XLV Symposium on Operational Research
SYM-OP-IS 2018
Zlatibor, 16-19 Septembar 2018

ASYMMETRIC CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOW

VLADIMIR ILIN[1], LUKA MATIJEVIĆ[2], TATJANA DAVIDOVIĆ[2], PANOS M. PARDALOS[3]

[1]Faculty of Technical Sciences, University of Novi Sad, v.ilin@uns.ac.rs
[2] Mathematical Institute of the Serbian Academy of Science and Arts, Belgrade, {luka,tanjad}@mi.sanu.ac.rs
[3]Faculty of Engineering, University of Florida, pardalos@ufl.edu

Abstract: *We consider an asymmetric vehicle routing problem that arises when delivering on-line ordered perishable goods to the multiple customers. The main goal is to minimize the total distance traveled while visiting all customers with a limited number of homogeneous vehicles within the pre-specified time window. Problem is formulated as mixed-integer linear program (MILP) with an assumption that the capacity of vehicles as well as the specified time window allow to serve all customers. The developed MILP is used within CPLEX commercial solver to examine its limits with respect to the size of instances that can be solved to optimality.*

Keywords: *Combinatorial optimization, Routing of homogeneous vehicles, Single depot, Minimization of total distance, Mixed-Integer linear program*

1. INTRODUCTION

Vehicle Routing Problem (VRP) is an important combinatorial optimization problem that arises in many everyday situations related to the transportation of goods [7,19,20]. It was proposed in [4] as an abstraction of vehicle scheduling problem. Since then formulation of VRP evolved in many directions based on imposed constraints and different objective functions. In recent review paper [2] 277 articles dealing with various VRPs have been analyzed and their classification with respect to different problem characteristics has been proposed. In this paper we consider three types of constraints added to the classical VRP: asymmetry, capacity and time window constraints.

Asymmetric capacitated VRPs are well-known in the literature and are explained in detail in several books [7,20]. The books show the mathematical formulations, relaxations and recent exact methods for various VRPs, including asymmetric capacitated VRP. VRP with time window was proposed by Russell [16] who presented an effective heuristic for the *M*-tour traveling salesman problem. Generally, there are two types of time windows that are extensively studied in the literature [13]: hard time windows and soft time windows. Hard time windows imply that time of arrival to the customer's address must belong to the given time window, early and late arrivals are not allowed. Soft time window implies that violation of the time window constraint, with certain penalties, is tolerable.

Asymmetric capacitated VRPs with time windows can be tackled with exact, heuristic, metaheuristics or hybrid methods. Baldacci et al. [1] provided a review of the recent developments on exact algorithms for solving the vehicle routing problems under capacity and time window constraints. A review of the recent heuristics and metaheuristics for various VRPs can be found in [6,10,11]. In addition, a recent article [17] proposed a hybrid algorithm for a class of VRPs, including asymmetric capacitated VRP. Moreover, different hybrid algorithms have been proposed in several papers and implemented for solving VRPs with time windows [14,22].

Most of the previously mentioned papers, as well as some others [5,12,21], have the objective to minimize total distances traveled. Decrease in total kilometers directly impacts money savings for the servicing company and indirectly impacts minimization of CO_2 emissions for the community. In order to describe environmental impacts more accurately, some of the researchers propose that the objective function should contain some additional components, such as total load transported [8,9]. There is also a recent paper [15] that explains how minimization of total distances traveled by vehicles, as well as minimization of total load transported, impact the decrease in environmental pollution. According to [18] significant savings in fuel consumption can be achieved by delivering heavy items before light ones in a tour. In this paper we adapted Mixed-Integer Linear Program (MILP) formulation from [15] to the considered asymmetric capacitated vehicle routing problem with time window. However, we used different objective function and we applied different solution approach.

The problem addressed in this paper is how to organize delivery of on-line ordered perishable goods to multiple customers scattered within the city and on the periphery of the city from one distribution center with homogenous fleet of vehicles. The problem can be labeled as a generalization of the well-known VRP. The main assumptions are that it is static and deterministic basic version of the problem in which the demands are known in advance and cannot be split, the vehicles are identical and are located at the single depot. We consider real-life problem that imposes three important constraints during transportation of goods: capacity of vehicles, one-way or temporarily closed directions within the city and predetermined time window for delivery. The first constraint introduces the capacity restrictions, i.e., the vehicles cannot carry more goods then it is defined by their capacity (prefix C to VRP). The second constraint introduces asymmetric weight (cost, distance) matrix (prefix A to CVRP). The third constraint specifies hard time window for vehicles to deliver the goods (suffix TW to ACVRP). Therefore, with previously defined constraints we extend the classical VRP problem to Asymmetric Capacitated VRP with Time Window (ACVRPTW) problem.

The main contribution of this paper is the development of mathematical formulation for the considered ACVRPTW in the form of MILP. This model is used within CPLEX commercial solver to evaluate the limits of the available hardware and software resources. The experimental evaluation is conducted on two real-life examples (one of small-size and one of larger dimension). It shows that the considered ACVRPTW is very complex optimization problem, as the larger example remained unsolved to optimality due to the memory limits of the available hardware.

This paper is organized as follows. After the introduction and the review of relevant literature presented in Section 1, the considered vehicle routing problem is described together with the Mixed-Integer Linear Programming formulation in Section 2. Section 3 contains the experimental evaluation performed on two case study instances using CPLEX commercial solver. Concluding remarks and directions for future work are given in the last section.

2. PROBLEM FORMULATION

The considered asymmetric vehicle routing problem (ACVRPTW) is defined on a complete direct graph $G(N, A)$, where $N=\{0,1,2,\ldots,n,n+1\}$ is the set of vertices (locations, customers) and $A=\{(i,j) \mid i, j \in N, i \neq j\}$ denotes the set of arcs with weights $d_{i,j}$ representing distances between customers i and j. Vertices 0 and n+1 refer to the single location (depot), the origin and destination of each route. This notation is adopted due to the simplicity of formulation.

Each vehicle starts from location 0 (depot), serves a determined subset of customers and then it finishes at location n+1 (depot). Each vehicle can perform maximum one tour, while each customer should be served exactly once by only one vehicle. The demands of customers are denoted with c_i, $i=1,2,\ldots,n$ and they represent the weight of goods to be delivered to each particular customer. The set V of v_{max} homogeneous vehicles (with same capacity Q and same speed s) is provided to serve all the customers' demands. Based on the vehicle speed, the travel time ($Tt_{i,j}$) between customers i and j can be calculated. The time to serve a customer j (to unload goods, deliver them to a customer and collect money) is considered as parameter St_j whose value is given in advance. In order to make the distribution of customers among vehicles more even, the parameter *Climit* (representing the maximum number of customers per vehicle) is introduced. Finally, the distribution of goods must be completed during the working hours, and therefore, *Tlimit* denotes the maximum allowed time for servicing customers (the pre-specified time window).

An example of the considered problem related to the delivery service in the city of Novi Sad is presented in Fig. 1.

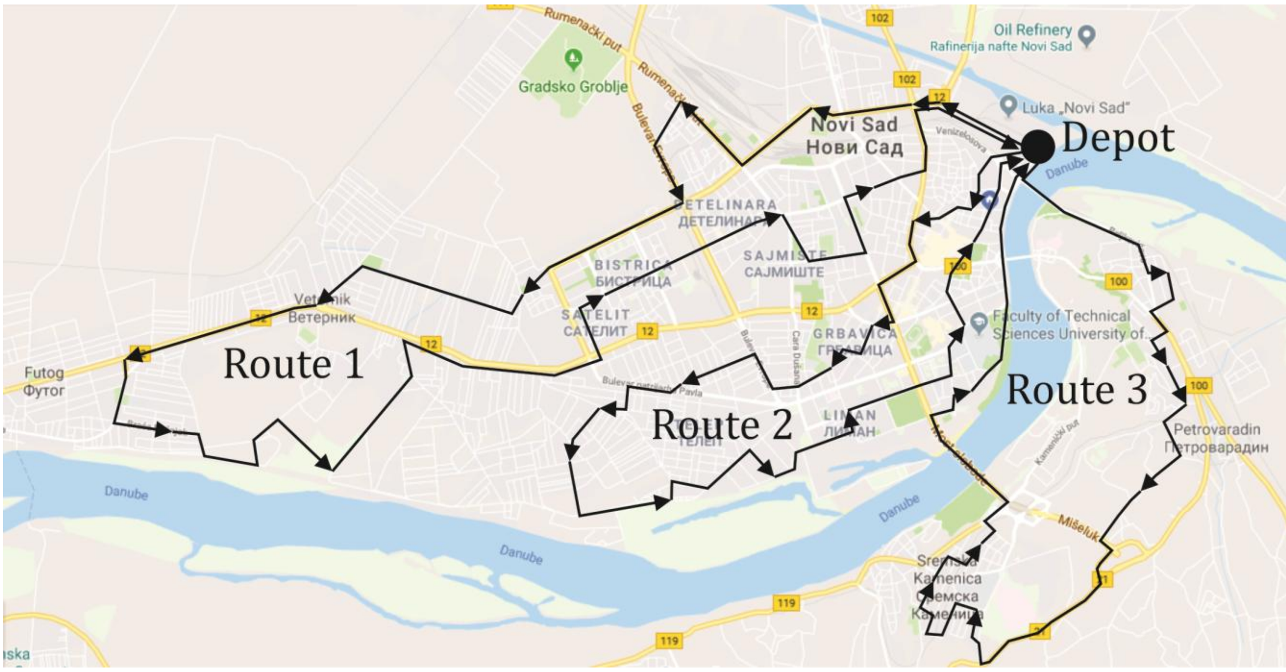

Figure 1: An example of the considered ACVRPTW

In order to provide the mathematical programming formulation of the considered problem, we need two sets of decision variables, binary variables x and real valued variables q defined as:

$$x_{i,j,v} = \begin{cases} 1, & \text{if the customer } j \text{ is visited immediately after } i \text{ by vehicle } k, \\ 0, & \text{otherwise;} \end{cases}$$

$q_{i,j,v}$ = load (quantity of goods)of vehicle v travelling from customer i directly to customer j.

The MILP formulation of our ACVRPTW can be described as follows:

$$\min \sum_{i=0}^{n} \sum_{j=1}^{n+1} \sum_{v=1}^{v_{max}} d_{i,j} x_{i,j,v} \tag{1}$$

s.t.

$$\sum_{i=0}^{n} \sum_{v=1}^{v_{max}} x_{i,j,v} = 1, \qquad 1 \le j \le n,\ \ i \ne j,\ \ 1 \le v \le v_{max}, \tag{2}$$

$$\sum_{i=0}^{n} x_{i,j,v} = \sum_{i=1}^{n+1} x_{j,i,v}\ , \qquad 1 \le j \le n,\ \ i \ne j,\ \ 1 \le v \le v_{max}, \tag{3}$$

$$\sum_{j=1}^{n+1} x_{0,j,v} \le 1, \qquad 1 \le v \le v_{max}, \tag{4}$$

$$\sum_{i=0}^{n} q_{i,j,v} - \sum_{i=1}^{n+1} q_{i,j,v} = c_j \sum_{i=0}^{n} x_{j,i,v}\ , \qquad 1 \le j \le n,\ \ i \ne j,\ \ 1 \le v \le v_{max}, \tag{5}$$

$$\sum_{i=0}^{n} q_{i,j,v} - \sum_{h=1}^{n+1} c_h x_{i,j,v} \geq c_j \sum_{i=0}^{n} x_{j,i,v} , \quad 1 \leq j \leq n, \; i \neq j, \; h \neq j, \; 1 \leq v \leq v_{max}, \tag{6}$$

$$q_{i,j,v} \leq (Q - c_i) x_{i,j,v} , \quad 0 \leq i \leq n, \quad 1 \leq j \leq n, \quad 1 \leq v \leq v_{max}, \tag{7}$$

$$q_{i,n+1,v} = 0, \quad 1 \leq i \leq n, \quad 1 \leq v \leq v_{max}, \tag{8}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{n+1} x_{i,j,v} \leq Climit, \quad 1 \leq v \leq v_{max}, \tag{9}$$

$$\sum_{i=0}^{n} \sum_{j=1}^{n+1} x_{i,j,v} (Tt_{i,j} + St_j) \leq Tlimit, \quad 1 \leq v \leq v_{max}, \tag{10}$$

$$x_{i,j,v} \in \{0,1\}, \quad 0 \leq i \leq n, \quad 1 \leq j \leq n+1, \quad 1 \leq v \leq v_{max}, \tag{11}$$

$$q_{i,j,v} \geq 0, \quad 0 \leq i \leq n, \quad 1 \leq j \leq n+1, \quad 1 \leq v \leq v_{max}, \tag{12}$$

The objective function (1) to be minimized represents the total distance traveled by all vehicles. Constraints (2) ensure that each customer is served exactly ones. The tours of each vehicle are defined by constraints (3) with the condition that each vehicle can make at most one tour (constraints (4)). Subtour elimination constraints are based on controlling the current load of vehicles [3]. Namely, when leaving the location of customer j, the load of a vehicle must decrease exactly for the demand of that customer (constraints (5)). In addition, when arriving to location of customer j the vehicle must contain enough goods to satisfy the demand of that customer (constraints (6)). Constraints (7) are related to the capacity constraints while constraints (7) impose that all customers must be served, i.e., each vehicle must be empty upon returning to the depot. Uniform distribution of customers among vehicles is ensured by constraints (9), while constraints (10) control the time limit for serving customers. Finally, constraints (11) and (12) control the nature of the decision variables. As can be seen from the proposed model, mathematical formulation of the considered ACVRPTW contains $n^2 v_{max}$ binary variables, $n^2 v_{max}$ real valued variables and $n-1+(n^2+4n)v_{max}$ constraints.

3. EXPERIMENTAL EVALUATION

We consider the following case study to evaluate the limits of the proposed model and the available hardware and software resources. Distribution company in the city of Novi Sad has a fleet of three vehicles and delivers goods to customers situated in the city and on the periphery of the city. Each customer has the opportunity to order different goods (including perishable goods) during one day and to expect delivery of ordered goods the next day.

Each vehicle in the fleet has a capacity of 3,5 tones and that is more than enough to deliver all ordered goods. Therefore, the main assumption is that all customers' demands can be served. There are two shifts during a day for delivering goods to customers. The first shift starts at 10 am and finishes at 3 pm. The second shift starts at 4 pm and finishes at 8 pm. The orders to be delivered during the first shifts are served by three vehicles, while for the second shift only two vehicles are used. The maximum number of customers in first shift is 55, while in the second shift 45 customers can be served per day.

The experiments are performed on Intel Core i7-2600 processor on 3.40GHz with 12GB RAM memory under Linux operating system. CPLEX 12.6.2 commercial solver is used as an exact solution method. The obtained solutions are presented in Table 1.

Table 1: CPLEX results for the two tested examples

Example	Obj. value(% gap)	CPU time [s]
EX1 (first shift)	60.99(18.77)	129600.00
EX2 (second shift)	50.69(00.00)	1102.19

For the larger example (first shift), CPLEX was not able to provide the optimal solution due to the memory limit. Therefore, in the second column of Table 1 we present the upper bound obtained after 36 hours of execution and the corresponding gap in the parentheses. The smaller example (second shift) is solved to optimality for little more than 18 minutes. The main conclusion from these results is that real-life examples are too large to be solved optimally by the currently available hardware and software resources. It implies that the application of alternative (heuristic, metaheuristic, approximation) methods is necessary and indicates directions for future research related to the considered ACVRPTW.

4. CONCLUSION

We considered the asymmetric capacitated vehicle routing problem with pre-specified time interval for serving the customers. This variant of the vehicle routing problem arises from the real-life application of delivering on-line ordered perishable goods to multiple customers with homogenous fleet of vehicles. The mathematical formulation of the considered problem is proposed in this paper in the form of Mixed-Integer Linear Program. The proposed formulation is used within the CPLEX commercial solver to examine the limits of the available hardware and software resources. Experiments performed on two real-life case study examples shows that the instance with 55 customers to be served by 3 vehicles already exceeds the memory limits of the available computing platform. Therefore, the obvious direction of future research is the application of metaheuristic methods. In addition, one could consider more general cases involving dynamic and/or stochastic variants of the problem, i.e., the cases when some of the parameters are not given in advance or some of the customers are not at home to collect the ordered goods.

Acknowledgement

This research was partially supported by Serbian Ministry of Education, Science and Technological Development under the grants nos. OI-174033 and III-044006.

REFERENCES

[1] Baldacci, R., Mingozzi, A., Roberti, R. (2012). Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints. European Journal of Operational Research, 218(1), 1-6.

[2] Braekers, K., Ramaekers, K., Van Nieuwenhuyse, I. (2016). The vehicle routing problem: State of the art classification and review. Computers & Industrial Engineering, 99, 300-313.

[3] Cinar, D., Gakis, K., Pardalos, P.M. (2016). A 2-phase constructive algorithm for cumulative vehicle routing problems with limited duration. Expert Systems with applications, 56, 48-58.

[4] Dantzig, G.B., Ramser, J.H. (1959). The truck dispatching problem. Management science, 6(1), 80-91.

[5] Du, T.C., Li, E.Y., Chou, D. (2005). Dynamic vehicle routing for online B2C delivery. Omega, 33(1), 33-45.

[6] Gendreau, M., Potvin, J.Y., Bräumlaysy, O., Hasle, G., Løkketangen, A. (2008). Metaheuristics for the vehicle routing problem and its extensions: A categorized bibliography. In Golden, B.L., Raghavan, S., Wasil, E.A. (Eds.), The vehicle routing problem: latest advances and new challenges, vol. 43 (143-169). Springer.

[7] Golden, B.L., Raghavan, S., Wasil, E.A. (Eds.). (2008). The vehicle routing problem: latest advances and new challenges, vol. 43, Springer Science & Business Media.

[8] Kara, I., Kara, B.Y., Yetis, M.K. (2007). Energy minimizing vehicle routing problem. Proceedings of the International Conference on Combinatorial Optimization and Applications (COCOA 2007), Xi'an, China, 62-71.

[9] Kopfer, H.W., Kopfer, H. (2013). Emissions minimization vehicle routing problem in dependence of different vehicle classes. Proceedings of the Third International Conference on Dynamics in logistics, (LDIC 2012), Bremen, Germany, 49-58.

[10] Labadie, N., Prins, C., Prodhon, C. (2016). Metaheuristics for Vehicle Routing Problems, Metaheuristics Sets, vol. 3. ISTE Ltd. and Willey & Sons Inc.

[11] Laporte, G., Ropke, S., Vidal, T. (2014). Heuristics for the vehicle routing problem. In Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications, (2nd ed.). 87-116. MOS-SIAM Series on Optimization, SIAM.

[12] Leggieri, V., Haouari, M. (2018). A matheuristic for the asymmetric capacitated vehicle routing problem. Discrete Applied Mathematics, 234, 139-150.

[13] Lin, C., Choy, K.L., Ho, G.T.S., Chung, S.H., Lam, H.Y. (2014) Survey of green vehicle routing problem: past and future trends. Expert Systems with Applications, 41(4), 1118-1138.

[14] Luo, J., Li, X., Chen, M.R., Liu, H. (2015). A novel hybrid shuffled frog leaping algorithm for vehicle routing problem with time windows. Information Sciences, 316, 266-292.

[15] Qiu, Y., Qiao, J., Pardalos, P.M. (2017). A branch-and-price algorithm for production routing problems with carbon cap-and-trade. Omega, 68, 49-61.

[16] Russell, R.A. (1977) An effective heuristic for the *M*-tour traveling salesman problem with some side conditions. Operations Research, 25(3), 517-524.

[17] Subramanian, A., Uchoa, E., Ochi, L.S. (2013). A hybrid algorithm for a class of vehicle routing problems. Computers & Operations Research, 40(10), 2519-2531.

[18] Suzuki, Y. (2011). A new truck-routing approach for reducing fuel consumption and pollutants emission. Transportation Research Part D: Transport and Environment, 16(1), 73-77.

[19] Toth, P., Vigo, D. (Eds.). (2002). The vehicle routing problem. SIAM.

[20] Toth, P., Vigo, D. (Eds.). (2014). Vehicle Routing: Problems, Methods, and Applications. (2nd ed.). MOS-SIAM Series on Optimization, SIAM.

[21] Wang, X., Ruan, J., Shi, Y. (2012). A recovery model for combinational disruptions in logistics delivery: Considering the real-world participators. International Journal of Production Economics, 140(1), 508-520.

[22] Zhang, D., Cai, S., Ye, F., Si, Y.W., Nguyen, T.T. (2017). A hybrid algorithm for a vehicle routing problem with realistic constraints. Information Sciences, 394, 167-182.