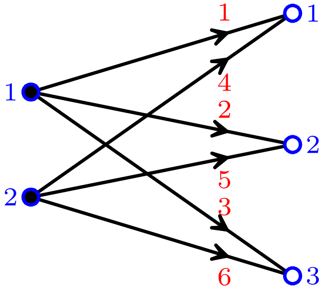
1. Product of Matrices

Example 2.2.6 (p. 40)

The *König digraph* of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

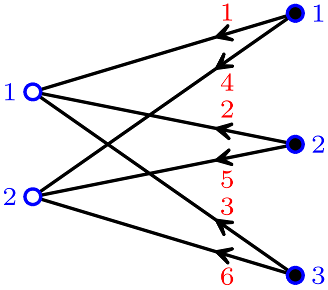
$G(A)$



The *König digraph* of the transpose matrix

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

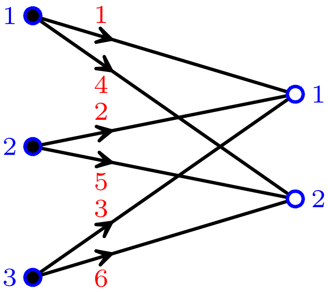
$G(A^T)$



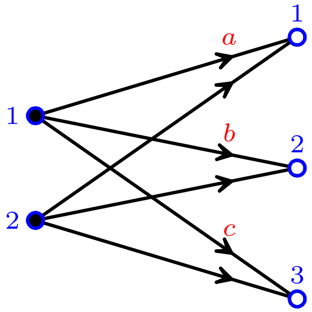
The *König digraph* of the transpose matrix

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

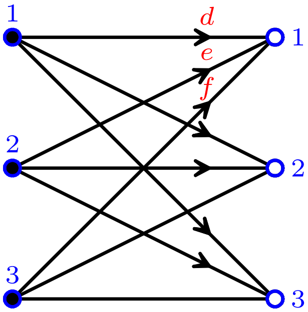
$G(A^T)$



Example 2.2.3 (pp. 37-38)



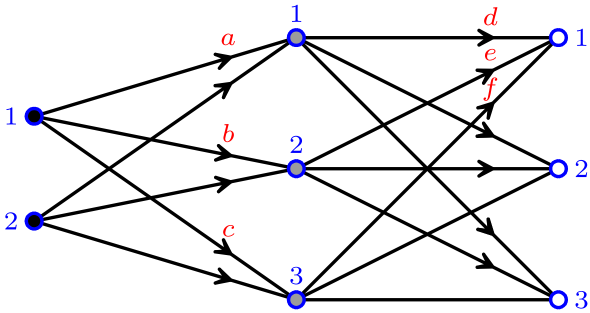
G_1



G_2

$$\begin{bmatrix} a & b & c \\ \square & \square & \square \end{bmatrix} \qquad \begin{bmatrix} d & \square & \square \\ e & \square & \square \\ f & \square & \square \end{bmatrix}$$

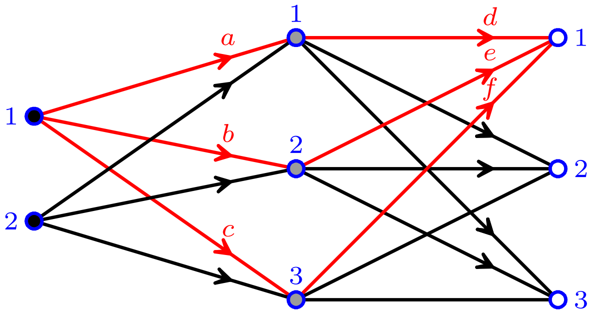
Composition of digraphs



$$G_1 * G_2$$

$$\begin{bmatrix} a & b & c \\ \square & \square & \square \end{bmatrix} \cdot \begin{bmatrix} d & \square & \square \\ e & \square & \square \\ f & \square & \square \end{bmatrix}$$

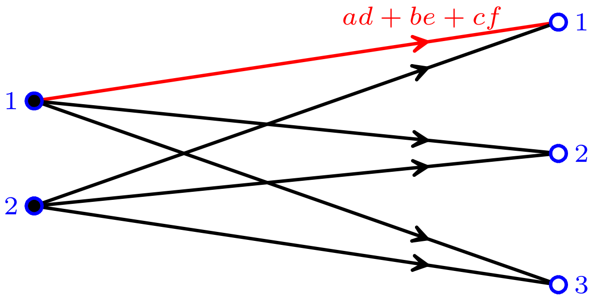
Product of matrices



$$G_1 * G_2$$

$$\begin{bmatrix} ad+be+cf & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Product of matrices

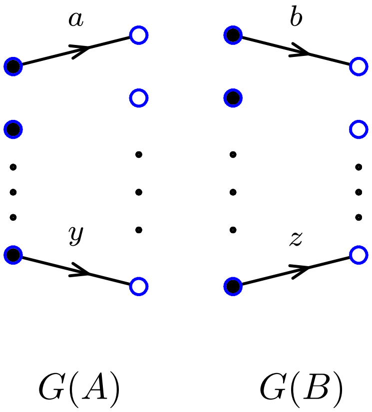


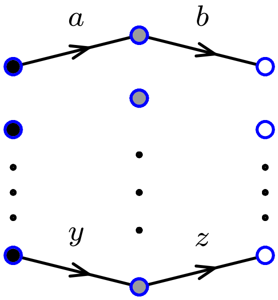
$$G_1 \cdot G_2$$

$$\begin{bmatrix} ad + be + cf & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

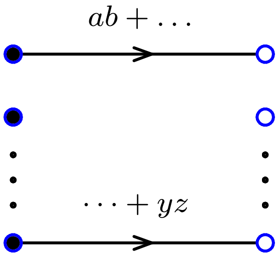
The anticommutativity property (p. 41)

$$(AB)^T = B^T A^T.$$

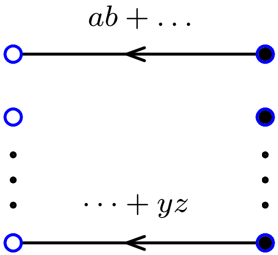

a
b
y
z
$G(A)$
$G(B)$



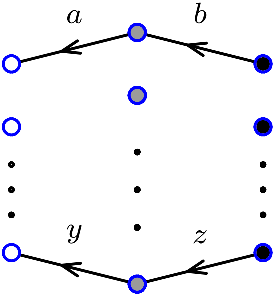
$$G(A) * G(B)$$



$G(AB)$

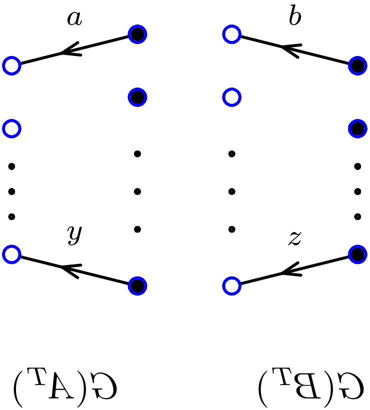


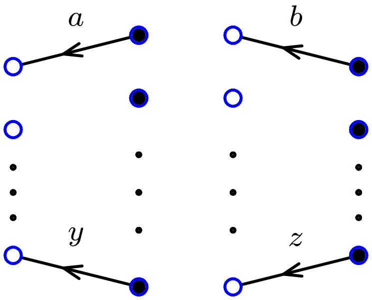
$$G\big((AB)^T\big)$$



$$G(B^T) * G(A^T)$$

This formula is written from right to left with an intention!


a
b
y
z
G(A^T)
G(B^T)



$G(A^T)$ $\qquad$ $G(B^T)$

$$G\big((AB)^T\big) = G(B^T \cdot A^T)$$

$$\Downarrow$$

$$(AB)^T = B^T A^T.$$

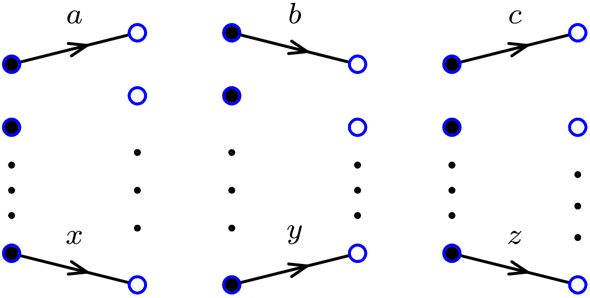
Using induction we get the more general product-transposition rule:

$$(A_1 A_2 \cdots A_k)^T = A_k^T \cdots A_2^T A_1^T.$$

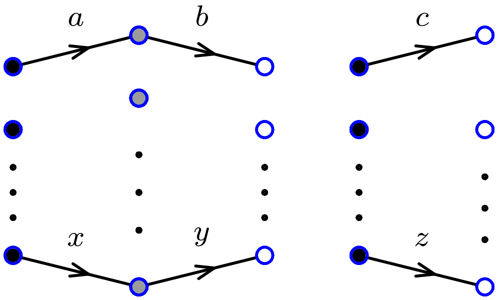
The associative law for matrix multiplication

Theorem 2.2.5 (p. 39)

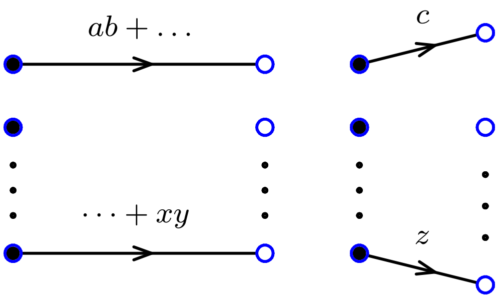
$$(AB)C = A(BC).$$



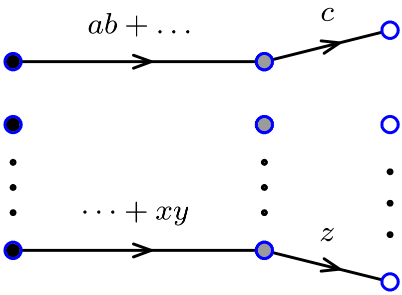
$G(A)$ $G(B)$ $G(C)$



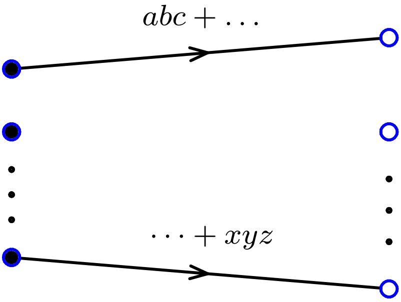
$G(A) * G(B)$ $\qquad$ $G(C)$



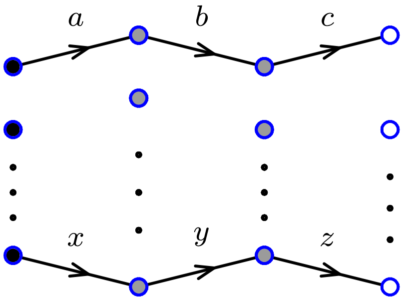
$G(AB)$ $G(C)$



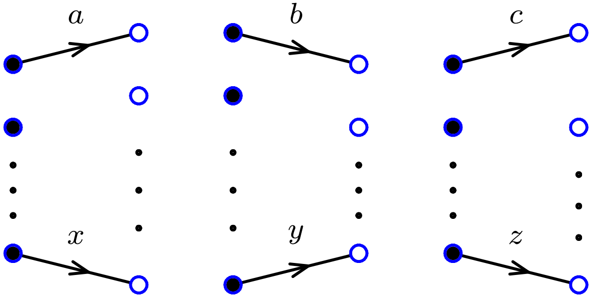
$$G(AB) * G(C)$$



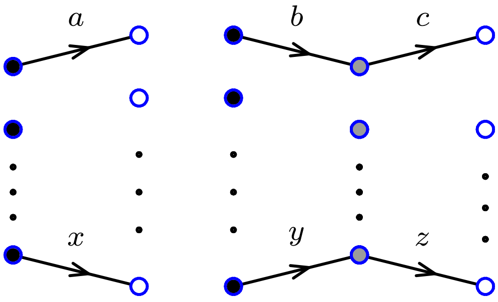
$$G\big((AB)C\big)$$



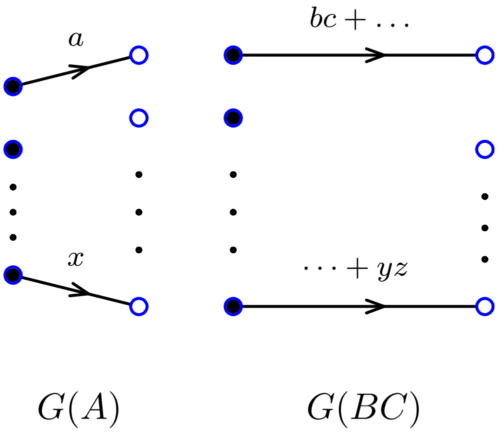
$$G(A) * G(B) * G(C)$$

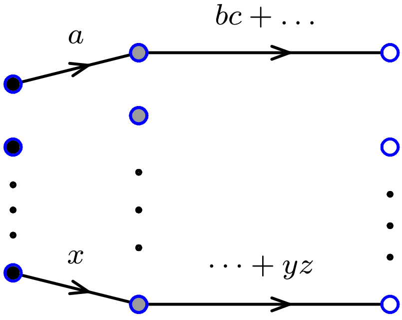


$G(A)$ $G(B)$ $G(C)$

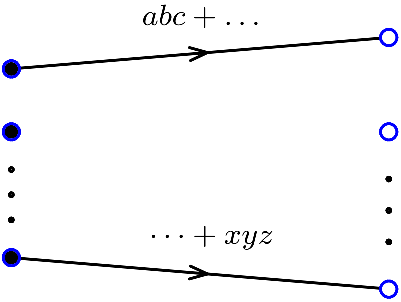


$G(A)$ $\qquad$ $G(B) * G(C)$

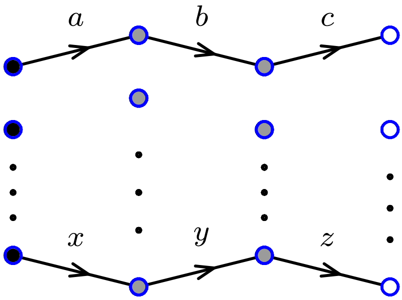

a
x
$bc + \dots$
$\dots + yz$
$G(A)$
$G(BC)$



$$G(A) * G(BC)$$



$$G\big(A(BC)\big)$$



$$G(A) * G(B) * G(C)$$

$$G\big((AB)C\big) = G\big(A(BC)\big)$$

$$\Downarrow$$

$$(AB)C = A(BC)$$

and both $(AB)C$ and $A(BC)$ can be obtained by inspecting paths of length 3 in

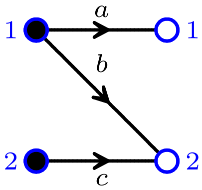
$$G(A) * G(B) * G(C).$$

Example 3.1.3 (p. 52)

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

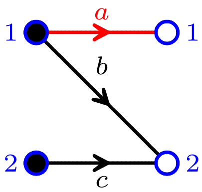
Find A^k.

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



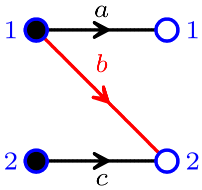
$G(A)$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



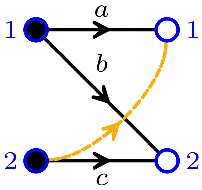
$G(A)$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



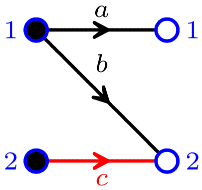
$G(A)$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



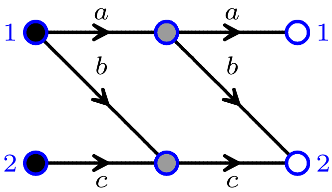
$G(A)$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$



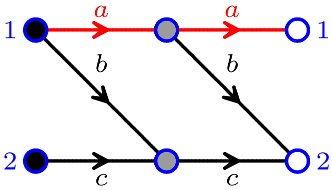
$G(A)$

$$A^2 = \begin{bmatrix} a^2 & ab + bc \\ 0 & c^2 \end{bmatrix}$$



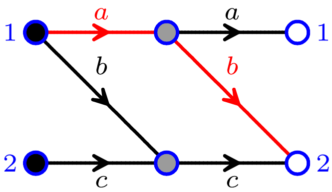
$$G(A) * G(A)$$

$$A^2 = \begin{bmatrix} a^2 & ab+bc \\ 0 & c^2 \end{bmatrix}$$



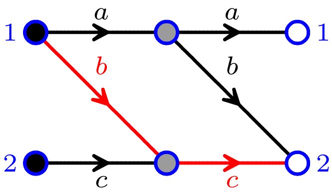
$$G(A) * G(A)$$

$$A^2 = \begin{bmatrix} a^2 & ab + bc \\ 0 & c^2 \end{bmatrix}$$



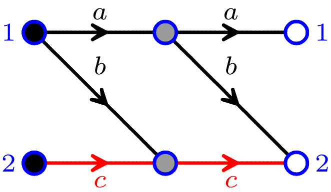
$$G(A) * G(A)$$

$$A^2 = \begin{bmatrix} a^2 & ab + bc \\ 0 & c^2 \end{bmatrix}$$



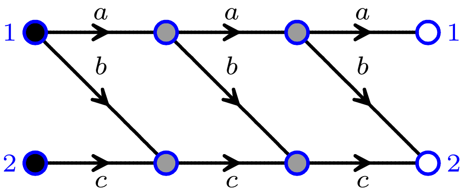
$$G(A) * G(A)$$

$$A^2 = \begin{bmatrix} a^2 & ab+bc \\ 0 & c^2 \end{bmatrix}$$



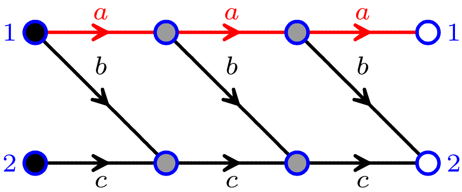
$$G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



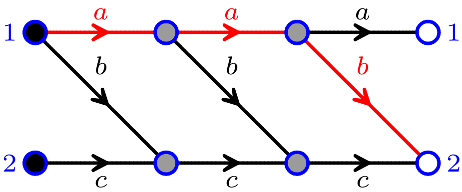
$$G(A) * G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



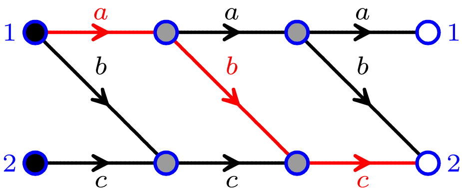
$$G(A) * G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



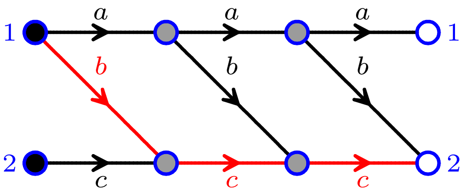
$$G(A) * G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



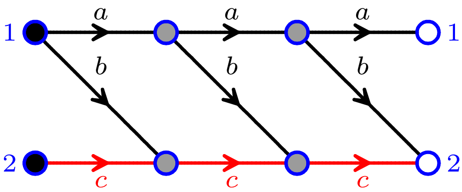
$$G(A) * G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



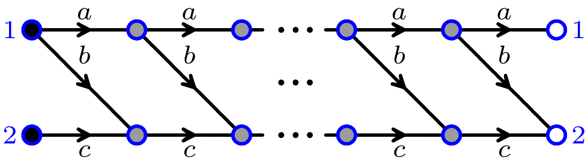
$$G(A) * G(A) * G(A)$$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$



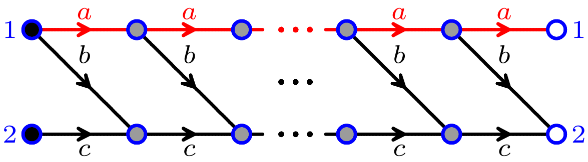
$$G(A) * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



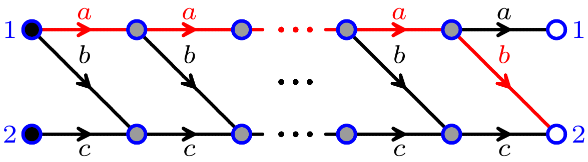
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



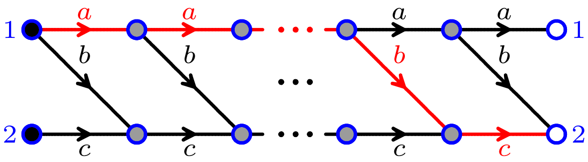
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



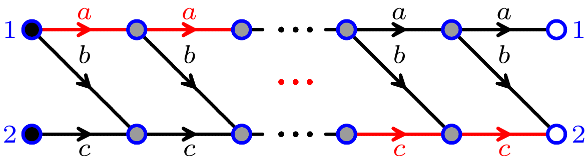
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



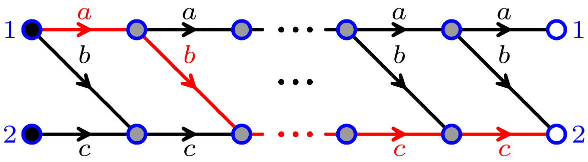
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



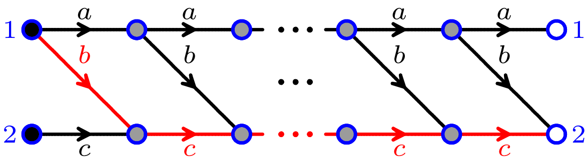
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



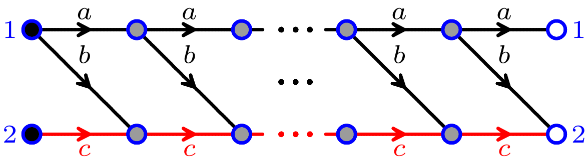
$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



$$G(A) * G(A) * \cdots * G(A) * G(A)$$

$$A^k = \begin{bmatrix} a^k & a^{k-1}b + a^{k-2}bc + \cdots + abc^{k-2} + bc^{k-1} \\ 0 & c^k \end{bmatrix}$$



$$G(A) * G(A) * \cdots * G(A) * G(A)$$