Ul. Kralja Petra I br. 149/12, Serbia
e-mail: julialex@ikomline.net

ABSTRACT. The industrialized organization of building, contrary to the traditional system, means a division of work and implementation of the mechanization, and by doing so replacing the work which is done with hands to work with machines. The industrial era started in Great Britain in the second half of the 18. century; In the following century it expanded to other parts of Europe and the USA, in the 20. century also on the area of Asia and the Pacifik, and it lasts until this day in other parts of the world.

According to some authors, the industrial era begins in the 18. century with the first industrial revolution which brought the process of production of iron and the use of energy coming from the water vapor, and continued in the second half of the 19. century with the era of using steel and electiral energy.

Industrialization represents a result of changes which turned the agricultural economy in to an industrial one. Products made in house workshops started to be produced in factories, so the productivity and technical eficiency have grown significantly.

The modernization of technology in the industry of the eighties of the 19. century significantly expanded its influence on architecture, and so starts the the industrialization process of building architectural objects which is manifasted through the use of construction machinery – a machine in the industrial production, transport and installation of different kinds of materials, mass (serial) production and standardization, "modern" relation of what is built towards nature, as well as prefabricated building.

After 80 years of a modernist vision of the famous architect, a modern house is still far away from a final industrial product. Its tradition is making it deeply resistant towards this approach. Maybe not for long, considering the fact that aspects of a modern prefabricated house are even more imposing as responsible for the use of an eficient eco-house of today. The newest examples prove that the old modernists were right.

The prefabricated house was a theme that was constantly discussed in architecture of the XX century, and it was often ridiculed and described as an "oldest newest idea" of Modern architecture[2].

By propagating the industrial technology, the modernists saw the concept of a prefabricated house as a natural and logical consequence of the industrialization itself – the house should have been the next one on the list for a serial production. Since 1919., Le Corbusier wrote about "a massive production of houses", which should follow the logic of the modern production of automobiles, airplanes and ships and take on a new role, the role of "machine for living" (Ward, 1992.;72)

[2] Modern architecture is the architecture of the XX century without ornaments, symbolism, vividness and metaphor, reduced to simple stereometric forms and based on a large use of technical – technological newness. (Milenkovic, 2001;)

After the I and II World war, a mass production of prefabricated houses and residental units, provided the opportunity for a quick solution of the residental problem out of step between the mass market and the great poverty. The houses are still usually being built at the location, with a way that is not far away from Roman period but during the course of time, more of its elements are coming to be serial produced in a factory. Actually, the evolution of a todays house can be looked at through a step by step larger application of prefabrication in the production of its architectural components – wood frame construction, door, window, fabricated wall panels, fabricated facade panels. The conventional construction of houses suffers from chronical problems (postponement due to the weather conditions, risk of having badly trained workers, non-eficient use of materials, creation of waste and polution at the location) the house of the future will, despite of its complexity, be ideal for experimenting in mass production.

Todays modern requirements, which have an exposed echological aspect, are again renewing the theme of the prefabricated house from the middle of the XX century.

The aspects of the modern prefabricated house impose as responsible for the use of an eficient eco-house of today.

In America the new architectural system took participation in science and technology – from solar panels to space stations which are used by NASA. The new technology is used for different climate and location limitations, as well as for other energy activities. A lot of other structures were proposed: objets of glass membrane, in America; polyhedral clusters as modules, hives in Jerusalem, Israel; octagonal plastic capsules in Stutgart, in Germany. A fantastic urban scenario-concept “capsule” apartmant was created, “a walking city”, “instant cities”, “plug-in cities”, in Japan, Europe and the USA.

At the beginning of the 21. century a lot of positive tranformations started, which were inspired by some advanced ideas. New radical forms emerge and new plastic materials and aluminum are being used for the already existing facades which resist the forces of the nature. Great flexibility is something that is their characteristic and different variations in the sence of the organization of function and form, and the glass facades make possible the optimal use of the energy of the Sun.

In an ideal way, the prefabrication combines traditional materials with the modern aesthetics in the creation of inovative solutions. The imperative is that houses are being developed so that they satisfy all needs of the buyer.

In the entire world praxis an idea was developed, for the needs of expanding the space of the already existing object, to use a parasite – a mini structure which is suitable for the planning in densely built city zones. The parasites are light, industrialized, modular or mobile flexible units which just like living organisms “feed” at the account of the infrastructure of the existing object and “live on the body of its host”. Their form is being determined depending on the set and aspirations in order of getting an sculptural expression. Also, transport containers – houses are being developed. They travel from one destination to another and can satisfy a divercity of functions. The containers are water proof, resistant to weather changes, eartquakes, strong winds – hurricanes and tornados.

Prefabrication comes into all spheres of society and finds applications in all different kinds of objects.

The development of society, the need for energy eficiency and a right matching in the environment caused the emergence of new systems of construction. Atractive objects with its design, functionality and low price fully satisfy all needs of demanding users.

**Key words:** industrialization, prefabricating, parasite – structure, mobile houses

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL-55. ID-24.**

# EXPERIMENTAL DEMONSTRATION OF HOMOCLIC CHAOS AND MIXED-MODE OSCILLATION

**Syamal K. Dana, S. Chakraborty**

Central Instrumentation,CSIR-Indian Institute of Chemical Biology
Jadavpur, Kolkata 700032, India
e-mail: syamaldana@gmail.com

**ABSTRACT.** In this talk, we present experimental observations of Shilnikov chaos and mixed-mode oscillations in Chua oscillator from a viewpoint of induced asymmetry. The original Chua circuit model is inversion symmetric. It has two inherent time scales in its double scroll dynamics: one representing both the scrolls around two symmetric equilibrium points and another largely different time scale due to the large cycle covering the two scrolls. Usually, these two time scales cannot help inducing homoclinic bifurcation. We introduce a third time scale in the Chua circuit by inducing asymmetry in the system model. In real systems, several sources of imperfection always exist, which breaks the inversion symmetry of a model system. These imperfections are usually modeled by adding a constant term to the normal form of a model flow. Using this basic concept, we first induce an asymmetry in the Chua circuit by forcing a DC voltage or adding a constant term in the model system and then investigate the role of the asymmetry in the origin of mixed-mode oscillations and Shilnikov chaos. The trajectory of Shilnikov chaos is seen as globally stable but the instability is confined to a local domain of a saddle focus. We characterize the complexity of the Shilnikov chaos using Poincáre section of the 3D homoclinic trajectories. We observed several regimes of homoclinic chaos and mixed-mode oscillations in parameter space as revealed by a bifurcation diagram. We explored several regimes of Shilnikov chaos in electronic experiment. A video demonstration of the experimental Shilnikov chaos is to be presented. We introduce an alternative method how to induce the similar asymmetry effect by coupling a second Chua oscillator in a rest state. By resting state, we mean the second oscillator is in a steady state. The asymmetry is controlled in the first oscillator by varying the coupling resistance, and thereby we are able to observe similar scenarios of Shilnikov chaos as seen in a DC forced Chua oscillator.

Note: *This talk is dedicated to Jules Henry Poincáre (1854-1912) and L.P.Shilnikov (1934-2011).*

## References

[1] S.Chakraborty, S.K.Dana, Shilnikov chaos and mixed-mode oscillation in Chua circuit, CHAOS **20**, 023107 (2010).
[2] S.K.Dana, P.K.Roy, Bursting near homoclinic bifurcation in two coupled Chua oscillators, Int. J. Bifur. Chaos, 17(10), 3437 (2007).
[3] P.K.Roy, S.K.Dana, Gluing bifurcation in Chua oscillator, Int.J.Bifur.Chaos **16** (12), 3497 (2006).
[4] S.K.Dana, S. Chakraborty, G. Ananathakrishna, Homoclinic bifurcation in Chua's circuit, Pramana-J.Physics, **64** (3), 443 (2005).

**Keywords**: Shilnikov chaos, Homoclinic chaos, mixed-mode oscillation, Chua circuit.

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

**CL- 56. ID-83..**

# METHODS OF RELIABILITY ASSESSMENT OF DAMAGED PIPELINE CORROSIO

**Živče Šarkoćević[1], Miodrag Arsić[2], Srdjan Jović[3], Dragan Lazarević[1]**

[1]High Technical School of Professional Studies, Zvečan, Serbia
[2]Institute for Testing of Materials (IMS), Belgrade, Serbia
[3] Faculty of Technical Sciences, Kosovska Mitrovica, University of Priština
38 227 Zvečan, Ul. Nušićeva br. 6, Serbia,
e-mail: zivcesarkocevic@yahoo.com

**ABSTRACT.** Gas and oil pipeline systems that transport large quantities of oil and gas are vital for the economies of countries through which they also have a significant impact on the global economy.
A prerequisite for their safe operation is a guarantee of structural integrity and high reliability. The integrity of the pipeline may be threatened errors that may appear in the stage design and construction of pipelines, pipe production and in the process of work. It is practically impossible to build a pipeline construction without error. However, all errors are not equally influence the integrity of the pipeline, it is important to distinguish permissible from impermissible errors.
Failures of oil and gas pipelines, caused by errors in the preparation of tubes and/or their damage in service, in addition to the impact on the reliability and safety in the work and have ecological impacts on the environment and the quality of land, water and air as well as the safety of the people.
The main objectives of large international companies that manufacture and transport of oil and gas are the identification of current approaches to assessing and inspection pipeline failures and future development trends in control technology. However, some of the existing control methods are used only when other methods can establish the existence of degradation as pipelines are very expensive. Therefore, to assess the reliability of pipelines apply many analytical methods in which to determine the maximum allowable working pressure sufficient to know the values of geometric defects.
This paper presents the criteria for determining the reliability of pipelines damaged by corrosion, based on the ability of pipelines to preserve the integrity of the Structural action of pressure. It would not be the sole criterion in determining the reliability of it relates to exposed pipes are significant secondary load (etc due to bending).
The aim of this study was to identify the different analytical methods used to assess the reliability of pipelines damaged by corrosion.

**Keywords**: pipe, corrosion, maximum allowable working pressure.

## References

[1] Arsić M. Choice of steel for welded tubes production. Welding Weld Struct, 2002; 47:33-37, Serbian.
[2] Šarkoćević Ž, Arsić M, Rakin M, Sedmak A. Fabrication of welded tubes by high strength steel and quality indicators. Struct Integr Life, 2008; 8:81-98.
[3] Sanjeev S, Ramachandra Murthy DS. On the accuracy of ductile fracture assessment of through-wall cracked pipes, Engineering Structures, 2007; 29:789–801.

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012)**, Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL- 57. ID-87.**

# DYNAMIC MODEL OF BUCKET WHEEL EXCAVATOR MAIN SUBSYSTEMS

**Z. Golubović[1], Z. Lekić[1], S. Makragić[1]**

[1] Faculty of Technical Science
The University of Pristina, Knjaza Milosa bb, 38220, Kosovska Mitrovica
e-mail: zoran@vgn.rs, zlatibor.lekic@gmail.com, makragic@nadlanu.com

In this paper, dynamics models of bucket wheel excavator (BWE) superstructure in the vertical plane, the boom lifting drive system and bucket wheel drive system describes in a mathematical model or single program. This model allows evaluating the effects of structural changes in one subsystem to its dynamic behavior as well as a dynamic behavior of the other subsystems on the BWE. Mathematical model solving provides the basic mechanical characteristics that define the dynamic behavior of a BWE in the process of digging. The obtained moments, angular velocity, displacement and force time functions are graphically represented. Results analysis of BWE dynamic calculations indicates that the appropriate reconstruction must be made to improve such a negative dynamic behavior in the process of digging.

**Keywords**: bucket wheel excavator, dynamic model, mathematical model, digging.

**References**

[1] Jevtić V, Golubović Z, Lekić Z, Mathematical modeling of resistance moment as the basic component for the dynamic behavior of the BWE, *XIV international conference on material handling and handling and warehousing*, Belgrade (1996).

[2] Jevtić V, Golubović Z, Lekić Z, Makragić S, The analysis of dynamic processes in the BWE working wheel drive system with a hydrodynamic coupling*, 6th Sever symposium on mechanical gears,* Subotica (1997). pp. 179-185

[3] Golubović Z, Jevtić V, Lekić Z, Dynamic behavior of planetary gear in BWE drive systems, *Yugoslav Congress of Theoretical and Applied Mechanics 97 XXII*, Vrnjačka Banja (1997).

[4] Jevtić V, Theoretical and experimental studies of flow stresses in lifting subsystem structural elements*, master's thesis*, Niš-Bohum (1979).

[5] Volkov D.P , Čerkasov V.A., Dynamics and strength of excavators, translated from the Russian, *Department of Mining Institute for Informatics and Economics* , Beograd (1989),

[6] Hitzschke K, Experimentelle Analyse der Belastung des Schaufelrades durch den Grabvorgang, *Hebeyeuge u. Fördermittel*, H.9. (1984)

[7] Raaz V. Assessment of the Digging Force and Optimum Selection of the Mechanical and Operational Parameters of Bucket Wheel Excavators for Mining of Overburden, *Coal and Partings, Krupp Fordertechnik*, Esen (1999).

[8] Batinić V, Determination of gear mesh stiffness in planetary gearing, *Military Technical Courier*, vol. 56, no. 2, (2008) pp. 227-236.

[9] Ljamić D, Arsić M, Ćirković B, Experimental analysis of BWE SchRs 650/5x24 bucket wheel load, *Mining Machinery - scientific Meeting*, (1995) Beograd

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)
**CL- 58. ID-88.**

# INFLUENCE OF DISC BRAKE INSTALLATION ONTO THE ROPE DRUM ON DINAMIC BEHAVIOR OF BUCKET WHEEL EXCAVATOR

**Z. Golubović[1], Z. Lekić[1], S. Makragić[1]**

[1] Faculty of Technical Science
The University of Pristina, Knjaza Milosa bb, 38220, Kosovska Mitrovica
e-mail: zoran@vgn.rs, zlatibor.lekic@gmail.com, makragic@nadlanu.com

One of the intelligent retrofit solutions for bucket wheel excavator (BWE) is installation of an additional holding brake system independently of the drives. The main function of the brake in boom lifting subsystem of BWE, during the process of horizontal cut digging, is to achieve sufficient breaking torque which will hold boom and working wheel. Within classical solutions of lifting subsystem, it is positioned to the input shaft of the gear box, which in the process of digging gives a certain freedom of gear movement. When the brake is activated, only two or three gear teeth are loaded. This process leads to wear on their hips, so they lose their involutes shape. In addition, as the latest gears pair is outside the gearbox housing, weather conditions affect the removal of grease from their contact surfaces. That enhances their wear. Disc brake installation onto the rope drum will primarily eliminated gears moving and completely will unload the last couple of gears. As installation of additional brake system to boom lifting subsystem affect the dynamic behavior of BWE in the digging process will be presented in this document.

**Keywords**: disc brake, bucket wheel excavator, dynamic behavior, gears.

## References

[1] Jevtić V, Golubović Z, Lekić Z, Mathematical modeling of resistance moment as the basic component for the dynamic behavior of the BWE, *XIV international conference on material handling and handling and warehousing*, Belgrade (1996).

[2] Jevtić V, Golubović Z, Lekić Z, Makragić S, The analysis of dynamic processes in the BWE working wheel drive system with a hydrodynamic coupling*, 6th Sever symposium on mechanical gears,* Subotica (1997). pp. 179-185

[3] Golubović Z, Jevtić V, Lekić Z, Dynamic behavior of planetary gear in BWE drive systems , *Yugoslav Congress of Theoretical and Applied Mechanics 97 XXII*, Vrnjačka Banja (1997).

[4] Golubović Z, Contribution of drive system reconstruction feasibility for middle class BWE, *master's thesis*, Kosovska Mitrovica (2009).

[5] Gnilke M, Intelligent retrofit solutions for bucket wheel excavators, *MAN TAKRAF Fördertechnik GmbH*, (2006)Lauchhammer,

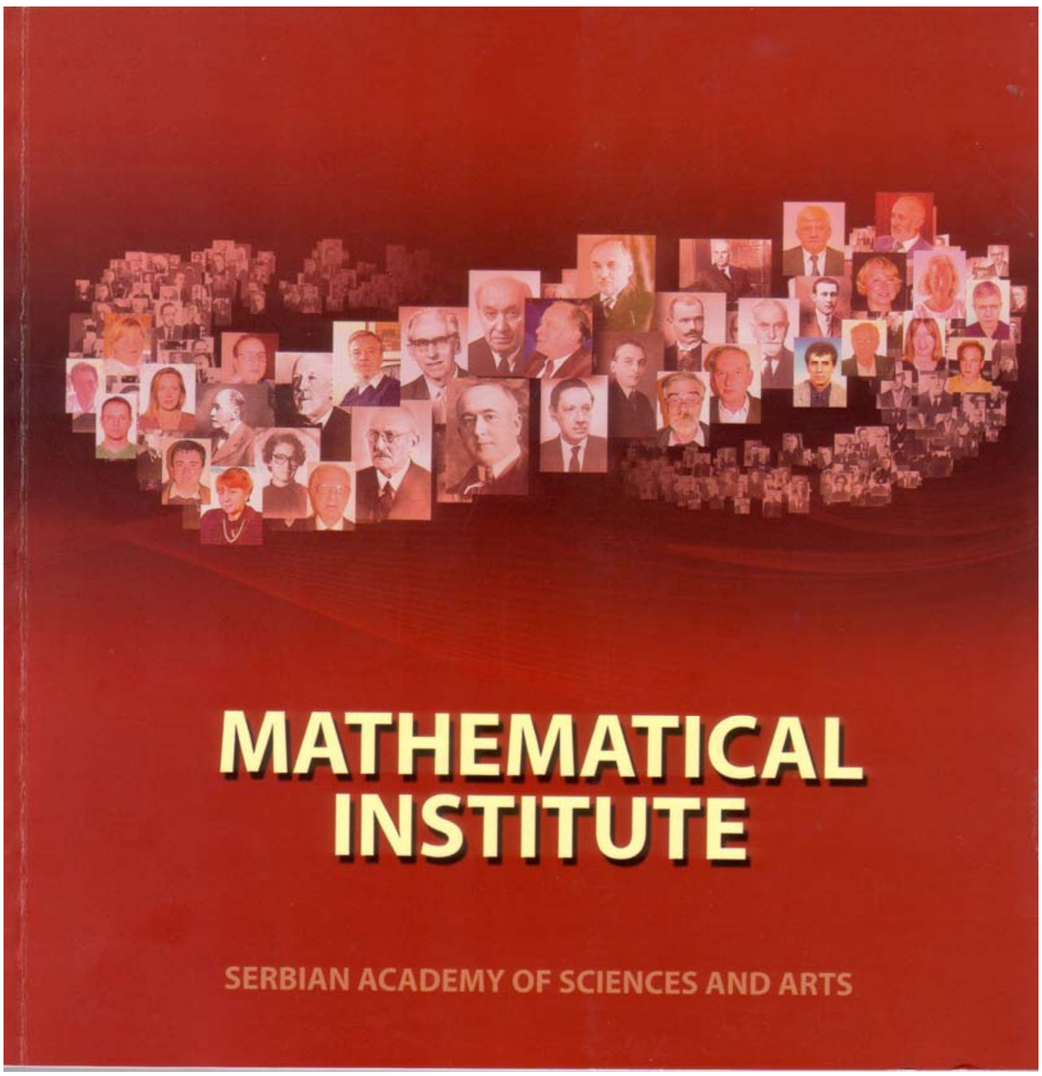
[6] SIBRE Siegerland-Bremsen GmbH, *Product catalogues* D-35708 Haiger, (2011) Germany

[7] Jevtić V, Theoretical and experimental studies of flow stresses in lifting subsystem structural elements*, master's thesis*, Niš-Bohum (1979).

[8] Volkov D.P , Čerkasov V.A., Dynamics and strength of excavators, translated from the Russian, *Department of Mining Institute for Informatics and Economics* , Beograd (1989),

[9] Bošnjak S, Zrnić N, Oguamanam D, On the dynamic modeling of bucket wheel excavators, *FME Transactions 34* (2006), pp. 221-226.

# APPENDIX I




Venue of Symposium – Mathematical Institute SANU
Ul. Knez Mihailova 36/III, Auditoria I

## The Founders of the Institute

Dr. Bilimovič Anton

Dr. Kašanin Radivoj

Dr. Gavrilović Bogdan

Dr. Milanković Milutin

Dr. Mišković Vojislav

Dr. Saltikov Nikola

Dr. Karamata Jovan

Naučno društvo Srbije | Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html
Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

# APPENDIX II

## The 50th Anniversary Conference of the ICNO-ENOC in Rome 2011.

| | |
|---|---|
| **ICNO I** | **September 1961 Kiev (USSR) Chair: Mitropolsky** |
| **ICNO II** | **September 1962 Warsaw (Poland) Chair: Ziemba** |
| **ICNO III** | **May 1964 Berlin (GDR) Chair: Reißig** |
| **ICNO IV** | **September 1967 Prague (Czechoslovakia) Chair: Djadkov** |
| **ICNO V** | **August 1969 Kiev (USSR) Chair: Mitropolsky** |
| **ICNO VI** | **September 1972 Poznan (Poland) Chair: Ziemba** |
| **ICNO VII** | **September 1975 Berlin (GDR) Chair: Schmidt** |
| **ICNO VIII** | **September 1978 Prague (Czechoslovakia) Chair: Pust** |
| **ICNO IX** | **September 1981 Kiev (USSR) Chair: Mitropolsky** |
| **ICNO X** | **September 1984 Varna (Bulgaria) Chair: Brankov** |
| **ICNO XI** | **August 1987 Budapest (Hungary) Chair: Farkas** |
| **ICNO XII** | **September 1990 Cracow (Poland) Chair: Gutowski** |
| **ENOC I** | **Hamburg, August 16 - 20, 1993, Chairman Professor Edwin Kreuzer** |
| **ENOC II** | **Prague, September 9 - 13, 1996, Chairman Professor Ladislav Pust, Secretary Professor Frantisek Peterka** |
| **ENOC III** | **Copenhagen, August 8 - 12, 1999, Chairman Professor Hans True** |
| **ENOC IV** | **Moscow, August 19 - 23, 2002, Chairman Professor Klimov** |
| **ENOC V** | **Eindhoven, August 7 - 12, 2005, Chairman Professor Dick van Campen** |
| **ENOC VI** | **St. Petersburg, June 30 - July 4, 2008, Chairman Professor Alexander Fradkov** |
| **ENOC VII** | **Rome, July 24 - 29, 2011, Chairman Professor Giuseppe Rega** |
| **ENOC VIII** | **Wien 2014.** |

Yu. A. Mitropolski

ICNO 1:

September 12-18, 1961, Kiev, USSR
IUTAM Symposium on the Theory of Non-Linear Vibrations
Academy of Sciences of the Ukrainian SSR
Institute of Mathematics
Chairman: Prof. Dr. Yu. Mitropolsky

Committee: N.N. Bogolyubov (USSR, Chair), M.L. Cartwright (UK), C. Hayashi (Japan), K. Klotter (USA), S. Lefschets (USA), Yu.A. Mitropolski (USSR), T. Vogel

Participants: 92 scientists from 13 countries, amongst them A.I. Lurie (USSR), A.N. Tikhonov (USSR), Yu.A. Mitropolski (USSR), M.L. Cartwright (UK), S. Lefschetz (USA), S. Smale (USA)

Naučno društvo Srbije | Serbian Scientific Society

http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

# APPENDIX III

# SYMPOSIUMS ON NONLINEAR MECHANICS IN SERBIA

Dear participants of the Sixth Symposium on Nonlinear Dynamics,

Symposium NONLINEAR DYNAMICS – Milutin Milanković, Belgrade October 2012 - НЕЛИНЕАРНА ДИНАМИКА 2012 organized by the Department of natural-mathematical sciences of Serbian Scientific Society is the eighth in Serbia in area of donlinear dynamics.

First symposium in area of Nonlinear mechanics was held in Aranđelovac in 1984 and was organized by Serbian society of mechanics. A leading scientist in the area of Nonlinear mechanics, academician RAS and NANU Yu.A. Mitropolskiy was an invited participant with a Plenary Lecture.

Symposium, entitled ***Sixth International Symposium on Nonlinear Mechanics Nonlinear Sciences and Applications Niš 2003*** (The 6$^{th}$ ISNM NSA NIŠ'2003), as was the case with the previous one, entitled '***Nonlinear Sciences at the Threshold of the Third Millenium***', is organized with the wish to unite in a single symposium, on the basis of the M***athematical Phenomenology*** of ***Mihajlo Petrovic Alas***, quite disparate sciences with the aim of integrating the knowledge of the participants of our symposium. This is a serious and long-term task of science.

***How did it all start?*** It started here, at the Faculty of Mechanical Engineering, University of Niš.

Prof. dr ***Danilo P. Rasković*** (Ph.D. in mechanical engineering and BS in mathematics), the first head of the Chair for Mechanics and Automatics, while teaching mechanics at the Department for Mechanical Engineering at the newly founded Faculty of Technical sciences, directed his youngest and most talented students towards studying nonlinear oscillations and nonlinear mechanics. Thereafter, he initiated a cooperation with the academician ***Jury Aleksejevitch Mitropolsky*** and a leading school of nonlinear mechanics, asymptotic methods and nonlinear oscillations at the Institute for Mathematics in Kiev, Ukraine. That is how a centre for nonlinear mechanics was established at the Chair for Mechanics, Faculty of Mechanical Engineering in Niš. They were supported by the Institute for Mathematics – SANU from Belgrade and by the scientist from all the Universities in Yugoslavia, especially Universities in Belgrade and Novi Sad. The organization of symposiums on nonlinear sciences followed.

The *first symposium on nonlinear mechanics* - ***Nonlinear Dynamics***, organized by YUSM and Serbian Society for Mechanics, was held in Arandjelovac. Prof. Jury Aleksejevitch Mitropolskiy gave the invited plenary lecture. All the members of the Chair for Mechanics and the Chair for Hydraulic Engineering of the Faculty of Mechanical Engineering University of Niš took part in this and other symposiums; they were co-organizers as well.

The *Second Yugoslav Symposium on Nonlinear Mechanics* entitled: **"*The First Yugoslav Conference on Nonlinear Deterministic and Stochastic Processes in Dynamical Systems with Applications YCNP Niš'91*",** organized by the Faculty of Mechanical Engineering was held in Niš. The Chairman of the Scientific

Committee was Prof. dr V. Vujičić, while the Chairman of the Organizing Committee was Prof. Katica (Stevanović) Hedrih. Proceedings of Abstracts was printed; the papers and invited lectures which were approved were printed in the first and the following issues of the **University Journal – *Facta Universitatis, new Series – Mechanics, Automatic Control and Robotics with Editor-in-Cief*** Katica (Stevanović) Hedrih**.** This Series was the third Series, which was being published, besides the *Series Mathematics, Informatics and Electronics, and Energetics*. William Nash, Kazuyuki Yagasaki ..., gave the invited lectures, at this international symposium.

The invited plenary lectures were given by the following Yugoslav scientists: *Veljko A. Vujičić, Vladan Djordjević, Ljubomir Grujić, Božidar Vujanović, and other..*

Prof**.** *dr Slobododan Laković*, the Dean of the Faculty of Mechanical Engineering University of Niš and a member of the Organizing Committee, has significantly contributed to the success of this symposium.

The ***Third Yugoslav Symposium on Nonlinear Mechanics*** was held in the form of a *Minisymposium*, as a part of the *XXII Yugoslav Conference on Mechanics in Ni*š in 1995. The Faculty of Mechanical Engineering in Niš, with co-organization by the Faculty of Civil Engineering organized this Congress as well.

The Chairman of the Organizing Committee was Prof. *Katica (Stevanović) Hedrih*. The Dean of the Faculty of Mechanical Engineering, was very hospitable. Academician ***Vladan Djordjević****,* the President of YUSM has contributed to the successful organization of this Congress and the Minisymposium. Professors *Yu. A. Mitropolskiy, V.V. Rumyantsev, Felix Chernousko, Anatoliy Martinyuk, Valentina Filchakova, Dan Stamatiu*, ... were guest at this symposium.

The ***Fourth Symposium on Nonlinear Mechanics*** was held in 1997, again in the form of a *Minisymposium*, as a part of the *XIII Yugoslav Congress on Theoretical and Applied Mechanics.* This Congress, held in the Congress Center – Zvezda in Vrnjacka Banja, was organized by the Yugoslav Society for Mechanics. The organization of this Symposium was helped by the Institute for Mathematics – SANU and the Faculty of Mechanical Engineering in Niš and in Belgrade.

The Chairman of the Scientific Committee was the academician ***Nikola Hajdin***, and the Chairman of the Organizing Committee was Prof. *Katica Hedrih*. Professors Anthony Kounadis, Guiseppe Rega, Anton Baltov, Ilya Blekhman .. ... were guest at this symposium.

The ***Fifth Symposium on Nonlinear Mechanics- Nonlinear Sciences at the Threshold of the Third Millenium*** was organized with the wish for it to become a tradition and to gather the connoisseurs of nonlinear phenomenology from disparate sciences and dynamic systems and for it to become renown all over the world.

The Chairmen of the Scientific Committee were academicians ***Jury A. Mitropolsky, V. M. Matrosov and V. Vujičić***, and the Chairmen of the Organizing Committee was Prof. *Katica Hedrih*. Academicians N. Hajdin, V.V. Rumyantsev and M. Prvanović and Professors D.S. Sophianopoulos, G.T.Michaltos, Ji Huan He, I. Finogenko, P.S. Krasil'nikov ..... were guests at this symposium.

The year of the Fifth Symposium was the year of the 10$^{th}$ **Jubilante issue** of the University Journal – ***Facta Universitatis, new Series – Mechanics, Automatic Control and Robotics***. These symposiums and the Journal Facta Universitatis are a permanent characteristic of the University of Niš, Faculty of Mechanical Engineering in Niš and scientific achievements of Yugoslav and Serbian scientists in international relations.

Last Mini-symposia Non-linear Dynamics at Third Serbian (28th Yu) Congress on Theoretical and Applied Mechanics, Vlasina lake, Serbia, 5-8 July 2011. Between invited lecturers were Professor Subhash C. Sinha, Director, Nonlinear Systems Research Laboratory at Auburn University and Founding Editor, ASME Journal of Computational and Nonlinear Dynamics and Professor John T. Katsikadelis, President of Hellenic Society of Mechanics.

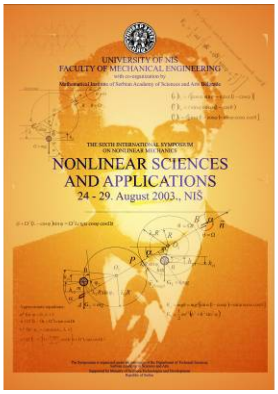
We would like to wish all participants of this Symposium a warm welcome to our country and our Serbian Scientific Society and Venue Symposia place at Mathematical Institute SANU.

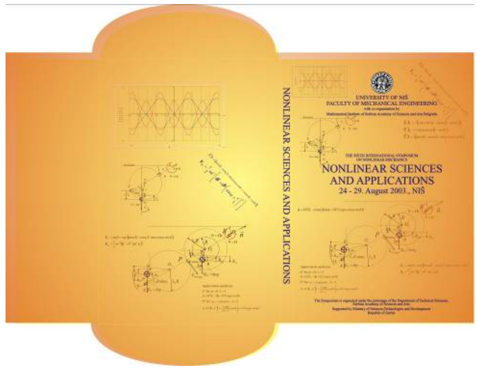
We would like to welcome you, hoping that creative efforts and achievements will prevail over preemption.

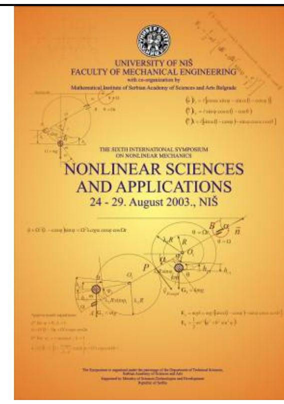
Katica (Stevanović) Hedrih

*Serbian Symposium on Nonlinear Mechanics, Aradjelovac, 1984;*
Invited Lecturer *Academician RAN and NANU Yu. A. Mitropolskiy* (KIEV) *in Niš with Serbian Scientists*

*Participants of*
**IUTAM SYMPOSIUM Non-linearity and Chaos in Engineering Dynamics UCL 1993 London UK**




*Invited Lecturer: Professor Willam Nash from MTI Masachusets at Yugoslav Conference on Deterministic and Stochstic Processes in Dynamical Systems with Applications Nis 1991*

**THE FIFTH YUGOSLAV SYMPOSIUM ON NONLINEAR MECHANICS -NONLINEAR SCIENCES AT THE THRESHOLD OF THE THIRD MILLENNIUM (YUSNM NIŠ '2000)**

*held in Niš, Yugoslavia at October 2-5, 2000. at Faculty of Mechanical Engineering, and it dedicated to the 40th Anniversary of the Faculty of Mechanical Engineering and Faculty of Civil Engineering and Architecture, as well as to the 35th Anniversary of the University of Niš. The Symposium is organized under the patronage of the Departmentof Technical Sciences, Serbian Academy of Sciences and Arts.*

*Academician RAS* ***V. Rumyantsev*** *(Moscow, Russia) in Niš (YUSNM NIŠ '2000-October 2-5, 2000.) at Faculty of Mechanical Engineering.*

*Participants of*

***The Fifth Yugoslav Symposium on Nonlinear Mechanics -NONLINEAR SCIENCES AT THE THRESHOLD OF THE THIRD MILLENNIUM (YUSNM NIŠ '2000)***

*from Russia, Greece, China, Bulgaria, Rumania, Ukraine and Yugoslavia. mr D. Jovanović (Niš,YU), Professor P. Krasil'nikov (Moscow, Russia), Professor Ji Huan He mr D. Jovanović (Niš,YU), , Academician UHEAS K. Hedrih (Niš, YU), Academician SASA M. Prvanović (Belgrade, YU), Professor G. Michaltsos (Athens, Greece), Z. Vosika (Belgrade YU), Professor D. Sophianopoulos (Athens, Greece), Academician SASA N. Hajdin (Belgrade, YU), Ass. Professor P. Rajkovi ć (Niš, YU), Professor G. T. Konstantakopoulos (Athens, Greece), Academician RAS V. Rumyantsev (Moscow, Russia), Professor D. Miki či ć (Belgrade, YU), Academician ANS V.A. Vujičić,....*

*Participants of the* 6th International Symposium on Nonlinear Mechanics
Nonlinear Sciences and Applications 6th INM NSA NIŠ 2003.
(in middle professor A. Vatsala –USA, V. Lakshmikanthan-prsident of IFNA, Professor Leela-USA, Professor T. Kawaguchi-presinet of Tensor Society-Japan, Professor L. Bareteu- Romania, Professor F. Peterka-Prague, J. Warminski –Lublin, Professor U. Gabbert-Magdeburg and T. Nestorovic-now Professor in Bochum,... and Serbian participants )


***Participants of the*** **6$^{th}$ International Symposium on Nonlinear Mechanics Nonlinear Sciences and Applications 6$^{th}$ INM NSA NIŠ 2003.**

**Participants of 6$^{th}$ INM NSA NIŠ 2003.**
D. Jovanović. K. (Stevanović) Hedrih (Niš), J. Warminski (Lublin),
F. Peterka (Prague) and G. Rega (Roma)

**6$^{th}$ International Symposium on Nonlinear Mechanics**
**Nonlinear Sciences and Applications 6$^{th}$ INM NSA NIŠ 2003.**


Last Mini-symposia Non-linear Dynamics at Third Serbian (28th Yu) Congress on Theoretical and Applied Mechanics, Vlasina lake, Serbia, 5-8 July 2011. Between invited lecturers were Professor Subhash C. Sinha, Director, Nonlinear Systems Research Laboratory at Auburn University and Founding Editor, ASME Journal of Computational and Nonlinear Dynamics and Professor John T. Katsikadelis, President of Hellenic Society of Mechanics.

# APPENDIX IV

# Chair of Mechanics
# Faculty of Mechanical Engineering Niš
# (1963-2005)

**dr Ing. Dipl. Math. Danilo P. Rašković**
**(1910-1985)**
***The First Head of Chair of Mechanics and Automatic at Faculty of Mechanical Engineering in Niš (1963-1974)***

Naučno društvo Srbije Serbian Scientific Society
http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

## *Prof., Dr., Eng., B.Sc. Mathematician,*

# *DANILO P. RAŠKOVIĆ,*

## *full-professor at the Faculties of Mechanical Engineering in Belgrade, Niš, Kragujevac and Mostar, and the Faculties of Science in Belgrade and Novi Sad*

Danilo Rašković, a doctor of technical sciences and mathematician with a university degree, was the founder of the first scientifically based courses of mechanics at the Faculty of Mechanical Engineering in Belgrade. He also introduced courses on the subject of resistance of material, elasticity theory, and oscillation theory all of which he taught, too. He was the author of many high-circulation textbooks of high scientific level and good mathematical foundation. He introduced vector, matrix and tensor calculus in the studies of mechanics at the Faculty of Mechanical Engineering in Belgrade and, later on, did the same at the mechanical engineering faculties in Niš, Kragujevac and Mostar. He enabled the Faculty in Belgrade, and similar schools elsewhere, to produce highly qualified and educated engineers which was one his greatest contributions. He wrote the first university textbook in Serbia on oscillation theory containing his original accomplishments in the field. He achieved considerable scientific results in the fields of elasticity theory and oscillation theory. With a good human resource base at Niš Faculty, which he had set up, he started research work into the field of nonlinear mechanics. His scientific work is important because in all of his projects he succeeded in connecting theories of elasticity and oscillation, and engineering practice. He wrote 25 university textbooks which covered the entire field of mechanics and related areas. Almost all of them had been reprinted several times, with some of them having 20 reprints. His excellent textbooks were in use on the territory of the entire former Yugoslavia, which was in tatters under the powerful influence of fascism during the Second World War.

Thanks to Professor Danilo Rašković, the faculties of mechanical engineering of Serbia, Bosnia and Herzegovina, and all the other republics of the once unified Yugoslavia, which are now separate states, produced excellent mechanical engineers. Rašković was a patriot and an honourable man. He was the recipient of the October award of the city of Niš for his contributions to the development of science at the city's university.

This distinguished scientific figure of exquisite creative energy and inspired enthusiasm, a scholar deeply attached to the Yugoslav and Serbian scientific and cultural heritage, and an exquisite pedagogue of high moral principles is in the living memory of many generations of students whom he taught how to learn and love mechanics, as a basic scientific branch of mechanical engineering either directly, through his lectures, or through his various and numerous textbooks and compilation of problems. His disciples and colleagues are glad that he had the ability to pass onto them his great enthusiasm permeated with his sincere devotion for mechanics and his exquisite scientific eagerness.

Professor Danilo P. Rašković was born in 1910, in Užice. Upon completing elementary school and six grades of high school, he graduated from the Military Academy in 1930. As an engineering military officer he enrolled in the department of mechanical and electrical engineering at the Faculty of Engineering in Belgrade, in 1933. Having graduated in 1938, he enrolled in the department of theoretical mathematics at the Faculty of Philosophy and graduated from it in 1941. As a graduate mechanical engineer he was appointed assistant section head of the Military Technical Institute in Čačak. He remained in that position during 1941. In 1942 he was appointed assistant at the Faculty of Engineering in Belgrade where he earned hid doctorate's degree in the same year, upon presenting his thesis entitled *Tangential Strains of Normally Profiled Beams*.

Professor Rašković lectured mechanics, strains of materials and oscillation theory at the faculties of mechanical engineering in Belgrade, Niš, Kragujevac, Novi Sad and Mostar, as well as at the Faculty of Science in Belgrade, Faculty of Philosophy in Novi Sad, Faculty of Electronics in Niš, and at the Military Technical College in Belgrade. More details on the research work of Professor Rašković can be found in the Belgrade University Bulletin no.75 of 1957, issued on the occasion of his appointment as a full professor at the Faculty of Mechanical Engineering in Belgrade. During his university career, he was twice elected Vice-Dean of the Faculty of Mechanical Engineering of Belgrade University. In the mechanical engineering department at the Faculty of Engineering in Niš, he lectured statistics, kinetics, kinematics, dynamics, oscillation theory, resistance of material, theory of elasticity, as well as analytical mechanics, theory of nonlinear oscillations and continuum mechanics at the postgraduate level. He was the first head of the department of mechanics and automatics at the Faculty of Mechanical Engineering in Niš. He was an extremely inspired professor, scientist and practitioner much favoured among his students and respected by his colleagues both as a professor and an engineer, because he knew how to relate engineering theory to practice.

Professor Rašković was a very fertile writer. While still in the military service he wrote five professional papers. In the period before 1957, when he was appointed full professor, he published 26 scholarly papers. As a full professor he wrote 37 pieces of scientific work that were published in scientific journals of the Serbian Academy of Sciences and Arts, Polish Academy of Science, German Society of Mechanics ZAMM and some other foreign journals. He took part in a number of scientific meetings in the country and abroad. He reviewed papers for four leading referral journals in the world: *Applied Mechanic*s *Review* (USA), *Mathematical Review* (USA), *Zentralblatt für Mathematik* (Germany) and *Referativnii žurnal* (Moscow). Professor Rašković was a member of several professional and scientific societies/association in the country and abroad, the GAMM being one of them. He initiated the foundation of the Yugoslav Society of Mechanics during 1952.

He wrote a considerable number of university textbooks which ran through numerous editions. Some of them still hold records as for the number of editions and copies printed within the group they belong to. In addition, he wrote a series of textbooks on the subject of mechanics for secondary technical schools, as well as a number of chapters in professional technical handbooks, mimeographed course materials and textbooks for post-secondary schools of mechanical engineering. He also wrote several textbooks for postgraduate studies.

Among the publications for postgraduate studies the following should be mentioned: *Analytical Mechanics, Theory of Elasticity and Tensor Calculus.*

Most of his university textbooks and publications were at the time of their first edition the only professional literature on the subject, in the Serbian language. So, his publications played an important part in spreading of the knowledge in the field of technical mechanics among students, and mechanical and other kinds of engineers in Serbia and Yugoslavia. It is particularly worth mentioning that he has interpreted all the material by the most modern mathematical apparatus and has illustrated it by numerous examples from the engineering practice. Many of the cited university publications are being reprinted even nowadays and are still used by both students of engineering and engineers themselves.

Although it has been ten years since he left us, Professor Rašković is still present among new generations of students, and engineers, through his renowned textbooks that bear the memory of his merits and which have also left an indelible imprint on the development of mechanical engineering science and practice, and on the formation of many a generation of university professors. His life and work have set an example to future generations of students educated at the University of Niš and provided them with a creative impulse. He is an everlasting paradigm and a proof of how one's deeds can outlive one's physical existence by far.

In 1962 Professor Rašković, **as the head of mechanics department at the Institute of Mathematics of the Serbian Academy of Sciences and Arts**, organized research work in four different study groups, each one dealing with a particular subject, which were: ***Stability of motion*** - supervised by Dr Veljko Vujičić, ***Boundary layer theory -*** supervised by Dr Victor Saljnikov, ***Problems of anisotropic incompatible materials with finite strain*** - supervised by Dr Rastko Stojanović and ***Optimal problems of mechanics*** -supervised by Prof. Dr. Danilo Rašković.

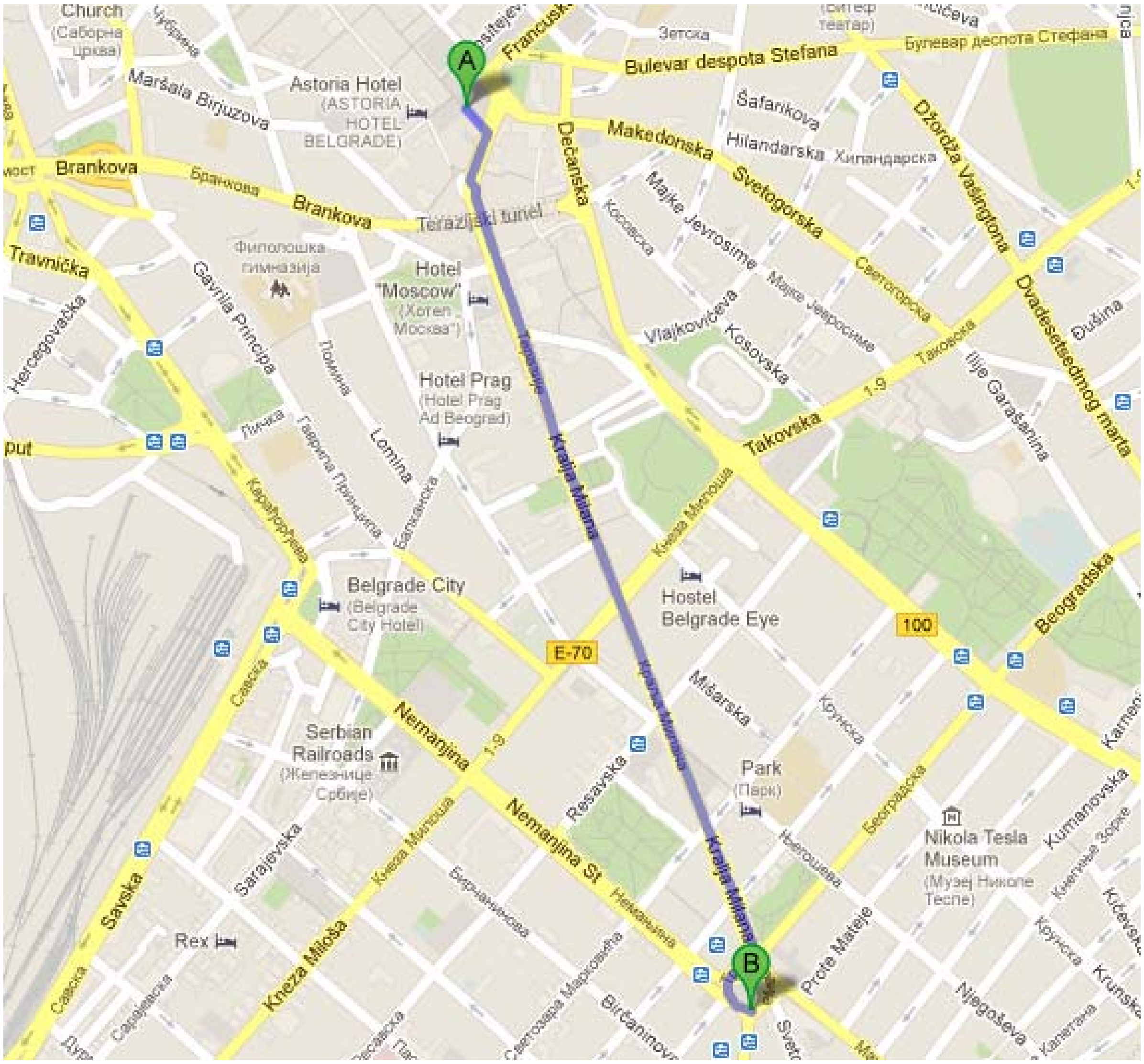
According to records from the mechanical engineering faculties in Belgrade and Niš, as well as those from the Zentralblatt's data base, he traveled abroad on several occasions in order to participate in international scientific gatherings or to expend his knowledge. In 1957 he went to Berlin to do his specialization studies with a piece of work which was published in the *Proceedings of the 20th International Congress of Applied Mechanics*. In September 1956, in Brussels, he participated in the working of the said congress. He took part in international congresses of applied mathematics and mechanics of the German society GAMM a few times: 1957 - in Hamburg and 1958 - in Saarbrücken. Also, in 1959, 1961 and 1962 he was delegate of the Yugoslav Society of Mechanics. In 1963, in Karlsruhe, he represented Mathematical Institute of the Serbian Academy of Sciences. In 1966, in Darmstadt, he "*produced a scientific statement in the field of oscillation theory"* and in 1968 in Prague, Czechoslovakia, he had a paper entitled *Second order acceleration (jerk) for the relative motion of a body expressed by a matrix method*.

He also participated, several times, in the working of the International Conference of Nonlinear Oscillation (ICNO): 1962 in Warsaw, as a delegate of the Council of Science of the People's Republic of Serbia; 1969 in Kiev; 1972 in Krakow, at the '72 ICNO.

Between the 1963/64 and 1973/74 academic years he was Head of the mechanics section of the mechanical engineering department at the Technical Faculty in Niš, while giving lectures on all subjects from the mechanics group. Simultaneously, he taught mechanics at technical faculties in Kragujevac and Mostar and, for a while, also the subject of applied mathematics at Novi Sad Faculty of Mathematics. He accepted the position in Niš after being acquitted of the duty as a lecturer at the Faculty of Mechanical Engineering in Belgrade. The said acquittal was brought in by the Faculty in Belgrade, and was registered under the no. 67/8, in January 1964. Comments on the controversial decision are left to the others. For further reference readers should look into the book (*).

In 1974/75 he was arrested in Mostar, Bosnia-Herzegovina, and unjustly sentenced. Following the experience, he worked on new editions of his high-circulation textbooks, out of which the 10th edition of *Mechanics I* for university studies deserves a special mention as does the 15th edition of his handbook containing tables from the strength of materials. Last months of his life he spent preparing his textbook *Elasticity Theory* for publishing. It came out in 1985 but he did not live to see it.

He died, unexpectedly, on January 29, 1985 in Belgrade.




**A – Knez Mihajlova 36, Mathematical Institute SANU**
**B – Hotel Slavija**

Naučno društvo Srbije — Serbian Scientific Society

http://afrodita.rcub.bg.ac.rs/~nds/ http://afrodita.rcub.bg.ac.rs/~nds/indexe.html

Symposium **Non-linear Dynamics – Milutin Milanković**
with Multi and Interdisciplinary Applications, **(SNDMIA 2012),** Belgrade, October1-5, 2012
(Eigth Serbian Symposium in area of Non-linear Sciences)

*Title:* **Symposium Nonlinear Dynamics – Milutin Milanković (Multidisciplinary and Interdisciplinary Applications), (SNDMIA 2012), Belgrade, October 1-5, 2012. (Eigth Serbian Symposium in area of Non-linear Sciences)**

**EDITOR-IN-CHIEFS OF PUBLICATION:** ***Katica R. (Stevanović) HEDRIH, ON174001 Project Leader and Žarko MIJAJLOVIĆ, Members of Serbian Scientific Society.***

***Publishers:*** Serbian Scientific Society, Street Šafarikova 7, 11000-Beograd.

**CORRESPONDING PERSON:** ***Prof. dr Slobodan Perović, President of Serbian Scientific Society.***

**Support:** Ministry of Education, Sciences and Technology Republic of Serbia and Project: ON174001 ***"Dynamics of hybrid systems with complex structures. Mechanics of Materials"*** trough Mathematical Institute SANU Belgrade and Faculty of Mechanical Engineering University of Niš.

**ANNOUNCEMENT:** P*arts of this published research results in this Booklet of Abstracts were supported by the Ministry of Education, Sciences and Technology of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 "Dynamics of hybrid systems with complex structures. Mechanics of Materials" and Faculty of Mechanical Engineering University of Niš.*

***Persons for Communications**: Katica R. (Stevanović) Hedrih.*

***Cover design and computer support**: **Katica (Stevanović) HEDRIH***

**ISBN 978-86-7746-344-1** **COBISS.SR – ID 193221132**
Number of Copies: 100 copies
***Printed by: SVEN Štamparija Niš,** Stojana Novakovića 10, 18 000 Niš,*
*Phone: +381 18 248142 Mob 064 11 56 986*

**CIP** - Каталогизација у публикацији
***Народна библиотека Србије, Београд***

531./534 (048) 530.182(048)

**Symposium Nonlinear Dynamics – Milutin Milanković (Multidisciplinary and Interdisciplinary Applications), (SNDMIA 2012), Belgrade, October 1-5, 2012.(Eigth Serbian Symposium in area of Non-linear Sciences).**
(2012: Beograd)
**Booklet of Abstracts / [Symposium] Nonlinear Dynamics –Milutin Milanković(Multidisciplinary and Interdisciplinary Applications), (SNDMIA 2012),** Beograd, October 1-5, 2012, / Organized by **Serbian Scientific Society**.
**Project ON174001 coordinated trough MATHEMATICAL INSTITUTE SANU,, 2012. Editors: Katica (Stevanović) HedriH and Žarko Mijajlović; (Niš: SVEN) – 186 str. Ilustr.27 cm.**
**Apstrackti na engleskom, srpskom, i ruskom jeziku.**
***Tiraž 100. – Bibliografija uz svaki rad***

**ISBN 978-86-7746-344-1** **COBISS.SR – ID 193221132**

1. MATHEMATICAL INSTITUTE SANU, Project ON174001 (Beograd)
2. Serbian Scientific Society, Seminar Nonlinear Dynamics - Milutin Milanković
3. **a)** Nelinearna dinamika – apstraktil; b)Nelinearna dinamika interdisciplinarne primene - Apstrakti **;**

**COBISS.SR – ID 193221132**

9 788677 463441

# Serbian Scientific Society

## Symposium Nonlinear Dynamics – Milutin Milanković

### Multidisciplinary and Interdisciplinary Applications

### (SNDMIA 2012), Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Symposium

# Nonlinear Dynamics – Milutin Milanković

## Multidisciplinary and Interdisciplinary Applications

## (SNDMIA 2012),

Belgrade, October 1-5, 2012.

(Eigth Serbian Symposium in area of Non-linear Sciences)

Editors: Katica R. (Stevanović) HEDRIH
Žarko Mijajlović

# Booklet of Abstracts

Beograd, October 1-5, 2012.
Venue: Mathematical Institute SANU