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OPEN LOCATION-DOMINATION NUMBER OF GENERALIZED PETERSEN GRAPHS

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Abstract: *In this paper one problem of domination for generalized Petersen graphs is considered. The exact value of Open location-domination number of generalized Petersen graphs is given*.

Keywords: *Generalized Petersen graphs, locating dominating set, open neighborhood locating dominating set, open location-domination number.*

1. INTRODUCTION

Let $G=(V,E)$ be an arbitrary graph and for any v V let us denote $N(v)$ and $N[v]$ open and closed neighborhoods of v. The open locating dominating set S of graph $G=(V,E)$ is set of vertices that dominates G and for any x,y V holds $N(x)\cap S\neq N(y)\cap S$. Set S will be denoted *OLD-set* of G. The cardinality of minimal such set S will be denoted as $\square_{old}(G)$.

The motivation for introduction of *OLD-set* and similar sets arose from security and protecting concerns. Different type of networks facilieties, or computer networks or network of routers could be theoretically represented by graphs. Aim is to define and determine the locations in such networks in order to identify and locate any „intruder“ or fault in some location in the network. Consider that in any location of the network, which means in any vertex of the corresponding graph there is some detecting device which can detect intruder in this and in all neighboring locations.

The locating dominating set is a set L V, where a detction device in location x L can determine if intruder is in the location or in $N(x)$, but could not determine in which element of $N(x)$. It follows, as introduced in (Slater 1983, 1987, 1988), L V is *locating dominating set* of G if L dominates G (i.e. ${}_{x\,L}N(x)=V$) and for any x,y V L holds $N(x)\cap S\neq N(y)\cap S$.

If a detection device can determine whether there is an intruder in the closed neighorhood of $N[x]$, but could not locate in which location, then we are interested in the *identifying code*. As introducesd in (Karpovsky *et al.* 1998), identifying code I is a vertex subset of V which dominates G, and for any x,y V holds $N(x)\cap S\neq N(y)\cap S$.

Finally, if a detection device can detetct an intruder in $N(x)$ without ability to detect it in x we are considering *open neighborhood dominating set*, as defined above. The problem of *OLD* sets was independently introduced by (Honkala *et al.* 2002) on k -cubes Q_k and generally on graps in (Seo and Slater 2010, 2011).

In (Lobstein) is presented a bibliography, currently with more then 350 entries, for work on distinguishing sets.

If two wertices x,y $V(G)$ such that $N(x)=N(y)$ exist, it follows that $N(x)\cap S=N(y)\cap S$ for any S V and G could not have an *OLD* set. This is proposed in

Proosition 1.1. (Seo and Slater 2010) A graph G has an *OLD* set if and only if G has no isolated vertex and $N(x)\neq N(y)$ for all pairs x,y of distinct vertices.

For a tree there is more detailed characterization presented in the following proposition.

Proosition 1.2. (Chellali *et al.* 2014, Seo and Slater 2011) For a tree T of order $n\geq 3$, T has an *OLD* set if and only if T does not contain a strong support vertex, where strong support vertex is a vertex which has two vertices of degree 1 as neighbors.

Some other connection between values $\gamma_{old}(G)$ and oreder of G are given in (Chellali *et al.* 2014).

Proposition 1.3. Assume $k \geq 2$, and suppose $k+1 \leq n \leq 2^k - 1$, then there exists a connected graph G of order n with $\gamma_{old}(G) = k$.

In the special case where graph G is a tree there are following results.

Theorem 1.4. (Seo and Slater 2011) If tree T of order $n \geq 5$ has an *OLD* set, then $n/2 + 1 \leq \gamma_{old}(T) \leq n-1$.

Theorem 1.5.(Seo and Slater 2013) For $n \geq 5$ and $n/2 + 1 \leq j \leq n-1$ there is a tree $T_{n;j}$ of order n with $\gamma_{old}(T_{n;j}) = j$.

Naturally, finding $OLD(G)$ is hard, and corresponding optimization problem is NP-hard which was proved in (Seo and Slater 2010).

In the paper (Chellali *et al.* 2014), authors characterize graph G of order n with $OLD(G) = 2, 3,$ or n and graph with minimum degree $\delta(G) \geq 2$ that are C_4 -free with $\gamma_{old}(G) = n-1$.

In the case of finite graphs G, there are some theoretcal results concerning bounds for values of $\gamma_{old}(G)$ in some cases.

Theorem 1.6. (Chellali *et al.* 2014) Let G be a connected graph with minimum degree $\delta(G) \geq 3$, and C_4 -free. Then $\gamma_{old}(G) \leq n - \rho(G)$, where $\rho(G)$ is the maximum number of vertices which are pairwise at distance at least 3.

Theorem 1.7. (Chellali *et al.* 2014, Seo and Slater 2010) For a graph G of order n and maximum degree $\Delta(G)$, if G has an *OLD* set, then $\gamma_{old}(G) \geq \frac{2n}{\Delta}$.

Theorem 1.8. (Henning and Yeo 2014) If G is a cubic graph of order n, then $\gamma_{old}(G) \leq \frac{3n}{4}$.

Theorem 1.9. (Seo and Slater 2010) If G is a regular graph of degree r, then $\gamma_{old}(G) \geq \frac{2 \cdot |V(G)|}{r+1}$.

This paper considers the strong metric dimension of a special class of graphs, so called generalized Petersen graphs.

2. Generalized Petersen graph $GP(n;m)$

Generalized Petersen graphs were first introduced by Coxester in (Coxester 1950) and were named by Watkins in (Watkins 1969). It is the smallest network in terms of node degree, diameter and network size. Due to its unique and optimal properties, several network topologies based on generalized Petersen graph have been proposed and investigated in the literature (Ohring and Das 1996). Various properties of $GP(n;m)$ have been recently theoretically investigated in the following areas: strong metric dimension (Kratica *et al.* 2017), component connectivity (Ferrero and Hanusch 2014), decycling number (Gao *et al.* 2015), and other. Various problems in networks can be studied by graphs theoretical methods. Dominations have become one of the major areas in Graph Theory after more than 30 years' development (Huang, J., Xu, J.M. 2007).

Definition 2.1. For $1 \leq m \leq n-1$, the *generalized Petersen graph* $GP(n;m)$ is a graph on $2n (n \geq 3)$ vertices with $V = \{\{u_i; v_i\} | 0 \leq i \leq n-1\}$ and the edge set $E = \{\{u_i, u_{i+1}\}, \{u_i, v_i\}, \{v_i, v_{i+m}\} | 0 \leq i \leq n-1\}$, where vertex indices are taken modulo n.

It is a 3-regular graph and contains $2n$ vetices and $3n$ edges.

A standard visualization of a generalized Petersen graph consisits of two types of vertices namely u_i in the outer rim and v_i in the inner rim. There are three types of edges consisiting of the outer rim edges u_i, u_{i+1}, the inner rim edges v_i, v_{i+m} and the „spokes“ u_i, v_i which form a 1-factor between the inner rim and the outer rim, see the Figure 1. The outer rim is always a cycle while the inner rim may consist of several isomorphic cycles. A generalized Petersen graph $GP(n;m)$ is given by two parameters n and m, where n is the number of vertices in each rim and m is the „span" of the inner rim.

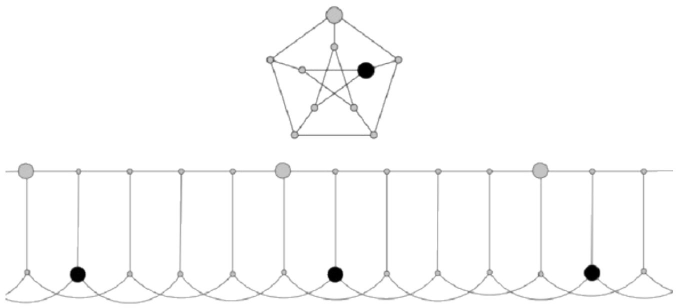

Figure 1. The Petersen graph $GP(5,2)$

3. Our result

In the next Theorem we have proved that the minimum cardinality of the OLD -set of generalized Petersen graph $GP(n;k)$ is equal to the parameter n.

Theorem 3.1. $\gamma_{old}(GP(n;k))=n$.

Proof: *Step 1*: $\gamma_{old}(GP(n;k))\leq n$

Let $S=\{v_i|i=0,\ldots,n-1\}$. Since $N(u_i)=\{u_{i-1},u_{i+1},v_i\}$ and $N(v_i)=\{u_i,v_{i-k},v_{i+k}\}$ then $S\cap N(u_i)=\{v_i\}$ and $S\cap N(v_i)=\{v_{i-k},v_{i+k}\}$, for each $i=0,\ldots,n-1$. It is easy to see that:

- For each w $V, S\cap N(w)\neq$;
- Since $n>2k$ then for each w_1,w_2 $V, S\cap N(w_1)\neq S\cap N(w_2)$.

Therefore, S is a open lpcating-dominating set of $GP(n;k)$, so consequently $\gamma_{old}(GP(n;k))\leq n$.

Step 2: $\gamma_{old}(GP(n;k))\geq n$

It is easy to see that $GP(n;k)$ is a regular graph of degree 3, with 2 n vertices. Then, by Theorem 1.9 it holds $\gamma_{old}(GP(n;k))\geq \dfrac{2\cdot 2\cdot n}{1+3}=n$.

This proves that $\gamma_{old}(GP(n;k))=n$ holds for the generalized Petersen graph $GP(n;k)$.

5. CONCLUSION

In this paper we considered one of the problems of domination for generalized Petersen graphs. The exact value of the Open location-domination number of generalized Petersen graphs is obtained.

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